Rivington's Mathematical Series

A TREATISE ON ARITHMETIC

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ON

ARITHMETIC

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of the book is intended to supply gradual tests of the student's progress.

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PREFACE

TO THE SECOND EDITION.

In the revision of this work I have been assisted by the advice of many eminent Teachers. The *Unitary Method* seems to be approved by all who have tested it by use with their pupils. I have added five hundred exercises to the papers at the end of the book, and I have inserted about the same number in various parts of the work. I have given a different method of extracting the Cube Root of a number, and I have added a few remarks on Ratio and Proportion. In other respects the book remains much as it was in the first edition.

J. H. S.

CAMBRIDGE.

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PURE ARITHMETIC.

COMMERCIAL ARITHMETIC.

ARITHMETIC.

I. On the Method of Representing Numbers by Figures.

1. ARITHMETIC is the science which teaches the use of numbers.

2. The number *one*, or *unity*, is taken as the foundation of all numbers, and all other numbers are derived from it by the process of *addition*.

Thus:

Two is the number that results from adding *one* to *one*;
Three is the number that results from adding *one* to *two*;
Four is the number that results from adding *one* to *three*;
and so on.

3. By means of the symbols or figures

1 2 3 4 5 6 7 8 9,

called the NINE SIGNIFICANT DIGITS, together with the symbol or figure 0, called ZERO, we can represent numbers of any magnitude.

4. First, each of the significant digits, standing by itself, represents a number *greater by one* than the number represented by the digit that immediately precedes it in the list of digits.

Thus 7 represents a number greater by *one* than the number represented by 6.

5. The symbol $+$, read PLUS, is used to denote the operation of ADDITION.

The symbol $=$ stands for the words "is equal to," or "the result is."

Since

$2=1+1$, where unity is written *twice*,
$3=2+1=1+1+1$, where unity is written *three* times,
$4=3+1=1+1+1+1$, where unity is written *four* times,

and so on.

6. Numbers between nine and a hundred are represented by *two* figures, the one on the left-hand signifying how many *groups of ten units* are contained in the number represented, and the one on the right-hand signifying how many single units are contained in the number, in addition to the groups of ten units.

Thus, in the expression 69,

the figure 6 represents six groups of ten units,
the figure 9 represents nine single units.

These groups of ten units are for brevity called *Tens*, and the single units are for brevity called *Units*.

Numbers between ninety-nine and a thousand are represented by *three* figures.

In the expression 745,

the figure 7 represents seven groups of a hundred units,
the figure 4 represents four groups of ten units,
the figure 5 represents five single units.

In the expression 3475,

the figure 3 represents three groups of *a thousand* units.

In the expression 23475,

the figure 2 represents two groups of *ten thousand* units.

In the expression 123475,

the figure 1 represents one group of *a hundred thousand* units.

In the expression 9123475,

the figure 9 represents nine groups of *a million* units; and so on.

7. To put the matter briefly: when we express a number in figures, and tell off the figures *from right to left*,

the *first* figure represents a number of *units*,
the *second* figure represents a number of *tens*,
the *third* figure represents a number of *hundreds*,

the *fourth* figure represents a number of *thousands*,
the *fifth* figure represents a number of *tens of thousands*,
the *sixth* figure represents a number of *hundreds of thousands*,

the *seventh* figure represents a number of *millions*,
the *eighth* figure represents a number of *tens of millions*,
the *ninth* figure represents a number of *hundreds of millions*,

the *tenth* figure represents a number of *thousands of millions*,
the *eleventh* figure represents a number of *tens of thousands of millions*,
the *twelfth* figure represents a number of *hundreds of thousands of millions*,

the *thirteenth* figure represents a number of *billions*.

8. When the symbol o appears in an expression, it shows that the number, represented by the expression, contains no single units, tens, hundreds, &c., according as the o is placed in the first, second, third place, the order of place being reckoned from right to left.

Thus:

20 represents the number which contains two groups of ten units and no single units;
300 represents the number which contains three groups of a hundred units, and no group of ten, and no single units;
4007 represents the number which contains four groups of a thousand units, and no group of a hundred, and no group of ten, and seven single units.

NUMERATION.

9. *To write in words the meaning of a number, expressed in figures, is called* NUMERATION.

The remarks, which we have already made, ought to enable the learner to write in words all numbers expressed by ONE, TWO, or THREE figures.

Thus:

the number expressed by 8 is written EIGHT;
the number expressed by 27 is written TWENTY-SEVEN;
the number expressed by 304 is written THREE HUNDRED AND FOUR.

10. Next take the case of numbers expressed by FOUR, FIVE, or SIX figures, as 4237, 23509, 402675.

Draw a line, separating the *three figures on the right* of each expression from the rest of the expression, and over the figure or figures on the left of the line write the word *Thousand*, thus:

Thousand			Thousand			Thousand	
4	237	;	23	509	;	402	675.

Then the meaning of each expression can be written at once in words, thus:

Four thousand, two hundred and thirty-seven;
Twenty-three thousand, five hundred and nine;
Four hundred and two thousand, six hundred and seventy-five.

11. Next take the case of numbers expressed by SEVEN, EIGHT, or NINE figures, as, for instance, the number expressed by 347295328.

Draw a line, separating the three figures on the right from the rest of the expression, and a second line, marking off the next three figures. Over these write the word *Thousand*, and over the figures on the left of this second line write the word *Millions*, thus:

Millions	Thousand	
347	295	328.

Then we can write the meaning in words, thus:

Three hundred and forty-seven *millions*,
two hundred and ninety-five *thousand*,
three hundred and twenty-eight.

Again, to express in words 20040030, write it thus:

Millions	Thousand	
20	040	030

and the number expressed in words is—

Twenty *millions*,
forty *thousand*
and thirty.

12. If more than nine figures are in the given number, mark off the figures by *threes*, as before, and

over the *fourth* parcel write the word *thousand*,
over the *fifth* parcel write the word *billions.*

Thus, to express in words 24003269407032, proceed thus:

Billions	Thousand	Millions	Thousand	
24	003	269	407	032

and the number expressed in words is—

Twenty-four *billions*,
three thousand two hundred and sixty-nine *millions*,
four hundred and seven *thousand*
and thirty-two.

Note. 1 followed by *three* zeros, 1000, represents a thousand.
1 followed by *six* zeros, 1000000, represents a million.
1 followed by *twelve* zeros, 1000000000000, represents a billion.

Examples. (i)

Write in words the numbers expressed by the following figures:—

(1) 7, 13, 45, 59, 326, 4578.
(2) 90, 110, 207, 4300, 4036, 4306.
(3) 780, 609, 5360, 2020, 1101.
(4) 36497, 49532, 654321, 743269.
(5) 45000, 32600, 75230, 500000.
(6) 8572914, 3469218, 4629817.
(7) 9000000, 29000000, 715000000.
(8) 910307240, 307004205, 380503040.
(9) 243759268342, 307405006270.
(10) 41723568271943 5, 203056300072010.

NOTATION.

13. To represent by figures a number, expressed in words, is called NOTATION.

The method to be employed is this:

Prepare the divisions in which the figures representing *thousands*, *millions*, &c., are to be placed, thus:

Billions	Thousand	Millions	Thousand	

and place in each division, as well on the right and left of the outermost lines, the figures required.

Thus, to represent by figures forty-seven thousand, three hundred and nine, we proceed thus:

Thousand	
47	309

and the number expressed in figures is 47309.

Again, to represent by figures four thousand three hundred and two millions, eighteen thousand and fifty-three, we proceed thus:

Thousand	Millions	Thousand	
4	302	018	053

and the number expressed in figures is 4302018053.

Examples. (ii)

Express in figures the following numbers:—

(1) Nine; twelve; seventeen; nineteen; thirteen; sixteen; eleven.

(2) Twenty-three; twenty-seven; thirty-five; thirty-eight; forty-four; forty; twenty-six; thirty-four.

(3) Sixty-seven; seventy-five; sixty-two; eighty-three; seventy-four; ninety-two; sixty-eight; ninety-five.

(4) Seventy-six; twenty-two; fifty; fifteen; twenty-eight; sixty-one; forty-nine; eighteen; ninety; seventy-three.

(5) One hundred and seven; one hundred and thirty; two hundred and forty-six; three hundred and seventy-two; six hundred and eight; seven hundred and forty; nine hundred and ninety.

(6) Eight hundred and thirty-six; seven hundred and forty-seven; four hundred and ten; nine hundred and thirteen; seven hundred and fifty; three hundred and eighty-four.

(7) Eight hundred and eighteen; eight hundred and eight; two hundred and six; four hundred and thirty; five hundred and twelve; seven hundred and eighty-seven.

(8) Seven thousand eight hundred and forty-five; nine thousand six hundred and thirty-seven; twelve thousand; eight thousand four hundred; six thousand and three; eighty-five thousand and forty.

(9) Five thousand four hundred and seventy; three thousand six hundred and fifty; eight thousand seven hundred and eighty; one thousand two hundred and forty-seven; four thousand eight hundred and eight.

(10) Six thousand and four; seven thousand and twenty-two; three thousand five hundred; nine thousand and forty-seven; two thousand and seventeen; nineteen thousand four hundred and two.

(11) Seventy thousand and seven; sixty thousand and sixty; fourteen thousand and fourteen; seventy thousand and seventeen; twelve thousand three hundred and three; sixteen thousand and five.

(12) Three hundred and fifty-six thousand seven hundred and twenty-eight; six hundred and forty thousand eight hundred and forty-two; nine hundred thousand; eight hundred thousand and forty.

(13) Seven millions; four millions five hundred and seventy-six thousand eight hundred and sixty-five; seventy-five millions eight hundred and six thousand, nine hundred and forty.

(14) Three hundred and fifteen millions; five millions and forty thousand; eight millions and seven hundred; eighteen millions and twenty; seven hundred millions and two.

(15) Three hundred and fifteen thousand six hundred and seventy-four millions, eighteen thousand and three; thirty-five thousand six hundred millions, five hundred and twenty.

(16) Seven billions; five billions, eight hundred thousand millions six hundred thousand and forty-seven; eight billions, forty-three thousand and seven.

(17) Three hundred and five billions, five thousand and four millions, six thousand and three; fifty-three billions, fifty-three millions, fifty-three thousand and fifty-three.

(18) Nine billions and nine; ninety billions and nine hundred; nineteen billions and nineteen thousand; one billion. one million, one thousand, one hundred and one.

ROMAN NUMERALS.

14. In the Roman system of Notation, which is still used frequently in inscriptions, in references to chapters of books, and for other purposes, the symbols chiefly employed were I, V, X, L, C, D, M.

These symbols, standing by themselves, represented respectively the numbers one, five, ten, fifty, a hundred, five hundred, and a thousand. Intermediate numbers were represented by means of an arrangement that the numbers represented by the symbols I and X when standing on the *right* of a higher symbol were to be *added* to the number represented by that symbol, and when standing on the *left* were to be *subtracted* from it.

Thus:

VI represented the number *six*,
IV represented the number *four*,
and LX represented the number *sixty*,
XL represented the number *forty*.

The following table will explain the method for numbers up to a thousand:

1	I.	11	XI.	21	XXI.	110	CX.
2	II.	12	XII.	30	XXX.	150	CL.
3	III.	13	XIII.	40	XL.	188	CLXXXVIII.
4	IV.	14	XIV.	44	XLIV.	200	CC.
5	V.	15	XV.	50	L.	300	CCC.
6	VI.	16	XVI.	60	LX.	400	CCCC.
7	VII.	17	XVII.	70	LXX.	500	D.
8	VIII.	18	XVIII.	80	LXXX.	600	DC.
9	IX.	19	XIX.	90	XC.	900	DCCCC.
10	X.	20	XX.	100	C.	1000	M.

Examples. (iii)

Write in words:—

(1) XXVII. (2) XLIX. (3) LXVIII.
(4) LXXIII. (5) XCII. (6) CXLIV.
(7) CLXIII. (8) CXCIX. (9) DCLXIV.
(10) MDCCCLXXII.

Write in Roman Numerals:—

(1) 37. (2) 59. (3) 62. (4) 87. (5) 95.
(6) 139. (7) 145. (8) 179. (9) 846. (10) 1763.

II. Addition.

15. If we combine two or more groups of units, so as to make one group, the number of units in this single group is called the SUM of the numbers of units in the original groups.

To find the sum of 5 and 3, we reason thus:

$$\begin{aligned} \text{Since}\quad 3 &= 1+1+1, &\text{(Art. 5)}\\ 5+3 &= 5+1+1+1 &\\ &= 6+1+1 &\text{(Art. 4)}\\ &= 7+1 &\\ &= 8. & \end{aligned}$$

16. By practice we become able to express the result of adding a number less than ten to another number, without breaking up the number, which we have to add, into units.

Thus we say

7 and 5 make 12,
15 and 8 make 23;

and so on.

Again, if we have three or four numbers, each less than ten, to add together, we perform the process mentally; thus, to add 4, 7, 9 and 6 together we say 4 and 7 make 11, 11 and 9 make 20, 20 and 6 make 26.

17. We now proceed to explain the process of addition in the case of higher numbers.

Suppose we have to add together the four numbers 2475, 397, 486, and 3007.

We arrange them thus,

$$\begin{array}{r} 2475 \\ 397 \\ 486 \\ 3007 \\ \hline 6365 \end{array}$$

placing the figures that represent *units* in each number in the same vertical line, and those that represent *tens* in the same vertical line, and similarly for those that represent *hundreds* and *thousands*. We then draw a horizontal line under the last number, and under this line we place the number representing the sum of the given numbers, which is found in the following way:

Adding 7, 6, 7 and 5 units, the sum is twenty-five units, that is 2 tens and 5 units: we place the five under the line of units, and carry on the 2 tens for addition to the line of tens.

Adding 2, 0, 8, 9 and 7 tens, the sum is twenty-six tens, that is 2 hundreds and 6 tens: we place the 6 under the line of tens, and carry on the 2 hundreds for addition to the line of hundreds.

Adding 2, 0, 4, 3 and 4 hundreds, the sum is thirteen hundreds, that is 1 thousand and three hundreds: we place the 3 under the line of hundreds, and carry on the 1 thousand for addition to the line of thousands.

Adding 1, 3 and 2 thousands, the sum is six thousands, and we place 6 under the line of thousands.

Examples. (iv)

Add together

(1) 4 and 7, 3 and 13, 5 and 15, 9 and 27.

(2) $\begin{array}{r} 62 \\ 36 \\ \hline \end{array}$ (3) $\begin{array}{r} 40 \\ 27 \\ \hline \end{array}$ (4) $\begin{array}{r} 36 \\ 24 \\ \hline \end{array}$

(5)	(6)	(7)
237	209	562
349	140	70
823	600	106

(8)	(9)	(10)
459	5462	24609
6	723	3470
237	8004	40052
4269	9217	6207

(11)	(12)	(13)	(14)
429	364	253	140
347	629	189	49
425	488	567	257
269	976	278	6
538	853	384	428

(15)	(16)	(17)	(18)
6842	8750	8604	6843
5679	4623	4007	4297
8526	7988	5290	326
5037	6543	3046	52
2409	5729	7259	7008

(19) $64+43+7+85+9$.

(20) $247+356+28+423+97+12$.

(21) $425+3742+4236+39+847$.

(22) $7288+976+45+623+4000$.

(23) $8+97623+3407+5260+86$.

(24) $41537+9215+48+6077+23+2413$.

(25) $275413+3126+725+5007$.

(26) $74259+346274+30000+1000001+207$.

(27) $4692+72430+80000729+40+600000000$.

(28)	(29)	(30)
46243	748325	5629
35297	54297	426580
825649	532684	37259
246728	20047	506
815	4207	670492
42376	617043	37987
645980	3025	6493

(31)	(32)	(33)
256497	654297	625493
648098	248643	75862
720430	380469	5436
630689	472586	87294
407246	582987	4859
864928	639458	862
254384	498468	13

(34)	(35)	(36)
7462594	4697498	6572043
8625837	527	2869257
4398025	4307046	436
6702403	27209	698206
5124917	152372	45297
6219806	4058	3526084
4390143	7265204	57002
7409425	4372943	852968

(37) Seven hundred and forty; forty thousand and fifteen; six hundred and forty-seven; fifty-three thousand three hundred and three; seventeen thousand five hundred and forty-six.

(38) Five hundred and eight; six thousand and nine; fifty-five thousand and fourteen; eight hundred and nineteen; seven hundred thousand and six; two thousand and twelve.

(39) Six hundred and forty-five thousand, eight hundred and forty-five; seventy thousand and forty-seven; sixty thousand and forty; seven hundred and fifty thousand; three hundred thousand and fifteen.

(40) Two hundred and one millions, ninety-six thousand, three hundred and forty-two; fifty-four thousand, three hundred and four; eighteen millions, six thousand and three; five hundred thousand and forty; eight millions and eight.

III. Subtraction.

18. If from a number we take away a smaller number, the process is called *Subtraction.*

Strictly we ought to take away each of the units, of which the smaller number is composed, separately from the larger number: thus, to subtract 3 from 5, we reason thus:

3 =

if we take away one of these units from 5, we have 4 left; if we take away the second unit from 4, we have 3 left; if we take away the third unit from 3, we have 2 left.

The Symbol $-$, read *minus*, is used to denote the operation of Subtraction. Thus the operation of subtracting 3 from 5, and its connection with the result, may be briefly expressed thus:

$$5 - 3 = 2.$$

19. By practice we become able to subtract a number, less than ten, from another number, without breaking up the smaller number into units; thus we say,

$$7 - 4 = 3,$$
$$18 - 5 = 13,$$
$$49 - 8 = 41;$$

and so on.

20. Before we proceed to explain the process of Subtraction in the case of higher numbers, we must notice the principle on which a certain step in the process is founded.

If we are comparing two numbers, with a view to discover the number, by which one exceeds the other, we may add ten single units to the greater, if we also add one group of ten units to the less, and we may add ten groups of ten units to the greater, if we also add one group of a hundred units to the less; and so on.

Suppose, for example, we want to find the number, by which 56 exceeds 29, we might reason thus,

56 = five tens together with six units.
29 = two tens together with nine units.

To the former add *ten single units*, and to the latter add *one group of ten units.*

Then the resulting numbers will be,

in the first case five tens together with sixteen units,
in the second case three tens together with nine units.

Hence the excess of the former over the latter will be the number, made up of two tens together with seven units, and will therefore be represented by 27.

Let us now take an example, to show the *practical* way of performing the operation of subtraction, accompanied by a complete explanation of the process.

Suppose we have to take 589 from 926;

From	926
Take	589
Remainder	337

We arrange the numbers, placing the figures that represent units in each in the same vertical line, and doing the same with those that represent tens and hundreds.

We then reason thus: we cannot take 9 units from 6 units; we therefore add *ten units* to the 6 units, making *sixteen* units, and we take 9 units from the sixteen units, and set down the result, which is 7 units, under the line of units.

Having increased the upper number by ten units, we add, by way of compensation, 1 *ten* to the lower number, changing 8 tens into 9 tens. We proceed thus: we cannot take 9 tens from 2 tens; we therefore add *ten tens* to the 2 tens, making *twelve* tens, and from these we take 9 tens, and set down the result, which is 3 tens, under the line of tens.

Having increased the upper number by *ten tens*, we add, by way of compensation, 1 *hundred* to the lower number, changing 5 hundreds into 6 hundreds.

We then take 6 hundreds from 9 hundreds, and set down the result, which is 3 hundreds, under the line of hundreds.

Examples. (v)

Find the difference between the following pairs of numbers:

(1) 13 and 6. (2) 15 and 7. (3) 23 and 4. (4) 3 and 32.

(5)	(6)	(7)	(8)	(9)
57	96	74	87	92
23	42	39	58	47

(10)	313 247	(11)	704 195	(12)	630 548	(13)	7426 3618
(14)	6239 4127	(15)	4729 501	(16)	6258 36	(17)	65472 4001
(18)	357 249	(19)	4625 1846	(20)	72649 43821	(21)	20004 17243

(22) 437—56 (23) 529—483 (24) 827—795

(25) 3000—958 (26) 7040—583 (27) 6259—479

(28) 58623—7428 (29) 64295—53296 (30) 70000—68904

(31) 52764 and 34297. (32) 42456 and 102479.

(33) 624300 and 1400072. (34) 99999 and 100000.

(35) A million and a thousand.

(36) A hundred millions and a hundred thousand.

(37) Ten thousand millions and a thousand and one.

(38) What number must be taken from 26 to leave 18?

(39) What number must be taken from 427 to leave 401?

(40) What number must be taken from three thousand and fifteen to leave two thousand four hundred and five?

(41) By how many does a thousand exceed four hundred and seven?

(42) The greater of two numbers is 427 and the sum of the numbers is 586, what is the smaller of the two numbers?

(43) What number must be added to 7428 to make 8047?

IV. Multiplication.

21. Multiplication is the process, by which we find the sum of two, three, four or more numbers, which are *equal.*

Thus, if we have to find the sum of three numbers each equal to 7 we call the process *the* MULTIPLICATION *of* 7 *by* 3.

This sum is called the PRODUCT of the multiplication of 7 by 3.

The number 3 is called the MULTIPLIER.

The number 7 is called the MULTIPLICAND.

The following table must be committed to memory.

The Multiplication Table.

Twice	Three times	Four times	Five times	Six times	Seven times
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2 .. 4	2 .. 6	2 .. 8	2 .. 10	2 .. 12	2 .. 14
3 .. 6	3 .. 9	3 .. 12	3 .. 15	3 .. 18	3 .. 21
4 .. 8	4 .. 12	4 .. 16	4 .. 20	4 .. 24	4 .. 28
5 .. 10	5 .. 15	5 .. 20	5 .. 25	5 .. 30	5 .. 35
6 .. 12	6 .. 18	6 .. 24	6 .. 30	6 .. 36	6 .. 42
7 .. 14	7 .. 21	7 .. 28	7 .. 35	7 .. 42	7 .. 49
8 .. 16	8 .. 24	8 .. 32	8 .. 40	8 .. 48	8 .. 56
9 .. 18	9 .. 27	9 .. 36	9 .. 45	9 .. 54	9 .. 63
10 .. 20	10 .. 30	10 .. 40	10 .. 50	10 .. 60	10 .. 70
11 .. 22	11 .. 33	11 .. 44	11 .. 55	11 .. 66	11 .. 77
12 .. 24	12 .. 36	12 .. 48	12 .. 60	12 .. 72	12 .. 84

Eight times	Nine times	Ten times	Eleven times	Twelve times
1 is 8	1 is 9	1 is 10	1 is 11	1 is 12
2 .. 16	2 .. 18	2 .. 20	2 .. 22	2 .. 24
3 .. 24	3 .. 27	3 .. 30	3 .. 33	3 .. 36
4 .. 32	4 .. 36	4 .. 40	4 .. 44	4 .. 48
5 .. 40	5 .. 45	5 .. 50	5 .. 55	5 .. 60
6 .. 48	6 .. 54	6 .. 60	6 .. 66	6 .. 72
7 .. 56	7 .. 63	7 .. 70	7 .. 77	7 .. 84
8 .. 64	8 .. 72	8 .. 80	8 .. 88	8 .. 96
9 .. 72	9 .. 81	9 .. 90	9 .. 99	9 .. 108
10 .. 80	10 .. 90	10 .. 100	10 .. 110	10 .. 120
11 .. 88	11 .. 99	11 .. 110	11 .. 121	11 .. 132
12 .. 96	12 .. 108	12 .. 120	12 .. 132	12 .. 144

22. Let it be carefully observed that Multiplication is a short form of addition. When we say that 3 times 4 is twelve, we assert that, if 4 and 4 and 4 be *added* together, the result is 12.

Each of the numbers 3 and 4 is called a FACTOR of the product 12.

Again, if we had to find the value of 4 times 67, we might proceed thus:

$$\begin{array}{r} 67 \\ 67 \\ 67 \\ 67 \\ \hline 268 \end{array}$$

Now since the figures in each vertical line are the same, we may save ourselves the trouble of addition, by learning, from the Multiplication Table, the numbers that result from adding the same number four times. Then we may write the operation in a shorter form, thus:

$$\begin{array}{r} 67 \\ 4 \\ \hline 268 \end{array}$$

The process will stand thus:

Four times 7 is twenty-eight; we set down 8 in the place of units, and carry on 2 for addition to the line of tens. Four times 6 tens is 24 tens, and adding 2 tens, the result is twenty-six tens, that is two hundreds and six tens; we set down 6 in the place of tens, and 2 in the place of hundreds, and the final result is 268.

Here 67 is called the Multiplicand,
4 is called the Multiplier,
268 is called the Product.

23. The symbol $\times$, placed between two numbers, expresses that the second is multiplied by the first, and the whole operation in the example just given is briefly expressed thus:

$$4 \times 67 = 268.$$

24. Next observe, that the multiplier and multiplicand may change places, without altering the value of the Product.

Thus $3 \times 4 = 4 \times 3$, or 3 times $4 = 4$ times 3.

For 3 times $4 = 4 + 4 + 4$

$$\left.\begin{array}{l} = 1 + 1 + 1 + 1 \\ + 1 + 1 + 1 + 1 \\ + 1 + 1 + 1 + 1 \end{array}\right\} \text{I.}$$

And 4 times $3 = 3+3+3+3$

$$\left.\begin{array}{l} =1+1+1 \\ +1+1+1 \\ +1+1+1 \\ +1+1+1 \end{array}\right\} \text{II.}$$

Now the results obtained from I. and II. must be the same, for the horizontal columns of the one are identical with the vertical columns of the other.

25. If we multiply a number by 10, the product is obtained by annexing 0 to the number, that is,

$$10 \times 364 = 3640.$$

If we multiply a number by 100, the product is obtained by annexing 00 to the number, that is,

$$100 \times 364 = 36400.$$

So by annexing 000 to a number we multiply it by 1000, and so on.

If we have to multiply a number by 20, we may first multiply it by 2 and then annex 0 to the result, and the final result will be the product required.

Again, if we have to multiply a number by 200, we may first multiply it by 2 and then annex 00 to the result.

The method of expressing the result of multiplications of this kind in practice is as follows :—

We multiply 4276 by 700 and 14239 by 6000 thus:

$$\begin{array}{r} 4276 \\ 700 \\ \hline 2993200 \end{array} \qquad\qquad \begin{array}{r} 14239 \\ 6000 \\ \hline 85434000 \end{array}$$

Examples. (vi)

Find the Product in the following cases of Multiplication.

(1) 3 times 15. (2) 4 times 76. (3) 5 times 98.
(4) 10 times 87. (5) 6 times 114. (6) 7 times 123.
(7) 11 times 843. (8) 12 times 947.
(9) Multiply 25 by 2, 3, 7, 9.

(10) Multiply 127 by 5, 8, 10, 11, 20, 70.
(11) Multiply 2467 by 4, 6, 11, 12, 500, 7000.
(12) Multiply 37429 by 9, 11, 12, 50000, 80000000.

26. Ex. (1). *To multiply* 347 *by* 23.

The form of the operation is

$$\begin{array}{r} 347 \\ 23 \\ \hline 1041 \\ 694 \\ \hline 7981 \\ \hline \end{array}$$

The explanation is this:

The multiplier is made up of two parts 3 and 20, we therefore multiply 347 first by 3, and then by 20, and add the two results.

Now $3 \times 347 = 1041$.
and $20 \times 347 = 6940$.

In setting down this second result *we omit the zero*, because it will have no effect on the addition which has to be performed.

Examples. (vii)

Multiply

(1) 23 by 15.	(2) 37 by 29.	(3) 45 by 36.
(4) 70 by 26.	(5) 125 by 24.	(6) 327 by 42.
(7) 205 by 43.	(8) 307 by 98.	(9) 2684 by 35.
(10) 57296 by 27.	(11) 84293 by 88.	(12) 7629302 by 76.

Ex. (2). *To multiply* 34007 *by* 213.

$$\begin{array}{r} 34007 \\ 213 \\ \hline 102021 \\ 34007 \\ 68014 \\ \hline 7243491 \\ \hline \end{array}$$

Here, when we multiply 34007 by 200, the result is 6801400, and *we omit the two zeros* at the end, being careful to put the 4 in the place of hundreds.

Observe that, in all cases, the first figure on the right of each partial product will be in the same vertical line with the figure, by which we are multiplying: thus, in the example just given, the 4 in the third product is in the same vertical line with the 2, by which we multiplied.

Ex. (3). *To multiply* 30047 *by* 21009.

```
    30047
    21009
   ------
   270423
  30047
 60094
---------
631257423
---------
```

Here the first figure on the right of the second product stands in the place of thousands, because we are then multiplying 30047 by 1000.

Ex. (4). *To multiply* 70407 *by* 3700.

```
   70407
     3700
 --------
 49284900
211221
---------
260505900
---------
```

Examples. (viii)

Multiply

(1) 326 by 532. (2) 704 by 176.
(3) 809 by 506. (4) 917 by 406.
(5) 5376 by 423. (6) 7846 by 340. (7) 7859 by 5006.
(8) 85639 by 598. (9) 79802 by 4007.
(10) 30207 by 5060. (11) 642867 by 90807.

(12) 8637405 by 40300. (13) 970052 by 40072.
(14) 980740 by 3406. (15) 9864302 by 300071.
(16) 870120506 by 700403. (17) 32804070 by 409300.
(18) 742349 by 947. (19) 578628 by 6205. ✓
(20) 428734 by 8057. (21) 984236 by 5009.
(22) 872469 by 70043. (23) 385704 by 36479.
(24) 423796 by 57243. (25) 620072 by 400205.
(26) 270403 by 502049. (27) 427964 by 582978.

Ex. (5). *To find the* continued product *of* 14, 8, *and* 70.

Here we first multiply 14 by 8, and then multiply the product by 70, thus:

$$\begin{array}{r} 14 \\ 8 \\ \hline 112 \\ 70 \\ \hline 7840 \end{array}$$

that is, $14 \times 8 \times 70 = 7840$.

Examples. (ix)

Find the continued product of

(1) 18, 19 and 20. (2) 436, 73, 12 and 5.
(3) 3476, 2300, 70010 and 2003.

27. When a number is multiplied *by itself* once, twice, three times, . . . the resulting products are called the second, third, fourth, . . . POWERS of the number. The process is called *Involution*, and the Power, to which the number is raised, is expressed by the number of times the number has been employed as a factor in the operation.

The term *square* is usually employed instead of *second* power.

The term *cube* is usually employed instead of *third power.*

Thus 144 is the square of 12, because $12 \times 12 = 144$.
64 is the cube of 4, because $4 \times 4 \times 4 = 64$.
81 is the fourth power of 3, because $3 \times 3 \times 3 \times 3 = 81$.

Examples. (x)

Find the squares of

(1) 15.	(2) 24.	(3) 40.	(4) 57.
(5) 69.	(6) 72.	(7) 87.	(8) 100.
(9) 114.	(10) 237.	(11) 625.	(12) 897.
(13) 789.			

And the cubes of

(14) 11.	(15) 13.	(16) 25.	(17) 47.
(18) 68.	(19) 93.	(20) 100.	(21) 257.
(22) 356.	(23) 539.	(24) 704.	(25) 987.

V. Division.

28. DIVISION is the process by which, when a *product* is given, and we know *one* of the factors, the *other* factor is determined.

The product is, with reference to this process, called the DIVIDEND.

The given factor is called the DIVISOR.

The factor, which has to be found, is called the QUOTIENT.

29. The operation of division is denoted by the sign $\div$

Thus $12 \div 3$ signifies that 12 is to be divided by 3.

The same operation is denoted by writing the Dividend over the Divisor with a line drawn between them, thus $\frac{12}{3}$.

In this Chapter we shall treat only of cases in which the Dividend contains the Divisor an exact number of times.

30. *For small numbers, the Multiplication Table affords the means of solving questions in Division.*

For instance, since $12 = 4 \times 3$,

$12 \div 4 = 3$, and, $12 \div 3 = 4$;

and since $96 = 12 \times 8$,

$96 \div 12 = 8$, and, $96 \div 8 = 12$.

31. When we divide one number by another, we find how many times the latter is contained in the former, and therefore any process by which we can discover how many times one number is contained in another will furnish a rule for division. Such a process is explained by the examples, which we shall now give.

Ex. (1). *Divide* 408 *by* 17.

$$\begin{aligned} &\text{Since } 17 \times 20 = 340, \\ &\text{and } \quad 17 \times 30 = 510, \end{aligned}$$

it is plain that 17 is contained in 408 more than *twenty* times, and less than *thirty* times.

If then we take away 340 from 408, and find how many times 17 is contained in the number that remains, we shall find how many times more than *twenty*, the Divisor is contained in the Dividend 408.

Now $408 - 340 = 68$, and this number contains 17 just *four* times.

Hence 17 is contained in 408 twenty times, and also four times, that is, the Quotient resulting from the division of 408 by 17 is 24.

This process is represented more briefly thus:

$$\begin{array}{r l} 17\,)\,408\,(\,20+4 & \\ 340 & \\ \hline 68 & \\ 68 & \\ \hline \end{array}$$

Hence $408 \div 17 = 24$.

And yet more briefly, availing ourselves of the notation by which the *local* value of digits is represented, and we are enabled to omit zeros,

$$\begin{array}{r l} 17\,)\,408\,(\,24 & \\ 34 & \\ \hline 68 & \\ 68 & \\ \hline \end{array}$$

Ex. (2). Suppose we have to divide 89012 by 17:

```
Divisor Dividend Quotient
 17 ) 89012 (5236
      85
      --
       40
       34
       --
        61
        51
        --
        102
        102
        ---
```

We first find how often 17 is contained in 89, and as it is contained five times, we set down 5 as the first figure in the quotient, then multiply 17 by 5, and subtract the result 85 from the 89: to the remainder 4 we annex the next figure in the dividend; then as 17 is contained in 40 twice, we set down 2 as the second figure in the quotient, then multiply 17 by 2, and subtract the result 34 from the 40; and proceed by similar steps to the end of the operation.

Ex. (3). Divide 920575 by 23.

```
23 ) 920575 (40025

       057
        46

        115
```

Here, when we bring down 0, the *third* figure of the dividend, 23 is not contained in it; we therefore set down 0 as the second figure of the quotient, and when we bring down 5, the *fourth* figure of the dividend, 23 is not contained in 5; we therefore set down another 0 as the third figure of the quotient. When we then bring down 7, the next figure of the dividend, 23 is contained in 57 twice, and the operation proceeds easily.

Examples. (xi)

Divide

(1) 18 by 6. (2) 27 by 9.
(3) 84 by 7. (4) 132 by 12.
(5) 182 by 13. (6) 238 by 17.
(7) 456 by 19. (8) 3708 by 36.
(9) 3996 by 37. (10) 6409 by 493.
(11) 431376 by 817. (12) 976272 by 946.
(13) 19249470 by 342. (14) 86366784 by 358.
(15) 224009433 by 489. (16) 4690325214 by 618.
(17) 2880376 by 1369. (18) 107810526 by 6142.
(19) 98955005667 by 4123. (20) 4076361 by 2019.
(21) 13312053 by 237. (22) 505350366 by 89.
(23) 360919856 by 83. (24) 4600304 by 907.
(25) 218860161 by 689. (26) 337103025 by 861.
(27) 39916424548 by 1001. (28) 152847420 by 5060.
(29) 26540538445 by 7649.
(30) 116558439800 by 17072.
(31) 350880088323434 by 74291.
(32) 369187022085112 by 65432.
(33) 8377413561524590 by 98989.
(34) 58376823669 by 642867.
(35) 2959990965442 by 9864302.
(36) 2614491091180 by 8723694.

32. If any two of the three numbers that form the Divisor, Dividend, and Quotient be given, we can find the third.

For Dividend ÷ Divisor = Quotient.
Dividend ÷ Quotient = Divisor.
Divisor × Quotient = Dividend.

Examples. (xii)

(1) The Dividend is 1171692, the Divisor 342, find the Quotient.

(2) The Dividend is 149201, the Quotient 23, find the Divisor.

(3) The Divisor is 987, the Quotient 64852, find the Dividend.

SHORT DIVISION.

33. When the Divisor is not greater than 12, the process of division may be greatly abridged.

Suppose we have to divide 92368 by 8.

The operation is set down in the following form:

$$\begin{array}{r|l} 8 & 92368 \\ \hline & 11546 \text{ Quotient.} \end{array}$$

The following is the process.

Since 8 is contained *once* in 9, with 1 as remainder, we set down 1 under the 9, and mentally prefix the remainder 1 to the 2, reading the result as 12: then since 8 is contained *once* in 12, with 4 as remainder, we set down 1 under the 2, and prefix 4 to the 3, reading the result as 43; then since 8 is contained *five times* in 43, with 3 as remainder, we set down 5 under the 3, and prefix 3 to the 6, reading the result as 36: then since 8 is contained *four times* in 36, with 4 as remainder, we set down 4 under the 6, and prefix 4 to the 8, reading the result as 48: then since 8 is contained *six times* in 48, with no remainder, we set down 6 under the 8, and our operation is completed.

Next, suppose we have to divide 11042304 by 12.

The operation is set down thus:

$$\begin{array}{r|l} 12 & 11042304 \\ \hline & \ \ 920192 \text{ Quotient.} \end{array}$$

The following is the process:

We must take three figures before we obtain a number which contains 12; then we say 12 is contained *nine times* in 110, with 2 to carry on; then 12 is contained *twice* in 24, and there is nothing to carry on; then 12 is *not contained at all* in 2, we therefore set down 0 under the 2, and carry on 2; then 12 is contained in 23 *once*, with 11 to carry on; then 12 is contained in 110 *nine times*, with 2 to carry on; lastly 12 is contained in 24 *twice* exactly.

Examples. (xiii)

Divide

(1) 7652 by 2. (2) 725961 by 3.
(3) 8650232 by 4. (4) 8749320 by 5.
(5) 7463424 by 6. (6) 3504221 by 7.
(7) 713406960 by 9. (8) 4362017 by 11.
(9) 7912464 by 12. (10) 4000623070905 by 9.
(11) 7642300721 by 11. (12) 36089882405604 by 12.

Divide each of the following numbers by 2, 3, and 4 separately:

(13) 4263924. (14) 620437548. (15) 27540918264.

Divide each of the following numbers by 5, 8, and 9 separately:

(16) 46528920. (17) 981754200. (18) 234567000.

Divide each of the following numbers by 7, 11, and 12 separately:

(19) 7971348. (20) 29574468. (21) 67363878I2.

CASTING OUT THE NINES.

34. A test of correctness of the result of multiplication is furnished by a curious property of numbers, which is explained in Hamblin Smith's *Algebra*, p. 323. We give the test in the form of a rule.

Divide the sum of the digits in the Multiplicand by 9, and set down the remainder. Divide the sum of the digits in the Multiplier by 9, and set down the remainder. Multiply the two remainders together, divide the result by 9, and set down the remainder. If the process be correct, this remainder will be the same as the remainder obtained by taking the sum of the digits in the Product and dividing it by 9.

For example, if we multiply 76371 by 854 the product is 65220834.

Sum of digits in Multiplicand = 24,
and 24 ÷ 9 gives remainder 6

Sum of digits in Multiplier = 17,
and 17 ÷ 9 gives remainder 8

First remainder × second remainder = 48,
and 48 ÷ 9 gives remainder 3

Sum of digits in the Product = 30,
and 30 ÷ 9 gives remainder 3

We need not enter upon a discussion of the cases in which this test may fail. It is sufficient for the beginner to know that he may employ it with advantage to secure accuracy in his work.

Examples. (xiv)

Multiply, and test the result of multiplying

(1) 27543 by 497. (2) 34682 by 4021. (3) 7549 by 206.
(4) 20051 by 3070. (5) 4372 by 2483. (6) 5072 by 3123.

VI. On the Resolution of Numbers into Factors.

35. We shall discuss in this section an operation, which is the opposite of that which we call multiplication. In multiplication we determine the product of two given factors: in the operation, of which we have now to treat, the product is given, and the factors have to be found.

36. For small numbers the factors may be determined by inspection:

thus, the factors of 21 are 3 and 7,
the factors of 55 are 5 and 11.

37. When we have found two factors that make up a product, one or both of these factors may be themselves reducible to simpler factors.

Thus 9 and 6 are factors of 54;

and the factors of 9 being 3 and 3,
and the factors of 6 being 2 and 3,

the number 54 can be split up into *four* factors, 2, 3, 3, 3.

38. *Prime* numbers are those, which have no exact divisor but themselves and unity.

Thus 2, 3, 5, 7, 11, 13, 17, 19 are Prime Numbers.

Composite numbers are those, which can be resolved into factors, each of which is greater than 1.

Thus 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 are Composite Numbers.

39. Every composite number can be resolved into factors, which are prime numbers: thus

$$4=2\times 2;\ 6=2\times 3;\ 8=2\times 2\times 2;\ 9=3\times 3.$$

Hence, in resolving a large number into factors, we divide it by any small prime number, by which we know it is exactly divisible, and then divide the quotient by any small prime number by which it is exactly divisible, and proceed in this way, till the quotient is 1; then the divisors are the factors required.

Thus, to find the factors of 2520:

$$\begin{array}{r|r}
2 & 2520 \\ \hline
2 & 1260 \\ \hline
2 & 630 \\ \hline
3 & 315 \\ \hline
3 & 105 \\ \hline
5 & 35 \\ \hline
7 & 7 \\ \hline
 & 1
\end{array}$$

Hence $2520=2\times 2\times 2\times 3\times 3\times 5\times 7$.

In practical arithmetic we seldom require to find *all* the factors of a composite number, but very frequently we want to know whether a number is exactly divisible by *a particular number.*

The student will find it of use to remember the following properties of numbers.

A number is exactly divisible

by 2 when its last figure is 0 or an *even* digit, as 426;
3 when the sum of its digits is divisible by 3, as 579;
4 when its last *two* figures are divisible by 4, as 2364;
8 when its last *three* figures are divisible by 8, as 25256;
5 when its last figure is 0 or 5, as 30 and 135;
9 when the sum of its digits is divisible by 9, as 275265;
10 when its last figure is 0.
11 when the difference between the sum of the digits in the *odd* places (reckoning from the right) and the sum of the digits in the *even* places is either 0 or divisible by 11.

Thus 24794 and 829191 are divisible by 11.

Examples. (xv)

Find whether the following numbers be exactly divisible by 2, 3, 4, 5, 8, 9, 10 or 11.

(1) 117.	(2) 288.	(3) 495.
(4) 1050.	(5) 23472.	(6) 42345.
(7) 27464.	(8) 32495.	(9) 84732.
(10) 6480.	(11) 619182718.	

NOTE.—We have inserted these remarks at this point, because, in attempting to resolve a large number into factors, it is well to know whether the attempt to divide it by 2 or 3 or 5, &c., will be successful.

The student may now, following the instruction given in Art. 39, work another set of Examples

Examples. (xvi)

Resolve into *prime* factors:

(1) 18.	(2) 24.	(3) 27.	(4) 32.
(5) 36.	(6) 39.	(7) 42.	(8) 51.
(9) 54.	(10) 57.	(11) 72.	(12) 85.
(13) 91.	(14) 99.	(15) 100.	(16) 105.
(17) 108.	(18) 112.	(19) 132.	(20) 176.
(21) 288.	(22) 432.	(23) 525.	(24) 625.
(25) 729.	(26) 999.	(27) 1296.	(28) 1760.
(29) 5760.			

40. The process of Multiplication may often be made shorter when the Multiplier is a composite number, by resolving it into *two or more factors.*

Thus if we have to multiply 2579825 by 56, we may resolve 56 into the factors 8 and 7, and proceed thus,

$$\begin{array}{r} 2579825 \\ 8 \\ \hline 20638600 \\ 7 \\ \hline 144470200 \end{array}$$

The advantage of this method will be more apparent when we come to multiplication of sums of money, weights, and measures.

Examples. (xvii)

Multiply, after resolving the multiplier into factors not greater than 12,

(1)	347 by 14.	(2)	423 by 22.
(3)	5462 by 27.	(4)	8497 by 36.
(5)	8573 by 49.	(6)	28472 by 56.
(7)	49273 by 63.	(8)	90728 by 132.
(9)	90725 by 360.	(10)	40207 by 108.
(11)	36729 by 1320.	(12)	704075 by 14400.

41. So also we may often simplify the process of division, when the Divisor, though greater than 12, can be made up by factors each not greater than 12. For we can divide the Dividend first by one of these factors, and then divide the Quotient by a second factor, and so on.

Suppose we have to divide 47268540 by 45.

Here 45 can be made up of the factors 9 and 5.

$$\begin{array}{r|r} 9 & 47268540 \\ \hline 5 & 5252060 \\ \hline & 1050412 \end{array}$$

Examples. (xviii)

Apply the process just explained in the division of

(1)	34608 by 14.	(2)	6791040 by 15.
(3)	752364576 by 18.	(4)	1143995886 by 27.
(5)	285216822 by 33.	(6)	2095501072 by 49.
(7)	4157028792 by 56.	(8)	1200130008 by 84.
(9)	22039992 by 108.	(10)	57667632 by 132.
(11)	472634500 by 125.	(12)	565184160 by 720.
(13)	537062400 by 14400.		

VII. Inexact Division.

42. Hitherto we have chosen Examples, in which the Divisor is contained an *exact* number of times in the Dividend.

Now suppose we have to divide 23 by 7.

Since $3 \times 7 = 21$, it follows that we can divide 23 units into 3 parcels, each containing 7 units, and when we have done this, 2 units out of the 23 remain over.

In such a case we call 3 the Quotient, and 2 the Remainder.

Again, if we have to divide 72469 by 53, we proceed thus,

```
53 ) 72469 ( 1367
     53
     --
     194
     159
     ---
      356
      318
      ---
       389
       371
       ---
        18
        --
```

Hence the Quotient is 1367, and the Remainder 18.

NOTE.—If we multiply the Quotient by the Divisor, and add the Remainder to the product, the sum must be equal to the Dividend.

Examples. (xix)

Divide

(1) 3492 by 37. (2) 486296 by 41.
(3) 872968 by 47. (4) 57092 by 65.
(5) 7492736 by 71. (6) 82749325 by 98.
(7) 87467 by 103. (8) 978462 by 409.
(9) 8276253 by 723. (10) 974004562 by 1009.
(11) 48237654 by 4821. (12) 68725642903 by 6871.

43. When we employ, in cases of *inexact* division, the method of short division, after breaking up the divisor into component factors, as in Art. 41, the remainder will be found by a process now to be explained.

Ex. (1). Divide 43276 by 21.

$$21\left\{\begin{array}{r|l} 3 & 43276 \\ \hline 7 & 14425 \text{ and 1 unit over,} \\ \hline & 2060 \text{ and 5 parcels of 3 units, or, 15 units over.} \end{array}\right.$$

Whence the Quotient is 2060, and the Remainder is 15+1, or, 16.

Ex. (2). Divide 572948 by 125.

$$125\left\{\begin{array}{r|l} 5 & 572948 \\ \hline 5 & 114589 \text{ and 3 units over,} \\ \hline 5 & 22917 \text{ and 4 parcels of 5 units, or, 20 units over,} \\ \hline & 4583 \text{ and 2 parcels of 25 units, or, 50 units over.} \end{array}\right.$$

Whence the Quotient is 4583, and the Remainder is 50+20+3, or, 73.

Examples. (xx)

Divide, employing Short Division,

(1) 4153 by 15. (2) 587595 by 16.
(3) 42813 by 18. (4) 423672 by 21.
(5) 724972 by 25. (6) 569024971 by 27.

(7) 2825780 by 33. (8) 8642396 by 35.
(9) 356599 by 48. (10) 8274913 by 64.
(11) 230047914 by 77. (12) 419421 by 99.
(13) 44487 by 105. (14) 95379 by 189.
(15) 1194477 by 210.

44. In dividing a number by 10, we have merely to mark off the *last* figure, the other figures giving the quotient, and the figure marked off the remainder.

Thus $2460197 \div 10 = 246019$ with remainder 7.

Again, to divide 42395675 by 20, we might proceed thus,

10 | 42395675
2 | 4239567 and 5 units over,
2119783 and 1 parcel of 10 units over ;

whence the quotient is 2119783, and Remainder $10+5$, or 15.

But the operation is written more briefly thus :

2,0 | 4239567,5
2119783 and 15 remainder.

Again,

in dividing by 100, we mark off the *last two* figures,
in dividing by 1000, we mark off the *last three* figures

from divisor and dividend, and find the quotient and remainder by a similar process.

45. If any *three* of the four numbers, that form the Divisor, Dividend, Quotient and Remainder be given, we can find the *fourth*.

1. Let Divisor, Dividend, and Quotient be given. Multiply the Divisor by the Quotient, subtract the result from the Dividend, and you have the Remainder.

2. Let Divisor, Quotient, and Remainder be given. Multiply the Divisor by the Quotient, add the Remainder to the result, and you have the Dividend.

3. Let Divisor, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Divisor, and you have the Quotient.

4. Let Quotient, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Quotient, and you have the Divisor.

Examples. (xxi)

(1) The Divisor is 25, the Dividend 4276, the Quotient 171. Find the Remainder.

(2) The Divisor is 342, the Quotient 1381, the Remainder 67. Find the Dividend.

(3) The Divisor is 596, the Dividend 372149, the Remainder 245. Find the Quotient.

(4) The Quotient is 2910, the Dividend 8765237, the Remainder 317. Find the Divisor.

Examination Papers.

(A)

(1) Express in words, 4237496; and in figures, six hundred and fifty-three thousand eight hundred and twelve.

(2) Find the sum of 24753, 86729, 4237, and 80462.

(3) Find the difference between 86293 and 78464.

(4) Multiply 8627 by 493, and 50042 by 307.

(5) Divide 8423793 by 9, and 2659582 by 358.

(B)

(1) Write in figures, twenty-five millions two hundred and fifty-seven thousand six hundred and thirty; and in words, 402050407.

(2) From seventeen millions and seventeen take eight thousand and eight.

(3) Multiply 6549 by 4037, and 27004 by 3700.

(4) Divide 32456789 by 96, first by long division and then by short division, and shew that the results agree.

(5) Find the sum of one million and six, fifteen thousand and eleven, one hundred thousand and ten, and sixty thousand four hundred; and divide the result by 9.

(C)

(1) Write in words, 10010201401; and in figures, one million twenty-three thousand and one. Add together the two numbers, and from the sum subtract their difference.

(2) Multiply 740296 by 2089, and 426004 by 3704.

(3) Divide 78297426 by 35, employing short division.

(4) From one hundred and twenty-six millions four hundred and six thousand and three take ninety-five millions and four.

(5) Divide the product of 723 and 347 by 48.

(D)

(1) Express in figures the number represented by MDCCCLXXXVIII.

(2) Divide 987654321 by 132, using short division.

(3) Reduce to prime factors 56, 78, and 114.

(4) Multiply the sum of 86297 and 40025 by the difference between 789 and 694.

(5) By how many does one million exceed one hundred and one?

(E)

(1) Divide three hundred and fifty-three thousand and eight millions nine hundred and seventy-two thousand six hundred and two by 5406.

(2) Multiply 8976589 by 9876.

(3) Resolve into elementary factors (i.e. prime numbers) 40, 90, and 126.

(4) Express in Roman notation 24, 47, and 178.

(5) How many bricks may be taken away in 24 carts, each taking 500 bricks?

VIII.—On the Method of Finding the Highest Common Factor of Two or more Numbers.

46. A number is said to be a *Factor* of another number, when the latter is exactly divisible by the former. Thus 3 is a factor of 12.

A number is said to be a *Common Factor* of two or more numbers, when each of the latter is exactly divisible by the former. Thus 3 is a Common Factor of 9, 12, and 15.

The *Highest Common Factor* of two or more numbers is the highest number which will exactly divide each of them.

Thus 6 is the Highest Common Factor of 6, 12 and 18, and 9 is the Highest Common Factor of 27, 36 and 108.

The words Highest Common Factor we shall write briefly H. C. F.

For small numbers the H. C. F. may be found by inspection, and by way of practice the student may work the following examples, applying the tests of divisibility given in Art. 39.

Examples. (xxii)

Find the H. C. F. of

(1) 8 and 14.	(2) 12 and 30.
(3) 40 and 60.	(4) 36 and 90.
(5) 48 and 144.	(6) 7, 14, 21.
(7) 15, 27, 105.	(8) 32, 48, 128.
(9) 16, 64, 256, 1024.	(10) 24, 51, 105, 729.

47. In large numbers, the factors cannot often be determined by inspection, and *if we have to find the* H. C. F. *of two such numbers, we have recourse to the*

following Rule, of which a proof is given in Hamblin Smith's *Algebra*, Art. 128.

Divide the greater of the two numbers by the less, and the Divisor by the remainder, repeating the process, until no remainder is left: the last divisor is the H. C. F. *required.*

Thus, to find the H. C. F. of 689 and 1573, we proceed thus:

```
689 ) 1573 ( 2
      1378
      ----
       195 ) 689 ( 3
             585
             ---
             104 ) 195 ( 1
                   104
                   ---
                    91 ) 104 ( 1
                          91
                          --
                          13 ) 91 ( 7
                               91
                               --
```

Hence 13 is the H. C. F. of 689 and 1573.

Examples. (xxiii)

Find the H. C. F. of

(1) 384 and 1296.

(2) 2272 and 3552.

(3) 7455 and 47223.

(4) 12321 and 54345.

(5) 6906 and 10359.

(6) 1908 and 2736.

(7) 49608 and 169416.

(8) 126025 and 40115.

(9) 1581227 and 16758766.

(10) 35175 and 236845.

48. If the H. C. F. of *three* numbers be required, we first find the H. C. F. of two of the numbers. Then the H. C. F. of this result and the third number will be the H. C. F. required.

For example, if we require the H. C. F. of 351, 459 and 1017, we first find the H. C. F. of 351 and 459 to be

27, and then we find the H. C. F. of 27 and 1017 to be 9, which is therefore the H. C. F. required.

Examples. (xxiv)

Find the H. C. F. of

(1) 16, 20, 28.	(2) 14, 42, 56, 138.
(3) 365, 511, 803.	(4) 232, 290, 493.
(5) 492, 1476, 1763.	(6) 148, 444, 592, 703.

IX.—Lowest Common Multiple.

49. A number is called a *Multiple* of another number, when the former is exactly divisible by the latter. Thus 12 is a multiple of 3.

A number is said to be a *Common Multiple* of two or more numbers, when the former is exactly divisible by each of the latter. Thus 12 is a Common Multiple of 2, 3 and 4.

The *Lowest Common Multiple* of two or more numbers, is the lowest number, which is exactly divisible by each of them.

Thus 12 is the Lowest Common Multiple of 4, 6 and 12, and 60 is the Lowest Common Multiple of 15, 20 and 30.

The words Lowest Common Multiple we shall write briefly L. C. M.

50. To find the L. C. M. of two numbers we have the following Rule, of which the proof is given in Hamblin Smith's *Algebra*, Art. 169.

Divide one of the numbers by the H. C. F. *and multiply the quotient by the other number. The result is the* L. C. M

For example, to find the L. C. M. of 24 and 36.

The H. C. F. of 24 and 36 is 12.

$$\text{Now } 24 \div 12 = 2.$$

$$\therefore \text{ L.C.M. of } 24 \text{ and } 36 = 36 \times 2 = 72.$$

NOTE.—The symbol $\therefore$ stands for the word *therefore.*

Examples. (xxv)

Find the L. C. M. of

(1) 27 and 54.
(2) 88 and 108.
(3) 633 and 844.
(4) 195 and 735.
(5) 1000 and 2125.
(6) 3432 and 3575.
(7) 936 and 2925.
(8) 2304 and 4032.
(9) 2443 and 4537.

51. To find the L. C. M. of three or more numbers, we might find the L. C. M. of any two, and then find the L. C. M. of the resulting number and of a third of the original numbers, and so on, the final result being the L. C. M. required.

Thus to find the L. C. M. of 12, 20, 36 and 54, we might proceed thus:

the L. C. M. of 12 and 20 is 60,
of 60 and 36 is 180,
of 180 and 54 is 540;
∴ the L. C. M. of 12, 20, 36 and 54 is 540.

But *in practice* it is generally more convenient to proceed by the following Rule.

Set down the given numbers side by side; divide by any prime number, such as 2, 3, 5, 7, 11 . . . which will exactly divide two at least of the numbers; set down the quotients and the numbers that are not exactly divisible by the divisor, side by side: and proceed in this way till you get a line of numbers, which are prime to one another. Then the continued product of all the divisors and the numbers in this line will be the L. C. M. *required.*

Thus, to find the L. C. M. of 12, 20, 30, 54.

$$\begin{array}{r|l} 2 & 12,\ 20,\ 30,\ 54 \\ \hline 2 & 6,\ 10,\ 15,\ 27 \\ \hline 3 & 3,\ 5,\ 15,\ 27 \\ \hline 5 & 1,\ 5,\ 5,\ 9 \\ \hline & 1,\ 1,\ 1,\ 9 \end{array}$$

$\therefore$ L.C.M. $= 2 \times 2 \times 3 \times 5 \times 9 = 540$.

Examples. (xxvi)

Find the L. C. M. of

(1) 6, 9, 24, 40. (2) 8, 12, 22, 55.
(3) 12, 18, 96, 144. (4) 16, 30, 48, 56, 72.
(5) 84, 156, 63, 99. (6) 27, 33, 54, 69, 132.
(7) 17, 51, 119, 210. (8) 15, 26, 39, 65, 180.
(9) 44, 126, 198, 280, 330. (10) 50, 338, 675, 702, 975.

NOTE.—Any one of the numbers, which is exactly contained in any other of the numbers, may be omitted in finding the L. C. M. Thus in Example (6) it will be sufficient to find the L. C. M. of 54, 69, 132.

Examination Papers.

I.

(1) From ten millions ten thousand and ten take seven hundred and sixty-eight thousand and eight.

(2) Multiply two millions fourteen thousand and eight by seven hundred and sixty-nine thousand and seventy, and prove the result by division.

(3) Find the H. C. F. of 3432 and 3575.

(4) Find the L. C. M. of 48, 132, 63, 99.

(5) The remainder in a division sum is 2099, the divisor 2342, and the quotient 28701 ; find the dividend.

II.

(1) Multiply 72122 by 8193, and show that either of these numbers can be multiplied by the other in three lines of multiplication.

(2) Divide 1834267 by 36, using short division.

(3) Find the H. C. F. of 1875 and 2425.

(4) Find the L. C. M. of 6, 18, 22, 99.

(5) Write down all the numbers of 4 digits which you can form with the figures 3, 6, 0, 4, and add them together.

III.

(1) Subtract 847 from 2029, and explain the process.

(2) Divide 79875 by 63, by short division, and explain the process.

(3) Write in words 10020030 and 10020030040050.

(4) Find the H. C. F. of 1632 and 2976.

(5) Find the L. C. M. of 3, 8, 6, 14, 28, 32.

IV.

(1) Multiply 31 by 47, and explain the process.

(2) Divide thirteen thousand five hundred and seventy-seven millions, one hundred and thirteen thousand and four by seven millions, four hundred and three thousand and six.

(3) Find the H. C. F. of

$$13 \times 17 \times 19,\ 17 \times 19 \times 21, \text{ and } 19 \times 21 \times 13.$$

(4) Find the L. C. M. of

$$13 \times 17 \times 19,\ 17 \times 19 \times 21, \text{ and } 19 \times 21 \times 13.$$

(5) There are two numbers of which the product is 373625, and the greater number is 875; find the sum of the two numbers.

V.

(1) Divide one thousand and forty-six millions, six hundred and eighty-nine thousand, eight hundred and thirty-nine by eighty millions, five hundred and fourteen thousand, six hundred and three.

(2) Find the sum of the products, two and two, of 359, 237, 856.

(3) Find the H. C. F. of 1019527 and 1231845.

(4) Find the L. C. M. of 12, 18, 26, 117, and 312.

(5) Multiply 709514 by 381569, and express the result in words.

VI.

(1) Express in figures twenty-five millions thirty-one thousand and seven; and in words 176500301.

(2) Multiply 905741 by 518963, and express the result in words.

(3) The quotient obtained by dividing 42146675 by a certain number is 743; what is the divisor?

(4) Find the L. C. M. of 1, 3, 5, 9, 12, 14, 16, 96, 128.

(5) Find the H. C. F. of 42435, 195615, and 233910.

VII.

(1) What do you understand by 34?

(2) Express in figures two hundred and one millions ninety-six thousand three hundred and forty-two; and divide it by fifty-four thousand three hundred and twenty-one.

(3) Divide 964374 by 165, by short division, and explain the process.

(4) Find the continued product of 12, 37, and 59.

(5) Express in their elementary factors 28, 45, 135, and 243.

VIII.

(1) Add together three hundred and three millions sixty-six thousand and ninety-nine, and thirty-three thousand and sixty millions six hundred and sixty thousand nine hundred and ninety. Divide the sum by thirty-three, and express the quotient in words.

(2) Divide 830728 by 231, by short division, and explain the process.

(3) Find the continued product of 250, 375, and 14000.

(4) Express in their elementary factors 87, 176, and 576.

(5) Find the H. C. F. of 504, 5292, and 3040.

IX.

(1) Subtract two thousand two hundred and two millions two thousand and two from eight thousand six hundred and sixty-six millions sixty-six thousand and sixty-six. Divide the difference by sixty-four, and express the quotient in words.

(2) How many times can 369 be subtracted from 2738, and what will be the final remainder?

(3) The product of two numbers is 1270374, and half of one of them is 3129, what is the other?

(4) The digits in the units' and millions' places of a number are 2 and 7 respectively, what will be the digits in the same places when 999999 is subtracted from the number?

(5) Find the L. C. M. of 552, 575, and 920.

X.—On Fractions.

52. Numbers are the measures of quantities.

A *Quantity* is anything, which may be regarded as being made up of parts like the whole.

Thus a sum of money is a quantity, because we may regard it as made up of parts like the whole.

To measure any quantity we fix upon some known quantity of the same kind for our Standard or UNIT, and the NUMBER, which expresses how many times this Unit is contained in the quantity, is called the MEASURE of the quantity.

To put this in a more *practical* shape, we give the following illustration: We measure large sums of money by the *Unit* which we call *a Pound*, and when we say that a man's income is *five hundred a year*, we mean, that he receives yearly a sum of money, which contains the unit five hundred times, and we call the Number Five Hundred the *measure* of his income.

53. Now we can conceive that a unit of measurement may be divided into a number of parts of *equal* magnitude.

For instance, if we take a Pound as the *Unit*, by which we measure sums of money, we suppose this unit to be divided into twenty equal parts, and we call each of these parts *one-twentieth* of a Pound; *two* such parts will be two-twentieths, *three* will be three-twentieths of a Pound. Such parts are called Fractions of a Pound, or other Unit, and we give the following definition:

DEF. We form a Fraction by dividing a unit into some number of equal parts and by taking one or more of those parts.

The number of equal parts, into which the Unit is divided, is called the DENOMINATOR of the fraction, and the number expressing how many of these parts are taken to form the fraction is called the NUMERATOR of the fraction.

These operations are denoted by the following symbols: we represent a fraction by writing the numerator above the denominator, and separating them by a horizontal line.

Thus $\frac{3}{4}$ represents the fraction, of which the numerator is 3 and the denominator 4.

Such Symbols are called Fraction-Symbols, or, for brevity, Fractions.

54. The symbol $\frac{1}{2}$ is read one-half.

The symbol $\frac{1}{3}$ is read one-third.

The symbol $\frac{3}{4}$ is read three-fourths.

The symbol $\frac{6}{7}$ is read six-sevenths,

and so on.

55. The Numerator and Denominator of a Fraction are called the TERMS of the fraction.

A PROPER fraction is one, in which the Numerator is less than the Denominator, as $\frac{3}{5}$.

An IMPROPER fraction is one, in which the Numerator is greater than the Denominator, as $\frac{7}{4}$.

In our explanation of the fundamental operations performed with fractions we shall make use, as far as is possible, of *proper* fractions only.

56. To show that $\frac{2}{3} = \frac{8}{12}$.

Suppose a UNIT to be divided into 3 equal parts.

Then $\frac{2}{3}$ will represent 2 of these parts .. .. .. (1).

Next, let each of the 3 parts be subdivided into 4 equal parts.

Thus the UNIT has been divided into 12 equal parts, and $\frac{8}{12}$ will represent 8 of these subdivisions .. .. .. (2).

Now 1 of the parts in (1) is equal to 4 of the subdivisions in (2).

$\therefore$ 2 parts are equal to 8 subdivisions,

and $\therefore \frac{2}{3} = \frac{8}{12}$

We draw from this proof two inferences:

I. If the numerator and denominator of a fraction

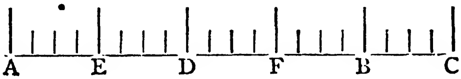
be *multiplied* by the same number, the value of the fraction is not altered.

Thus $\frac{3}{7} = \frac{12}{28}$, and $\frac{4}{15} = \frac{40}{150}$

II. If the numerator and denominator of a fraction be *divided* by the same number, the value of the fraction is not altered.

Thus $\frac{14}{20} = \frac{7}{10}$, and $\frac{90}{100} = \frac{9}{10}$

57. To make the important theorem established in Article 56 more clear, we shall give a *practical* proof that $\frac{4}{5} = \frac{16}{20}$, by taking a straight line as the unit of length.



Let the line AC be divided into 5 equal parts.

Then, if B be the point of division nearest to C,

AB is $\frac{4}{5}$ of AC.. .. ..(1).

Next, let each of the parts be subdivided into 4 equal parts.

Then AC contains 20 of these subdivisions,

and AB contains 16 of these subdivisions;

$\therefore$ AB is $\frac{16}{20}$ of AC .. ..(2).

Comparing (1) and (2), we conclude that

$$\frac{4}{5} = \frac{16}{20}$$

58. A fraction is in its *Lowest Terms* when the numerator and denominator have no common factor except unity:

Thus $\frac{2}{3}$, $\frac{5}{7}$, $\frac{17}{19}$ represent fractions in their lowest terms.

To reduce a fraction to its lowest terms we have the following Rule:

Divide the Numerator and Denominator by their H. C. F.

Thus, if we have to reduce $\frac{18}{81}$ to its lowest terms, we know that 9 is the H. C. F. of 18 and 81, and dividing the numerator and denominator by 9, we have as the resulting fraction $\frac{2}{9}$.

Again, to reduce $\frac{25}{500}$ to its lowest terms, we find 25 to be the H. C. F. of 25 and 500, and therefore $\frac{1}{20}$ will be the reduced fraction.

When we see, by inspection, or by an application of the tests of divisibility given in Art. 39, that a factor is common to both Numerator and Denominator, we may divide both by this factor and reduce the fraction to *lower* terms, without going through the process of finding the H. C. F.

Thus, to reduce the fraction $\frac{270}{9360}$, we see that both terms are divisible by 10, and $\therefore \frac{270}{9360} = \frac{27}{936}$

Now 27 and 936 are both divisible by 9 (Art 39),

$$\text{and } \therefore \frac{27}{936} = \frac{3}{104}$$

Examples. (xxvii)

Reduce to their lowest terms the following fractions:

(1) $\frac{24}{80}$ (2) $\frac{72}{280}$ (3) $\frac{42}{210}$ (4) $\frac{192}{570}$

(5) $\frac{5184}{6912}$ (6) $\frac{1680}{1920}$ (7) $\frac{6409}{7395}$ (8) $\frac{319}{5687}$

(9) $\frac{9495}{15615}$ (10) $\frac{3178}{5221}$

59. Two Fractions may be replaced by two equivalent fractions with a Common Denominator by the following rule:

Find the L. C. M. *of the denominators of the given fractions.*
Divide the L. C. M. *by the denominator of each fraction.*
Multiply the first Numerator by the first Quotient.
Multiply the second Numerator by the second Quotient.
The two Products will be the Numerators of the equivalent fractions, whose common denominator is the L. C. M. *of the original denominators.*

The same rule holds for three, four, or more fractions.

Ex. (1). Reduce to equivalent fractions with the lowest common denominator $\frac{3}{8}$ and $\frac{4}{7}$

Denominators 8, 7.
L. C. M. 56.
Quotients 7, 8.
New numerators 21, 32.
Equivalent fractions $\frac{21}{56}$ $\frac{32}{56}$

Ex. (2). Reduce to equivalent fractions with the lowest common denominator

$$\frac{2}{3} \quad \frac{4}{9} \quad \frac{13}{72}$$

Denominators 3, 9, 72.
L. C. M. 72.
Quotients 24, 8, 1.
New numerators 48, 32, 13.
Equivalent fractions $\frac{48}{72}$ $\frac{32}{72}$ $\frac{13}{72}$

Examples. (xxviii)

Reduce to equivalent fractions with the lowest common denominator

(1) $\frac{3}{4}$ $\frac{5}{7}$ (2) $\frac{4}{9}$ $\frac{5}{18}$ $\frac{7}{27}$ (3) $\frac{3}{5}$ $\frac{4}{7}$ $\frac{6}{11}$

(4) $\frac{5}{12}$ $\frac{13}{20}$ $\frac{17}{80}$ $\frac{19}{120}$ (5) $\frac{4}{7}$ $\frac{15}{17}$ $\frac{26}{51}$ $\frac{65}{102}$

(6) $\frac{1}{3}$ $\frac{3}{5}$ $\frac{1}{6}$ $\frac{5}{18}$ (7) $\frac{3}{10}$ $\frac{5}{27}$ $\frac{7}{90}$ $\frac{11}{360}$

60. To compare the values of two or more fractions, we convert them into equivalent fractions with a common denominator: then the comparison of the values of the original fractions can be made by comparing the numerators of the new fractions.

For example, to compare the values of $\frac{2}{3}$ $\frac{3}{4}$ and $\frac{5}{7}$

The equivalent fractions are $\frac{56}{84}$ $\frac{63}{84}$ $\frac{60}{84}$

The descending order of value of the numerators is 63, 60, 56;

∴ the descending order of value of the given fractions is

$$\frac{3}{4} \quad \frac{5}{7} \quad \frac{2}{3}$$

61. We may also compare fractions by reducing them to fractions with a common *Numerator*, and assigning the greatest value to that one of the resulting fractions which has the *least* denominator.

Thus to compare the values of

$$\frac{3}{5} \quad \frac{27}{31} \text{ and } \frac{81}{95}$$

The equivalent fractions are

$$\frac{81}{135} \quad \frac{81}{93} \text{ and } \frac{81}{95}$$

∴ the descending order of the given fractions is

$$\frac{27}{31} \quad \frac{81}{95} \quad \frac{3}{5}$$

Examples. (xxix)

Compare the values of

(1) $\frac{3}{7}$ $\frac{4}{5}$ $\frac{9}{13}$ (2) $\frac{5}{6}$ $\frac{7}{9}$ $\frac{12}{17}$

(3) $\frac{9}{11}$ $\frac{13}{15}$ $\frac{17}{21}$ (4) $\frac{3}{20}$ $\frac{7}{40}$ $\frac{11}{70}$

(5) $\frac{7}{33}$ $\frac{9}{43}$ $\frac{11}{53}$ (6) $\frac{2}{17}$ $\frac{5}{34}$ $\frac{7}{51}$

ADDITION OF FRACTIONS.

62. The rule for adding two or more fractions together is this:

Reduce the Fractions to equivalent fractions having the Lowest Common Denominator.

Then add *the numerators of the equivalent fractions and place the result as the Numerator of a fraction, whose Denominator is the common denominator of the equivalent fractions.*

The fraction will be equal to the sum of the original fractions.

For example, to find the sum of $\frac{1}{3}$ and $\frac{1}{4}$

$$\frac{1}{3} = \frac{4}{12} \text{ and } \frac{1}{4} = \frac{3}{12}$$

$$\therefore \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Examples. (xxx)

Find the sum of the following fractions:

(1) $\frac{1}{7}$ and $\frac{2}{5}$ (2) $\frac{2}{3}$ and $\frac{3}{14}$

(3) $\frac{2}{7}$ $\frac{3}{11}$ and $\frac{2}{33}$ (4) $\frac{4}{9}$ $\frac{5}{14}$ and $\frac{5}{42}$

(5) $\frac{5}{13}$ $\frac{2}{39}$ $\frac{25}{78}$ and $\frac{7}{156}$

(6) $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ and $\frac{1}{32}$

(7) $\frac{5}{11}$ $\frac{2}{27}$ $\frac{7}{55}$ and $\frac{4}{135}$

(8) $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{7}$ $\frac{1}{9}$ and $\frac{1}{11}$

(9) $\frac{2}{3}$ $\frac{3}{40}$. $\frac{7}{240}$ and $\frac{11}{2880}$

SUBTRACTION OF FRACTIONS.

63. The rule for subtracting a fraction from a greater fraction is this:

Reduce the fractions to equivalent fractions having the Lowest Common Denominator. Then subtract *the numerator of the smaller of the equivalent fractions from the numerator of the greater, and place the result as the Numerator of a fraction, whose denominator is the common Denominator of the equivalent fractions. This fraction will be equal to the difference of the original fractions.*

For example, to find the difference between $\frac{2}{3}$ and $\frac{5}{7}$

$$\frac{2}{3} = \frac{14}{21} \text{ and } \frac{5}{7} = \frac{15}{21}$$

$$\therefore \frac{5}{7} - \frac{2}{3} = \frac{15}{21} - \frac{14}{21} = \frac{1}{21}$$

Examples. (xxxi)

Find the difference of the following fractions:

(1) $\frac{4}{5}$ and $\frac{5}{6}$ (2) $\frac{3}{7}$ and $\frac{15}{19}$

(3) $\frac{11}{12}$ and $\frac{12}{13}$ (4) $\frac{13}{51}$ and $\frac{235}{357}$

(5) $\frac{17}{63}$ and $\frac{29}{108}$ (6) $\frac{9}{38}$ and $\frac{43}{209}$

(7) $\frac{146}{273}$ and $\frac{268}{637}$ (8) $\frac{199}{200}$ and $\frac{359}{360}$

(9) $\frac{347}{1242}$ and $\frac{835}{1998}$

MULTIPLICATION OF FRACTIONS.

64. A fraction is multiplied by a whole number by multiplying the numerator by that number and leaving the denominator unchanged.

Thus $\frac{2}{7}$ multiplied by 3 becomes $\frac{6}{7}$

For each of the symbols $\frac{2}{7}$ and $\frac{6}{7}$ implies that a unit has been divided into 7 equal parts, and three times as many of those parts are taken to form the fraction represented by the latter as are taken to form the fraction represented by the former.

65. *To prove that* $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$

$$\tfrac{2}{3} \text{ of } \tfrac{4}{5} = \tfrac{2}{3} \text{ of } \tfrac{12}{15} \qquad \text{Art. 56.}$$

Now suppose a unit to be divided into 15 equal parts.

Then $\frac{2}{3}$ of $\frac{4}{5} = \frac{2}{3}$ of 12 of such parts.

$= \frac{8}{12}$ of 12 of such parts. Art. 56.

$= 8$ of such parts. Art. 56.

But $\frac{8}{15} = 8$ of such parts, Art. 56.

$\therefore \frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$

Hence we derive the Rule for what is called MULTIPLICATION OF FRACTIONS.

We extend the meaning of the sign $\times$, and define $\frac{2}{3} \times \frac{4}{5}$ (which according to our definition in Art. 21 would have no meaning) to mean $\frac{2}{3}$ of $\frac{4}{5}$, and we conclude that $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$, which in words gives us this rule:

"*Take the product of the numerators to form the Numerator of the resulting fraction, and the product of the denominators to form the Denominator.*"

The same rule holds good for the multiplication of three or more fractions.

Before effecting the Multiplication, common factors should be removed from the Numerator and Denominator. It will be well for the learner to be familiar with the principles laid down in Art. 39.

For example, to find the value of $\frac{14}{25}$ of $\frac{35}{51}$ of $\frac{17}{49}$ we proceed thus:

$$\frac{14}{25} \text{ of } \frac{35}{51} \text{ of } \frac{17}{49} = \frac{14 \times 35 \times 17}{25 \times 51 \times 49}$$

$$= \frac{2 \times 7 \times 5 \times 7 \times 17}{5 \times 5 \times 3 \times 17 \times 7 \times 7}$$

and removing common factors from numerator and denominator,

$$= \frac{2}{5 \times 3}$$

$$= \frac{2}{15}$$

Examples. (xxxii)

Reduce to their simplest form

(1) $\frac{3}{7}$ of $\frac{5}{9}$

(2) $\frac{3}{4} \times \frac{5}{7} \times \frac{9}{11}$

(3) $\frac{42}{65} \times \frac{39}{56} \times \frac{24}{27}$

(4) $\frac{84}{85} \times \frac{102}{117} \times \frac{91}{132}$

(5) $\frac{17}{82} \times \frac{27}{38} \times \frac{123}{153}$

(6) $\frac{100}{161} \times \frac{35}{72} \times \frac{33}{40}$

(7) $\frac{9}{10}$ of $\frac{11}{12}$ of $\frac{13}{14}$ of $\frac{21}{29}$

(8) $\frac{202}{405}$ of $\frac{573}{657}$ of $\frac{248}{1719}$

(9) $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1535}{1068}$

DIVISION OF FRACTIONS.

66. A fraction is divided by a whole number by multiplying the denominator by that number and leaving the numerator unchanged.

Thus $\frac{2}{7}$ divided by 3 becomes $\frac{2}{21}$

For $\frac{2}{7}$ implies that a unit has been divided into 7 equal parts,

$\frac{2}{21}$ implies that a unit has been divided into 21 equal parts,

and hence each part in the former is three times as great as each part in the latter, and since the same number of

parts is taken in both cases, the latter fraction is one-third of the former.

67. *To show that* $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$

The quotient resulting from the division of $\frac{2}{3}$ by $\frac{4}{5}$ is such a number, that when it is multiplied by the divisor $\frac{4}{5}$ the product must be equal to the dividend $\frac{2}{3}$, that is

$$\frac{4}{5} \text{ of the Quotient} = \frac{2}{3}$$

$$\therefore \frac{5}{4} \text{ of } \frac{4}{5} \text{ of the Quotient} = \frac{5}{4} \text{ of } \frac{2}{3}$$

$$\therefore \frac{20}{20} \text{ of the Quotient} = \frac{5}{4} \text{ of } \frac{2}{3}$$

$$\therefore \text{the Quotient} = \frac{5}{4} \text{ of } \frac{2}{3}$$

$$\text{that is, } \frac{2}{3} \div \frac{4}{5} = \frac{5}{4} \text{ of } \frac{2}{3}$$

$$\text{or, } \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$$

Hence we obtain the following rule for what is called DIVISION OF FRACTIONS.

Invert the divisor, and proceed as in Multiplication.

$$\text{Thus } \frac{12}{49} \div \frac{16}{35} = \frac{12}{49} \times \frac{35}{16} = \frac{15}{28}$$

Examples. (xxxiii)

Divide

(1) $\frac{12}{49}$ by $\frac{3}{7}$ (2) $\frac{25}{39}$ by $\frac{10}{13}$

(3) $\frac{85}{256}$ by $\frac{51}{64}$ (4) $\frac{16}{261}$ by $\frac{4}{27}$

(5) $\frac{38}{275}$ by $\frac{133}{385}$ (6) $\frac{91}{369}$ by $\frac{78}{287}$

(7) $\frac{49}{550}$ by $\frac{343}{1450}$ (8) $\frac{625}{1544}$ by $\frac{1251}{2316}$

(9) $\frac{1535}{2421}$ by $\frac{921}{1076}$.

68. Having now established the elementary rules for operations performed with fractions, we proceed to notice some other points belonging to this branch of Arithmetic.

69. A whole number, or integer, can be written as a fraction, by putting 1 beneath the number as a denominator: thus 5 may be written as a fraction, thus, $\frac{5}{1}$.

Also, since $\frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4}$ and so on, it is clear that we can represent a whole number by a fraction, whose denominator is any whole number we please to select.

70. A *Mixed Number* is a number made up of an integer and a fraction, as $4\frac{2}{7}$ This may be read thus, *four and two-sevenths*, and must be regarded as the *sum* of 4 and $\frac{2}{7}$.

A mixed number can be brought into the form of an improper fraction, by multiplying the integer by the denominator of the fraction, adding to the product the numerator of the fraction, and making the sum the numerator of a fraction, of which the denominator is the denominator of the original fraction.

Thus $4\frac{2}{7} = \frac{30}{7}$

Conversely, an improper fraction can be reduced to a mixed number, by dividing the numerator by the denominator, setting down the quotient as the integral part, and making the remainder the numerator of the fractional part of the mixed number, the denominator being the denominator of the original fraction.

Thus $\frac{25}{7} = 3\frac{4}{7}$

For $\frac{25}{7} = \frac{21+4}{7} = \frac{21}{7} + \frac{4}{7} = 3 + \frac{4}{7} = 3\frac{4}{7}$

Examples. (xxxiv)

Convert into improper fractions

(1) $7\frac{4}{9}$ (2) $23\frac{15}{47}$ (3) $216\frac{13}{29}$ (4) $173\frac{19}{1000}$

and into mixed numbers

(5) $\frac{427}{10}$ (6) $\frac{3477}{1000}$ (7) $\frac{4293}{137}$ (8) $\frac{65943}{71}$

71. The rules for the Addition, Subtraction Multiplication and Division of Fractions are applicable to *Improper* Fractions.

Thus $\frac{7}{4} + \frac{13}{10} = \frac{35}{20} + \frac{26}{20} = \frac{61}{20} = 3\frac{1}{20}$

$\frac{9}{5} - \frac{13}{12} = \frac{108}{60} - \frac{65}{60} = \frac{43}{60}$

$\frac{13}{9} \times \frac{27}{26} = \frac{13 \times 27}{9 \times 26} = \frac{13 \times 9 \times 3}{9 \times 13 \times 2} = \frac{3}{2} = 1\frac{1}{2}$

$\frac{117}{110} \div \frac{91}{33} = \frac{117}{110} \times \frac{33}{91} = \frac{9 \times 13 \times 11 \times 3}{11 \times 10 \times 7 \times 13} = \frac{9 \times 3}{10 \times 7} = \frac{27}{70}$

72. In the application of the rules to *Mixed Numbers*, we *may* in all cases change the Mixed Numbers into Improper Fractions, and proceed as in the foregoing Examples. In Division we *must* proceed thus.

For example,

$$4\frac{5}{9} \div 12\frac{3}{10} = \frac{41}{9} \div \frac{123}{10} = \frac{41}{9} \times \frac{10}{123} = \frac{10}{27}$$

$$16 \div 12\frac{4}{5} = \frac{16}{1} \div \frac{64}{5} = \frac{16}{1} \times \frac{5}{64} = \frac{5}{4} = 1\frac{1}{4}$$

In Multiplication it is usually the best course: thus

$$7\frac{2}{3} \times 5\frac{4}{7} = \frac{23}{3} \times \frac{39}{7} = \frac{23 \times 13}{7} = \frac{299}{7} = 42\frac{5}{7}$$

In Addition it is often advantageous to proceed thus:

$$\begin{aligned} 4\tfrac{2}{3} + 3\tfrac{4}{7} &= 4 + \tfrac{2}{3} + 3 + \tfrac{4}{7} \\ &= 4 + 3 + \tfrac{2}{3} + \tfrac{4}{7} \\ &= 7 + \tfrac{14}{21} + \tfrac{12}{21} \\ &= 7 + \tfrac{26}{21} \\ &= 7 + 1\tfrac{5}{21} \\ &= 8\tfrac{5}{21} \end{aligned}$$

and, similarly, when three or more numbers are to be added, we may separate the fractions from the integers, and make a distinct operation for each class.

In Subtraction we can employ the same method, but a little care is necessary. Suppose we have to take

$$3\tfrac{4}{7} \text{ from } 4\tfrac{2}{3}$$

Reducing the *fractional* parts of the numbers to equivalent fractions with a common denominator, we have

$$3\tfrac{12}{21} \text{ and } 4\tfrac{14}{21}$$

We can now take the integral part of the first number from the integral part of the second, and the fractional part of the first from the fractional part of the second, and we have

$$4\tfrac{14}{21} - 3\tfrac{12}{21} = 1\tfrac{2}{21}$$

But suppose we have to take $3\frac{5}{7}$ from $10\frac{2}{5}$

Since $\qquad \frac{5}{7} = \frac{25}{35}$ and $\frac{2}{5} = \frac{14}{35}$

$$\tfrac{5}{7} \text{ is } \textit{greater} \text{ than } \tfrac{2}{5}$$

and we cannot take away the fractional part of $3\frac{25}{35}$ from the fractional part of $10\frac{14}{35}$

We escape from the difficulty by the device of adding *unity* to each expression, to $3\frac{25}{35}$ in the form of 1, and to $10\frac{14}{35}$ in the form of $\frac{35}{35}$

Thus $10\frac{14}{35} - 3\frac{25}{35} = 10\frac{49}{35} - 4\frac{25}{35} = 6\frac{24}{35}$

Take another illustration of a *practical* nature.

From $5\frac{1}{4}d.$ take away $3\frac{3}{4}d.$

We add four farthings, i.e. $\frac{4}{4}$ of a penny, to the former sum, and 1 penny to the latter, and reason thus:

$$5\tfrac{1}{4}d. - 3\tfrac{3}{4}d. = 5\tfrac{5}{4}d. - 4\tfrac{3}{4}d. = 1\tfrac{2}{4}d. = 1\tfrac{1}{2}d.$$

NOTE.—Since it is often *difficult to see whether one fraction is greater or less than another, it is generally*

best to reduce mixed numbers to improper fractions in the case of Subtraction.

Thus $10\frac{2}{5} - 3\frac{5}{7} = \frac{52}{5} - \frac{26}{7} = \frac{364}{35} - \frac{130}{35} = \frac{234}{35} = 6\frac{24}{35}$

Examples. (xxxv)

Simplify the following fractions:

(1) $4\frac{2}{7} \div 3\frac{7}{9}$ (2) $8\frac{3}{5} \div 6\frac{1}{7}$ (3) $104\frac{2}{9} \div 53\frac{7}{13}$

(4) $6\frac{2}{3} \times 9\frac{5}{6}$ (5) $14 \times 3\frac{2}{21}$ (6) $9\frac{4}{15} \times 19\frac{2}{7}$

(7) $2\frac{1}{3} + 3\frac{1}{4}$ (8) $5\frac{2}{7} + 6\frac{3}{11} + 1\frac{4}{15}$

(9) $16\frac{2}{5} + 4\frac{4}{9} + 17\frac{13}{45}$ (10) $4\frac{3}{7} - 2\frac{1}{5}$

(11) $14\frac{4}{9} - 5\frac{7}{8}$ (12) $6\frac{7}{13} - 5\frac{9}{14}$

The following examples should be carefully noticed.

I. From 17 take $4\frac{5}{21}$

$$17 - 4\frac{5}{21} = 16 + 1 - 4\frac{5}{21} = 16 - 4 + 1 - \frac{5}{21}$$
$$= 12 + \frac{16}{21} = 12\frac{16}{21}$$

II. From 317 take $\frac{5}{49}$

$$317 - \frac{5}{49} = 316 + 1 - \frac{5}{49} = 316 + \frac{44}{49} = 316\frac{44}{49}$$

III. Multiply $\frac{999}{1000}$ by 397.

Since $\frac{999}{1000} = 1 - \frac{1}{1000}$

$$397 \times \frac{999}{1000} = 397 - \frac{397}{1000} = 396 + 1 - \frac{397}{1000}$$
$$= 396 + \frac{603}{1000} = 396\frac{603}{1000}$$

73. A Compound Fraction is defined to be the fraction of a fraction.

Thus $\frac{2}{3}$ of $\frac{5}{7}$, and $\frac{3}{4}$ of $2\frac{1}{4}$ of $5\frac{2}{7}$ are compound fractions.

They are reduced to simple fractions by the process of Multiplication.

Thus $\frac{3}{4}$ of $2\frac{1}{4}$ of $5\frac{2}{7} = \frac{3}{4} \times \frac{9}{4} \times \frac{37}{7} = \frac{3 \times 9 \times 37}{4 \times 4 \times 7} = \frac{999}{112} = 8\frac{103}{112}$

74. A Complex Fraction is one, of which the Numerator or Denominator is itself a fraction or mixed number.

Thus $\dfrac{\frac{3}{4}}{7}$ $\dfrac{2}{\frac{5}{9}}$ and $\dfrac{4\frac{2}{7}}{5\frac{3}{8}}$ are complex fractions.

They are reduced to simple fractions by the process of Division.

Thus $\dfrac{\frac{3}{4}}{7} = \frac{3}{4} \div 7 = \frac{3}{4} \div \frac{7}{1} = \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$

and $\dfrac{2}{\frac{5}{9}} = 2 \div \frac{5}{9} = \frac{2}{1} \times \frac{9}{5} = \frac{18}{5} = 3\frac{3}{5}$

Examples. (xxxvi)

Simplify the following fractions :

(1) $\frac{3}{4}$ of $5\frac{1}{9}$ of $7\frac{1}{5}$ (2) $4\frac{3}{7}$ of $11\frac{1}{3}$ of 15

(3) $\frac{7}{8}$ of $2\frac{5}{9}$ of $3\frac{4}{7}$ of 90 (4) $\dfrac{\frac{2}{5}}{\frac{7}{8}}$ (5) $\dfrac{6\frac{2}{9}}{4\frac{2}{3}}$

(6) $\dfrac{14}{3\frac{1}{2}}$ (7) $\dfrac{30\frac{1}{4}}{11}$ (8) $\dfrac{16\frac{2}{3}}{\frac{25}{39}}$

ON THE USE OF BRACKETS.

75. When an expression is inclosed in a bracket ( ), it is intended to show that the whole of the expression is affected by some symbol, which precedes, or follows, the bracket.

Thus, $24 \times (3\frac{1}{2} + 7\frac{1}{4})$ means, that 24 times the sum of the numbers $3\frac{1}{2}$ and $7\frac{1}{4}$ is to be taken, which we may effect by combining $3\frac{1}{2}$ and $7\frac{1}{4}$ by addition, and multiplying the result by 24.

Again, $2\frac{5}{7} \div (4\frac{3}{4} - 2\frac{1}{2})$ signifies, that $2\frac{5}{7}$ is to be divided by the difference between $4\frac{3}{4}$ and $2\frac{1}{2}$; and therefore the result will be

$2\frac{5}{7} \div 2\frac{1}{4}$ or $\frac{19}{7} \div \frac{9}{4}$ or $\frac{19}{7} \times \frac{4}{9}$ or $\frac{76}{63}$

And, generally, we may say, that when numbers are included in a bracket, the expression, within the bracket, must be brought into the simplest form, before combining it with expressions, not in the bracket.

76. The methods of denoting a bracket are various; thus, the marks [ ] and { } are often employed. Brackets are made to inclose one another, as in the expression

$$3 \div [2 + 3 \div \{4 + 5 \div (2 + \tfrac{1}{3})\}]$$

In removing such brackets it is best to commence with the *innermost*, and to remove the brackets one by one, thus

$$\begin{aligned} &3 \div [2 + 3 \div \{4 + 5 \div (2 + \tfrac{1}{3})\}] \\ &= 3 \div [2 + 3 \div \{4 + 5 \div \tfrac{7}{3}\}] \\ &= 3 \div [2 + 3 \div \{4 + \tfrac{15}{7}\}] \\ &= 3 \div [2 + 3 \div \tfrac{43}{7}] \\ &= 3 \div [2 + \tfrac{21}{43}] \\ &= 3 \div \tfrac{107}{43} = \tfrac{129}{107} \end{aligned}$$

We have worked out this example at length because it will teach the learner how to simplify with neatness a peculiar class of fractions, called *Continued* Fractions, which appear in a form like the following:

$$\cfrac{1}{4 + \cfrac{1}{1 - \cfrac{1}{2 - \frac{9}{16}}}}$$

This fraction, by the aid of brackets, may be represented thus,

$$1 \div [4 + 1 \div \{1 - 1 \div (2 - \tfrac{9}{16})\}]$$

and then we can simplify it by the gradual removal of the brackets, the final result being $\frac{7}{51}$

77. There is another method of simplifying Complex and Continued Fractions, which we may explain by the following examples:

Ex. (1) To simplify $\dfrac{5}{2+\frac{3}{7}}$

Multiply all the terms of the fraction by 7, and it becomes

$$\frac{35}{14+3} \quad \text{or} \quad \frac{35}{17}$$

Ex. (2) To simplify $\dfrac{\frac{2}{3}}{5+\frac{3}{10}}$

Multiply all the terms by 30, and we get

$$\frac{20}{150+9} \quad \text{or} \quad \frac{20}{159}$$

Ex. (3) To simplify $\dfrac{\frac{2}{3}-\frac{3}{7}}{\frac{5}{6}-\frac{5}{14}}$

Multiply all the terms by 42, and we get

$$\frac{28-18}{35-15} \quad \text{or} \quad \frac{10}{20} \quad \text{or} \quad \frac{1}{2}$$

Ex. (4) To simplify $\dfrac{3}{3+\dfrac{4}{9+\frac{2}{7}}}$

$$\frac{3}{3+\dfrac{4}{9+\frac{2}{7}}} = \frac{3}{3+\frac{28}{65}} = \frac{195}{195+28} = \frac{195}{223}$$

Ex. (5) To simplify $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}$

$$\frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}} = \frac{1}{1+\dfrac{1}{1+\frac{2}{3}}} = \frac{1}{1+\frac{3}{5}} = \frac{5}{8}$$

Examples. (xxxvii)

Simplify the following fractions:

(1) $\dfrac{6}{5+\frac{3}{4}}$ (2) $\dfrac{7}{19-\frac{3}{11}}$ (3) $\dfrac{\frac{4}{5}}{7-\frac{2}{3}}$

(4) $\dfrac{6\frac{7}{8}}{11-\frac{5}{12}}$ (5) $\dfrac{\frac{4}{5}-\frac{3}{10}}{\frac{7}{10}-\frac{9}{20}}$ (6) $\dfrac{\frac{11}{16}-\frac{5}{24}}{\frac{7}{12}+\frac{3}{4}}$

(7) $\dfrac{2}{5+\dfrac{6}{9+\frac{3}{4}}}$ (8) $\dfrac{3}{2+\dfrac{1}{3+\frac{1}{3}}}$

(9) $\dfrac{5}{2-\dfrac{1}{4-\frac{2}{5}}}$ (10) $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{2}{5}}}}$

78. If two brackets stand side by side, *with no sign between them*, as $\left(\frac{2}{3}+\frac{3}{4}\right)\left(\frac{5}{6}-\frac{2}{7}\right)$ it is implied that the contents of one bracket are to be *multiplied* by the contents of the other.

The following examples are selected as containing peculiar forms of symbolic representation, which should be carefully noticed

(1) $\frac{3}{5}+\frac{4}{7}$ of $\frac{2}{3}-\frac{3}{10}$

- The first step here is to take the product of $\frac{4}{7}$ and $\frac{2}{3}$, so that the expression becomes $\frac{3}{5}+\frac{8}{21}-\frac{3}{10}$; then add the first two fractions together, and take $\frac{3}{10}$ from the sum.

(2) $\frac{3}{5}+\frac{4}{7}$ of $\left(\frac{2}{3}-\frac{3}{10}\right)$

First take the difference of $\frac{2}{3}$ and $\frac{3}{10}$, multiply the result by $\frac{4}{7}$, and add the product to $\frac{3}{5}$

(3) $\frac{2}{3}\times\frac{4}{5}\div\frac{6}{7}$.

First simplify $\frac{2}{3}\times\frac{4}{5}$, the result being $\frac{8}{15}$

Then divide $\frac{8}{15}$ by $\frac{6}{7}$, the result being $\frac{8}{15}\times\frac{7}{6}$, or $\frac{28}{45}$

(4) $\frac{2}{3} \div \frac{4}{5} \times \frac{6}{7}$

First simplify $\frac{2}{3} \div \frac{4}{5}$, the result being $\frac{2}{3} \times \frac{5}{4}$, or $\frac{5}{6}$

Then multiply $\frac{5}{6}$ by $\frac{6}{7}$, the result being $\frac{5}{7}$

Examples. (xxxviii)

Simplify the following expressions:

(1) $3\frac{2}{5} \div (2\frac{1}{3} + 1\frac{5}{7})$ (2) $(4\frac{3}{11} + 2\frac{1}{5}) \div 35\frac{3}{5}$

(3) $\dfrac{2}{1 + \dfrac{5}{7 + \frac{2}{3}}}$ (4) $\dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{2}}}$

(5) $5 \dfrac{3}{4 - \dfrac{5}{7 + \frac{2}{5}}}$ (6) $\frac{3}{7} + \frac{5}{9}$ of $\frac{3}{10} - \frac{2}{21}$

(7) $\frac{2}{3}$ of $\frac{5}{6} + \frac{3}{7} \div \frac{4}{5}$ (8) $(\frac{11}{13} \div \frac{3}{7})$ of $7\frac{7}{12} - 1\frac{3}{5}$

(9) $(\frac{4}{9} - \frac{3}{11})(2\frac{3}{4} + 3\frac{2}{3})$ (10) $(\frac{3}{13} - \frac{2}{39}) \div (\frac{5}{78} + \frac{7}{156})$

(11) $\dfrac{(2 + \frac{1}{5}) \div (3 + \frac{1}{7})}{(\frac{1}{2} - \frac{1}{3}) \times (4 - 3\frac{3}{7})}$ (12) $\dfrac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$

79. We shall conclude this Chapter with a set of Miscellaneous Examples on Fractions.

Examples. (xxxix)

(1) Add together

$$\frac{17}{33} \quad \frac{5}{12} \quad \frac{8}{44} \quad \frac{3}{28} \quad \frac{15}{56}$$

(2) Add

$$\frac{2}{5} \text{ of } \frac{3}{7} \text{ to } \frac{3}{7} \text{ of } 2\frac{1}{3}$$

and multiply the result by

$$(\frac{2}{3} \text{ of } \frac{5}{6}) \div (\frac{5}{4} + \frac{4}{5})$$

(3) Subtract

$$\frac{2}{3} \text{ of } \frac{5}{6} \text{ from } 1\frac{1}{2} \text{ of } \frac{4}{9}$$

and divide the result by

$$(\frac{2}{3} - \frac{4}{7}) \times (\frac{4}{5} - \frac{5}{8})$$

(4) Simplify the fractions $\frac{221}{629}$ $\frac{72816}{8528}$ and find their product.

(5) Divide the product of $3\frac{2}{5}$ and $3\frac{3}{7}$ by the product of $1\frac{5}{7}$ and $1\frac{13}{21}$

(6) Multiply together the fractions $4\frac{1}{3}$ $2\frac{3}{4}$ and add the result to $4\frac{3}{4} + 3\frac{1}{3}$

(7) Multiply the difference between $\frac{21}{71}$ and $\frac{281}{781}$ by the sum of $4\frac{7}{15}$ and $1\frac{3}{8}$; and multiply the result by the difference between $10\frac{2}{5}$ and $5\frac{2}{3}$

(8) Simplify

$$\left(\tfrac{1}{3} + \tfrac{4}{7}\right)\frac{20\frac{1}{4}}{3\frac{6}{7} + 2\frac{1}{4}}$$

(9) Simplify

$$\left(3\tfrac{4}{5} + 5\tfrac{1}{9} - \tfrac{1}{45}\right)\left(4\tfrac{1}{5} - 3\tfrac{1}{4}\right)$$

divided by

$$1\tfrac{5}{11} + 2\tfrac{1}{8} - \left(2\tfrac{9}{16} - \tfrac{1}{8} - \tfrac{1}{22}\right)$$

(10) Simplify

$$\left(1\tfrac{1}{3} + 2\tfrac{2}{7}\right)\left(\frac{5\frac{1}{16}}{4\frac{9}{7} + 1\frac{1}{4}}\right)$$

(11) Simplify

$$\left(7\tfrac{1}{9} + 1\tfrac{4}{5} - \tfrac{1}{45}\right)\left(2\tfrac{1}{4} - \tfrac{4}{5}\right)$$

divided by

$$4\tfrac{1}{8} - \tfrac{6}{11} - \left(2\tfrac{7}{8} - \tfrac{7}{16} - \tfrac{1}{22}\right)$$

(12) Simplify

$$\frac{6\frac{3}{4} - 1\frac{5}{14}}{2\frac{1}{6} + 1\frac{3}{7}} \qquad \text{and } \left(\tfrac{5}{7} \text{ of } 1\tfrac{6}{13}\right) \div \frac{2\frac{5}{7}}{3\frac{1}{4}}$$

(13) Simplify

$$\frac{1}{4 - \dfrac{1}{2 - \dfrac{1}{1 - \frac{5}{12}}}} \qquad \text{and } \frac{1}{4 + \dfrac{1}{1 - \dfrac{1}{2 - \frac{9}{16}}}}$$

(14) Simplify

$$\frac{10\frac{2}{5} - 1\frac{5}{7}}{7\frac{1}{8} + 3\frac{3}{40}} \qquad \text{and } \left(\tfrac{3}{7} \text{ of } 2\tfrac{1}{17}\right) \div \frac{1\frac{2}{3}}{2\frac{3}{7}}$$

(15) Simplify

$$\frac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{13}{15} + 7\frac{7}{8} - 6\frac{6}{7}} \qquad \text{and } 1\tfrac{21}{43} \times \tfrac{37797}{75008}$$

(16) Simplify

$$\frac{5 - \dfrac{1}{5 - \frac{1}{5}}}{3 - \dfrac{1}{3 - \frac{1}{3}}} \times \tfrac{9}{23} \text{ of } 7 \text{ and } \frac{6 + \dfrac{1}{6 - \frac{1}{6}}}{4 - \dfrac{1}{4 - \frac{1}{4}}} \times 10\tfrac{8}{9}$$

(17) Simplify

$$\frac{8\frac{3}{5} - 7\frac{3}{4} + 5\frac{2}{3} - 4\frac{1}{2}}{13 - 11\frac{9}{10} + 10\frac{7}{9} - 9\frac{17}{20}} \times \tfrac{2}{11} \text{ of } 365.$$

(18) Simplify

$$\frac{\frac{1}{21} \times 5\frac{17}{23} \times 6\frac{3}{11} + 6\frac{10}{51} \times 1\frac{23}{49} \div 2\frac{5}{17} + 1\frac{20}{49}}{9\frac{10}{57} \times 1\frac{22}{23} \div 5\frac{17}{38} + 3\frac{11}{78} \times 6\frac{17}{21} \div 7\frac{21}{52}} \times 12\tfrac{4}{9}$$

(19) Simplify

$$\frac{\frac{1}{23} \text{ of } 6\frac{13}{17} \text{ of } 24\frac{11}{13} - 4\frac{13}{18} \times 3\frac{33}{34} \div 3\frac{37}{96}}{8\frac{17}{19} \times 5\frac{14}{39} \div 4\frac{15}{32} - 7\frac{19}{20} \times 5\frac{11}{65} \div 14\frac{21}{25}} \times 4\tfrac{8}{23}$$

(20) Simplify

$$\frac{19}{7 \times \dfrac{2}{3 - 1\frac{2}{3}}} \times \tfrac{7735}{67184} \div \left(1\tfrac{3}{16} - \tfrac{47}{48}\right)$$

(21) Simplify

$$\frac{1}{2 + \dfrac{3}{4 + \frac{5}{6}}} \times \tfrac{48627}{4147} \div \left(1\tfrac{1}{2} - \tfrac{23}{33}\right)$$

(22) Simplify

$$\frac{\dfrac{7}{4 - \frac{5}{6}} - \dfrac{5}{6 - \frac{3}{8}}}{\dfrac{4}{7 - \frac{4}{7}} + \dfrac{2}{4 - \frac{2}{3}}} \times \frac{\dfrac{1}{\frac{1}{2} - \frac{27}{50}} - 13}{19 - \dfrac{1}{\frac{1}{2} - \frac{6}{31}}}$$

(23) Simplify

$$\frac{\dfrac{2}{3-\frac{4}{5}}+\dfrac{3}{4-\frac{5}{6}}}{\dfrac{3}{2-\frac{1}{4}}-\dfrac{1}{3-\frac{4}{5}}}\times\frac{\dfrac{1}{\frac{2}{3}-\frac{2}{9}}-\dfrac{1}{1\frac{1}{2}-\frac{19}{22}}}{\dfrac{1}{1\frac{1}{7}-\frac{1}{8}}-\dfrac{2}{6\frac{3}{5}-2\frac{5}{11}}}$$

XI. Decimal Fractions.

80. The Multiples of 10 are 10, 20, 30, 40, 50, and so on. (Art. 40.)

The Powers of 10 are 10, 100, 1000, 10000, and so on, and these are called the first, second, third, fourth .... powers of 10. (Art. 27.)

81. A Fraction, which has for its denominator one of the Powers of 10, is called a Decimal Fraction, or, for shortness' sake, a Decimal. All other fractions are, by way of distinction, called Vulgar Fractions.

82. To save the trouble of writing the denominators of decimal fractions, a method of notation is used, by which we can express the value of the denominator in every case.

This method will be best explained by the following examples:

·3 stands for $\frac{3}{10}$, and is read thus, *three-tenths.*

·25 stands for $\frac{25}{100}$, and is read thus, *twenty-five hundredths.*

·347 stands for $\frac{347}{1000}$, and is read thus, *three hundred and forty-seven thousandths.*

The figures which follow the Point · are those which form the *Numerator* of the fraction in each case.

The *number* of the figures, which follow the Point, corresponds to the number denoting the particular Power of 10, which forms the *Denominator* of the fraction in each case.

Now, as the first power of 10 is 1 followed by one zero, and the second power of 10 is 1 followed by two zeros, and the third power of 10 is 1 followed by three zeros, and so on, we can in every case write the denominator, by affixing to 1 a number of zeros, equal to the number of figures that follow the Point.

Thus, $\cdot 426789$ stands for $\frac{426789}{1000000}$

six zeros being affixed to the 1, because the number of figures that follow the Point is in this case *six*.

Again,

$$\cdot 07 \text{ stands for } \tfrac{7}{100}$$
$$\cdot 005 \text{ stands for } \tfrac{5}{1000}$$
$$\cdot 00025 \text{ stands for } \tfrac{25}{100000}$$

the zeros, which come between the Point and the figures 7, 5, and 25, not being set down in the numerators of the fractions, as having no effect on the value of the numerators, seeing that 07 and 7 stand for the same number, and that 005 and 5 stand for the same number.

But these zeros affect the value of the denominators, as for instance

$$\cdot 7 = \tfrac{7}{10} \text{ while } \cdot 07 = \tfrac{7}{100} \text{ and } \cdot 007 = \tfrac{7}{1000}$$

83. Zeros *affixed* to a decimal have no effect on its value: that is,

$$\cdot 7,\ \cdot 70,\ \cdot 700 \text{ are all equal;}$$

$$\text{for } \cdot 7 = \tfrac{7}{10}$$
$$\cdot 70 = \tfrac{70}{100} = \tfrac{7}{10}$$
$$\cdot 700 = \tfrac{700}{1000} = \tfrac{70}{100} = \tfrac{7}{10}$$

84. The method of representing Decimal Fractions is merely an extension of the method, by which Integers are represented, as will be seen from the following considerations.

As the *local* value of each digit increases tenfold, as we advance from right to left, so does the local value

of each decrease in the same proportion as we advance from left to right.

If, then, we affix a line of digits to the right of the units' place, each one of these having from its position a value one-tenth part of the value, which it would have if it were one place farther to the left, we shall have on the right hand of the units' place a series of Fractions, of which the denominators are successively 10, 100, 1000, ... while the numerators may be any numbers between 9 and zero.

Thus, $246 \cdot 4789$

$= 2 \times 100 + 4 \times 10 + 6 + \frac{4}{10} + \frac{7}{100} + \frac{8}{1000} + \frac{9}{10000}$

85. A number, made up of an integer and a decimal, as $4 \cdot 5$, may be expressed in a fractional form, by writing as the Numerator all the figures in the number, and as the Denominator 1 followed by as many zeros as there are figures *after the Point.*

Thus, $4 \cdot 5 = \frac{45}{10}$

for $4 \cdot 5 = 4 + \frac{5}{10} = \frac{40}{10} + \frac{5}{10} = \frac{45}{10}$

Again $14 \cdot 075 = \frac{14075}{1000}$

for $14 \cdot 075 = 14 + \frac{75}{1000} = \frac{14000}{1000} + \frac{75}{1000} = \frac{14075}{1000}$

Examples. (xl)

Express, by means of fraction-symbols in their lowest terms,

(1) $\cdot 5$. (2) $\cdot 25$. (3) $\cdot 75$. (4) $\cdot 375$.

(5) $\cdot 00243$. (6) $\cdot 0000725$. (7) $14 \cdot 8$. (8) $104 \cdot 235$.

(9) $50 \cdot 0004$. (10) $100 \cdot 001$.

Express in the abbreviated form

(11) $\frac{9}{10}$ (12) $\frac{37}{100}$ (13) $\frac{4579}{10000}$

(14) $\frac{3}{1000}$ (15) $\frac{17295}{100}$ (16) $\frac{59}{10000000}$

(17) $\frac{25679}{1000000}$ (18) $\frac{325793}{100000}$ (19) $\frac{19}{10000}$

86. We call

·5,	3·7,	15·9	decimal expressions of the *first* order,
·25,	4·39,	143·73	decimal expressions of the *second* order,
·043,	5·006,	27·009	decimal expressions of the *third* order;

the number of the order depending on the number of figures that *follow the Point.*

The number denoting the order we call the INDEX of the order: thus 1 is the index of the *first* order; 2 of the *second* order, and so on.

87. From what is stated in Art. 83 we learn that a decimal of any order may be made into an equivalent decimal of a *higher* order, by affixing one, two, three zeros according as the index of the higher exceeds the index of the lower by 1, 2, 3.

Thus ·43 may be made into an equivalent decimal of the *fifth* order, by affixing *three* zeros, thus, ·43000,

and ·047 may be made into an equivalent decimal of the *seventh* order, by affixing *four* zeros, thus, ·0470000.

ADDITION OF DECIMAL FRACTIONS.

88. To add ·27 to ·45 we might proceed thus,

$$\cdot 27 = \tfrac{27}{100}$$

$$\cdot 45 = \tfrac{45}{100};$$

$$\therefore \ \cdot 27 + \cdot 45 = \tfrac{27}{100} + \tfrac{45}{100} = \tfrac{72}{100} = \cdot 72.$$

But we obtain the same result, if we set down the decimals one under another, Point under Point, add the figures as if they stood for whole numbers, and place the Point in the result under the other Points, thus,

$$\begin{array}{r} \cdot 27 \\ \cdot 45 \\ \hline \cdot 72. \end{array}$$

89. If the decimals to be added be not of the same order, as for instance ·37 and ·049, we reason thus:

·049 is a decimal of the third order,

·37 is a decimal of the second order, but it can be made into an equivalent decimal of the third order, by affixing a cypher, thus, ·370.

Then we proceed to add the decimals thus,

$$\begin{array}{r} \cdot 370 \\ \cdot 049 \\ \hline \cdot 419 \\ \hline \end{array}$$

Now suppose we have to add more than two decimal expressions, as ·0074, ·72, ·05, and ·123456.

Of these four expressions the last is of the *sixth* order, and we may make the other three into equivalent decimals of the sixth order, and set them down thus:

$$\begin{array}{r} \cdot 007400 \\ \cdot 720000 \\ \cdot 050000 \\ \cdot 123456 \\ \\ \cdot 900856 \\ \hline \end{array}$$

When the learner is thoroughly acquainted with the principle, on which this process of addition depends, he may omit the affixed zeros, since they have no effect on the result, and may write the sum just worked out in the following way:

$$\begin{array}{l} \cdot 0074 \\ \cdot 72 \\ \cdot 05 \\ \cdot 123456 \\ \hline \cdot 900856 \\ \hline \end{array}$$

If the numbers to be added be made up of integers

combined with decimals, we keep the Points in a vertical line, and proceed as in addition of integers.

Thus to add 4·27, 15·004, ·9007, and 23, we proceed thus:

4·2700	or thus.	4·27
15·0040		15·004
·9007		·9007
23·0000		23·
43·1747		43·1747

Examples. (xli)

Find the sum of

(1) ·275 and ·425 (2) ·007 and ·2394.

(3) ·001 and ·0002.

(4) 13·279, 3·00046, 742·000372.

(5) ·000493, 3·24, 15, 42·6, 324·42037.

(6) 49·327, ·458, 8317·05, 341·875, 32·4962.

(7) 700·372, 894·0009, ·347, ·00082, 5370·006.

(8) 560·379, ·45687, 350·0036, 7·074, 52·257.

SUBTRACTION OF DECIMAL FRACTIONS.

90. If we have to find the difference between ·47 and ·35, where both decimals are of the same order, and ·47 is the larger of the two, we proceed thus:

From	·47
Take	·35
Result	·12

performing an operation like that of Subtraction of Integers, and keeping the Points in a vertical line.

That this method gives the correct result is evident, for

$$\cdot 47 - \cdot 35 = \tfrac{47}{100} - \tfrac{35}{100} = \tfrac{12}{100} = \cdot 12.$$

91. If we have to find the difference between ·888 and ·9, we may make the latter into a decimal of the third order, thus, ·900, and since this is larger than ·888, we proceed thus:

From	·900
Take	·888
Result	·012

If we have to find the difference between ·998 and 1, we observe that 1, being an integer, must be greater than ·998, which is a Proper Fraction, i. e. $\frac{998}{1000}$, and we proceed thus:

From	1·000
Take	·998
Result	·002

Examples. (xlii)

Find the difference between

(1) 56·429 and 5·218. (2) 9·005 and 7·462.

(3) 53·316 and 5·0867. (4) ·799 and ·8.

(5) 6·047 and 5·9863. (6) 850·007 and 270·8796.

(7) ·0000086 and ·00001. (8) ·00537 and ·000985.

(9) 10 and ·0002. (10) ·09999 and ·101.

MULTIPLICATION OF DECIMALS.

92. In finding the product of ·12 and ·11, we might proceed thus,

$$\cdot 12 \times \cdot 11 = \frac{12}{100} \times \frac{11}{100} = \frac{12 \times 11}{100 \times 100} = \frac{132}{10000} = \cdot 0132,$$

the result being a decimal of the *fourth* order.

Again, if we have to find the product of 4·32 and ·00012,

$$4 \cdot 32 \times \cdot 00012 = \frac{432}{100} \times \frac{12}{100000} = \frac{5184}{10000000} = \cdot 0005184,$$

the result being a decimal of the *seventh* order.

And, generally, the product of any two decimal expressions is a decimal expression of an order, whose index is the sum of the indices of the orders of the two expressions.

Hence we deduce the following rule for Multiplication of Decimals:

Multiply as in the case of integers, and mark off in the product a number of decimal places equal to the sum of the number of decimal places in the two factors.

For example, to multiply 2·4327 by 4·23.

$$\begin{array}{r} 2\cdot4327 \\ 4\cdot23 \\ \hline 72981 \\ 48654 \\ 97308 \\ \hline 10\cdot290321 \end{array}$$

Again, to multiply 43·672 by ·00000047.

$$\begin{array}{r} 43\cdot672 \\ \cdot00000047 \\ \hline 305704 \\ 174688 \\ \hline 2052584 \end{array}$$

We have now to mark off eleven decimal places from this product, and as the product contains only seven figures, we must prefix four zeros, and put the Point on the left of these, thus, ·00002052584, and this will be the required product.

One more case must be considered.

Suppose we have to multiply ·235 by ·48;

$$\begin{array}{r} \cdot235 \\ \cdot48 \\ \hline 1880 \\ 940 \\ \hline \cdot11280 \end{array}$$

This decimal of the *fifth* order is equivalent to a decimal of the *fourth* order $\cdot 1128$ (Art. 83), and this is the simplest form of the result.

Examples. (xliii)

Multiply

(1) $7\cdot 5$ by $4\cdot 7$. (2) $3\cdot 62$ by $5\cdot 23$. (3) $\cdot 427$ by $\cdot 235$.

(4) $\cdot 562$ by $\cdot 00074$. (5) $3\cdot 00704$ by $4\cdot 0205$.

(6) $\cdot 0009$ by 1000. (7) $623\cdot 4075$ by $24\cdot 0259$.

(8) $\cdot 00746$ by $\cdot 006235$. (9) $1432\cdot 6749$ by $\cdot 00004030705$.

(10) $50704\cdot 042$ by $\cdot 004007090061$.

Find the value of the following:

(11) $\cdot 407 \times 4\cdot 03 \times \cdot 006$.

(12) $1\cdot 01 \times 1000 \times \cdot 001$.

(13) $\cdot 52 \times \cdot 007 \times 4\cdot 3 \times \cdot 02$.

Find the continued product of

(14) $\cdot 07$, $4\cdot 6$, $\cdot 009$ and $52\cdot 47$.

(15) $42\cdot 6$, $\cdot 795$, $4\cdot 03$ and $\cdot 00074$.

(16) What is the cube of $2\cdot 74$?

(17) Raise $3\cdot 5$ to the fourth power.

DIVISION OF DECIMALS.

93. If we have to divide $\cdot 27$ by 3, we might proceed thus,

$$\cdot 27 \div 3 = \frac{27}{100} \div 3 = \frac{9}{100} = \cdot 09.$$

Again, if we have to divide $\cdot 00625$ by 25, we might proceed thus,

$$\cdot 00625 \div 25 = \frac{625}{100000} \div 25 = \frac{25}{100000} = \cdot 00025.$$

In both cases *the Quotient is a decimal of the same order as the Dividend.*

Hence we derive the following Rule.

If the Divisor be an integer, perform the operation of Division as if the Dividend were also an integer, and mark off in the Quotient as many decimal places as there are Decimal places in the Dividend.

For example, suppose we have to divide $\cdot00086751$ by 243.

$$\begin{array}{r l} 243)\ \cdot00086751\ (357 & \\ 729 & \\ \hline 1385 & \\ \\ 1701 & \\ 1701 & \\ \hline \end{array}$$

The Quotient is to be a decimal of the *eighth* order,

$\therefore$ the result is $\cdot00000357$.

94. Next observe that, if the Divisor be a *decimal* expression, *we can in every case change it into an Integer*, by a process which we shall now explain.

If we multiply a decimal expression

by 10, the effect is to move the Point one place to the right,

by 100, the effect is to move the Point two places to the right,

by 1000, the effect is to move the Point three places to the right.

and so on,

For instance, $123\cdot456 \times 10 = 1234\cdot56$,

and $123\cdot456 \times 100 = 12345\cdot6$.

The reason is obvious,

for $123\cdot456 \times 10 = \frac{123456}{1000} \times 10 = \frac{123456}{100} = 1234\cdot56$,

and $123\cdot456 \times 100 = \frac{123456}{1000} \times 100 = \frac{123456}{10} = 12345\cdot6$.

Hence we can transform any Divisor into an Integer,

by multiplying it by 10, 100, 1000,.... according as the Divisor is a decimal of the first, second, third....order.

For example, if the Divisor be ·000492, and we multiply it by 1000000, we transform it into the Integer 492.

Now we may multiply a Divisor by any number, if we multiply the Dividend by the same number.

For instance, if the Divisor be 8 and the Dividend 32, we may multiply each by 10,

so that the Divisor becomes 80, and the Dividend 320;

and whether we divide 32 by 8, or 320 by 80, the Quotient will be the same number, that is, 4.

95. We can now lay down a general Rule for Division of Decimals.

If the Divisor be a decimal, change it into an Integer by removing the Point a sufficient number of places to the right, and also remove the Point in the Dividend the same number of places to the right. Divide as in the case of integers. Then, if the dividend be an integer, the quotient will be an integer, and if the dividend be a decimal, the quotient will be a decimal of the same order.

The process will be better understood from the following examples.

Ex. (1). Divide ·625 by ·025.

$$\cdot625 \div \cdot025 = \frac{\cdot625}{\cdot025} = \frac{625\cdot}{025\cdot} = \frac{625}{25}$$

```
25) 625 (25
    50
    ---
    125
    125
    ---
```

Here the Quotient is an *Integer*, because the Dividend is an Integer;

∴ the Quotient required is 25.

Ex. (2). Divide 108·997 by 2·3.

$$108\cdot997 \div 2\cdot3 = \frac{108\cdot997}{2\cdot3} = \frac{1089\cdot97}{23\cdot} = \frac{1089\cdot97}{23}$$

```
23) 1089·97 (4739
    92
    ——
    169
    161
    ———
      89
      69
      ——
      207
      207
      ———
```

Here the Quotient is a decimal of the *second* order. because the Dividend is a decimal of the second order.

∴ the Quotient required is 47·39.

Ex. (3). Divide ·625 by ·00025.

$$\cdot625 \div \cdot00025 = \frac{\cdot625}{\cdot00025} = \frac{62500\cdot}{00025\cdot} = \frac{62500}{25}$$

```
25) 62500 (2500
    50
    ——
    125
    125
    ———
      00
      ——
```

Here the Quotient is an *Integer*, because the Dividend is an Integer;

∴ the Quotient required is 2500.

Ex. (4). Divide ·00169 by 1·3.

$$\cdot00169 \div 1\cdot3 = \frac{\cdot00169}{1\cdot3} = \frac{0\cdot0169}{13\cdot} = \frac{\cdot0169}{13}$$

```
13) ·0169 (13
     13
     ——
     39
     39
     ——
```

Here the Quotient is a decimal of the *fourth* order, because the Dividend is a decimal of the fourth order;

$\therefore$ the Quotient is $\cdot 0013$.

Ex. (5). Divide 625 by $\cdot 25$.

$$625 \div \cdot 25 = \frac{625}{\cdot 25} = \frac{62500\cdot}{25\cdot} = \frac{62500}{25}$$

$$\begin{array}{l} 25)\ 62500\ (2500 \\ \quad\ \ 50 \\ \quad\ \ \overline{125} \\ \quad\ \ 125 \\ \quad\ \ \overline{\quad 00} \end{array}$$

Here the Quotient is an *Integer*, because the Dividend is an Integer.

$\therefore 625 \div \cdot 25 = 2500$.

These are cases of *exact* division, that is, when, on the process of division being carried out, *there is no remainder.*

Examples. (xliv)

Divide

(1) $1\cdot 296$ by $\cdot 108$. (2) $17\cdot 28$ by $\cdot 0012$.

(3) $\cdot 00169$ by $1\cdot 3$. (4) 2921 by $\cdot 23$.

(5) $15633\cdot 0062$ by $362\cdot 9$. (6) 1 by $\cdot 0001$.

(7) $\cdot 03096$ by $\cdot 000072$. (8) $\cdot 7644$ by $\cdot 0052$.

(9) $\cdot 0000615228$ by 307. (10) $746\cdot 44808$ by $7\cdot 58$.

(11) $\cdot 24294591$ by $36\cdot 9$. (12) $63987\cdot 42$ by $\cdot 000073$.

(13) $\cdot 26986365$ by 3500. (14) $26986\cdot 14$ by $\cdot 00009$.

(15) $\cdot 00131053$ by $\cdot 0065$. (16) 617325 by $\cdot 00025$.

(17) $\cdot 830676$ by $\cdot 000231$. (18) $\cdot 00019517$ by 673.

(19) $1\cdot 0191$ by $\cdot 00079$. (20) $2078\cdot 61$ by 579.

(21) $241\cdot 16047$ by $\cdot 527$. (22) $\cdot 65220834$ by $\cdot 00854$.

(23) $4700460\cdot 66583$ by $\cdot 00518963$.

96. We next take the following example.

Divide 347 by ·64.

Here $347 \div \cdot 64 = \frac{347}{\cdot 64} = \frac{34700\cdot}{64\cdot} = \frac{34700}{64}$

and we proceed thus:

```
64) 34700 (542
    320
    ---
     270
     256
     ---
      140
      128
      ---
       12
```

We have then the Quotient 542, and Remainder 12.

If we wish to carry on the division further, we may do so, by placing a decimal point at the end of the Dividend, and affixing as many zeros as we please, observing that all the figures, which will come after those already in the Quotient, will be decimals.

The operation, completed from the outset, will stand thus,

```
64) 34700·0000 (542·1875
    320
    ---
     270
     256
     ---
      140
      128
      ---
       120
        64
       ---
        560
        512
        ---
         480
         448
         ---
          320
          320
          ---
```

Examples. (xlv)

Divide

(1) 7·45 by ·32. (2) 14·327 by 12·8.

(3) 43·26 by 12·5. (4) 7432·974 by ·225.

(5) 1·2 by 625. (6) ·217 by 1250.

97. The student is now to observe that, by employing Short Division, the example just worked out may be put in a very concise form. Thus taking up the work at the point where we have to divide 34700 by 64, we proceed thus:

$$\begin{array}{r|l} 8 & 34700\cdot0 \\ \hline 8 & 4337\cdot5000 \\ \hline & 542\cdot1875 \text{ Quotient.} \end{array}$$

So also, if we have to divide 43672·509 by 36, we proceed thus,

$$\begin{array}{r|l} 4 & 43672\cdot50900 \\ \hline 9 & 10918\cdot12725 \\ \hline & 1213\cdot12525 \text{ Quotient.} \end{array}$$

Again, to divide ·0000013932 by 32, we proceed thus:

$$\begin{array}{r|l} 4 & \cdot0000013932 \\ \hline 8 & \cdot0000003483000 \\ \hline & \cdot0000000435375 \text{ Quotient.} \end{array}$$

NOTE.—Division by 10, 100, 1000 . . . is effected by moving the decimal place in the Dividend one, two, three . . . places to the left.

Thus 24·6 ÷ 10 = 2·46.

·47 ÷ 100 = ·0047.

Examples. (xlvi)

Employ Short Division in finding the Quotient when we divide

(1) 426·478 by 16. (2) ·07849782 by 72.

(3) 362·47 by ·025. (4) ·00007263 by 4·5.

(5) 42·007437 by ·24. (6) ·00463 by 50.

(7) 2·4715 by ·000016. (8) 9000 by ·00036.

(9) ·001 by 100. (10) ·001001001 by 2000.

N.B.—The process of Division may often be shortened by multiplying the Dividend and Divisor by a number which will transform the Divisor into a power or a multiple of 10: thus, if we have to divide 24·46927151 by 12·5, we multiply both by 8.

Then $\frac{24\cdot46927151}{12\cdot5} = \frac{195\cdot75417208}{100} = 1\cdot9575417208.$

98. In the Examples hitherto given the cases are all those of *exact* division.

In all cases we may proceed with the division, till there is no remainder, or till certain figures in the Quotient recur again and again in the same order.

We shall give an example of this recurrence of figures in Art. 99, but first we must observe that we often require to find the Quotient *up to a certain place of decimals.*

For example, suppose we have to find the Quotient arising from the division of 2·47 by ·37, to four places of decimals.

$$2\cdot47 \div \cdot37 = \frac{2\cdot47}{\cdot37} = \frac{247}{37}$$

```
37) 247·0000 (6·6756
    222
    ---
     250
     222
     ---
      280
      259
      ---
       210
       185
       ---
        250
        222
        ---
```

Hence the Quotient, correct to four places of decimals, is 6·6756.

Examples. (xlvii)

Find the Quotient to three places of decimals when we divide

(1) 42·5 by ·0023. (2) ·197 by ·79.
(3) 37·9 by 409. (4) 27100 by ·00313.
(5) ·0269 by ·281. (6) 229 by ·007.

99. If we continue the division further in the Example given in Art. 98, we find the figures 756 coming again and again in the same order in the Quotient, so that the Quotient is 6·6756756756... without any termination.

Let us now take this example.

Divide 90 by ·0011.

$$\text{Here } 90 \div \cdot 0011 = \frac{90}{\cdot 0011} = \frac{900000}{11}$$

$$\begin{array}{r} 11\underline{)\,900000} \\ 81818 \end{array}$$

Up to this point the Quotient is an Integer: but, if we proceed further with the division, we shall obtain a decimal expression: thus if we affix two more zeros, preceded by a decimal point, to the dividend, we shall have

$$\begin{array}{r} 11\underline{)\,900000\cdot 00} \\ 81818\cdot 18 \end{array}$$

If we carry on the division to any extent we shall have the two figures 18 coming again and again in the same order. A decimal of this kind is called *Periodic*, *Circulating*, or *Recurring*.

100. The extent of the Period is denoted, by placing a dot over the *first*, and another dot over the *last* of the figures in it.

Thus $\cdot\dot{1}\dot{8}$ denotes a decimal *of an order such that it can be represented by no finite index*, since it runs on ·18181818... to an infinite number of figures.

So also, $6\cdot\dot{7}5\dot{6}$ stands for $6\cdot756756756$ .. ..

$\cdot\dot{0}4\dot{7}$ stands for $\cdot047047047$ .. .. ..

$\cdot4\dot{3}7\dot{2}$ stands for $\cdot4372372372$ .. .. ..

$26\cdot04\dot{7}\dot{9}$ stands for $26\cdot04797979$ .. .. ..

$\cdot0002\dot{6}$ stands for $\cdot000266666$ .. .. ..

101. A Vulgar Fraction may be converted into a Decimal Fraction by the following process.

Reduce the fraction to its lowest terms, and then find the Quotient resulting from the division of the numerator by the denominator, by the rule for division of decimals.

Thus to reduce $\frac{3}{8}$ to a decimal, we proceed thus :

$$\begin{array}{r} 8)\ 3\cdot000 \\ \hline \cdot375 \end{array}$$

$$\therefore \tfrac{3}{8} = \cdot375$$

Again to reduce $\frac{47}{32}$ to a decimal, we proceed thus :

32) 47·00000 (1·46875
32
150
128
220
192
280
256
240
224
160
160

$$\therefore \tfrac{47}{32} = 1\cdot46875.$$

Or we might work by Short Division, thus:

$$\begin{array}{r|l} 4 & 47\cdot00 \\ \hline 8 & 11\cdot75 \\ \hline & 1\cdot46875 \end{array}$$

Again, to reduce $\frac{1}{7}$ to a decimal, we proceed thus:

$$\begin{array}{r|l} 7 & 1\cdot00000000 \\ \hline & \cdot14285714\ldots\ldots \end{array}$$

$$\therefore \tfrac{1}{7} = \cdot\dot{1}4285\dot{7}.$$

102. *To shew that, when a Vulgar Fraction is reduced to a decimal, either the operation must terminate or the figures of the Quotient must recur in the same order.*

Consider the operation by which such a fraction as $\frac{1}{7}$ is reduced to a decimal. The only remainders that can occur are 0, 1, 2, 3, 4, 5, 6. If the remainder 0 should occur, the division terminates: if not, we can only have *six* different remainders, and when any of these occurs a second time, we must have a recurrence of the former remainders in the same order.

When a fraction in its lowest terms is reduced to a decimal and produces a recurring decimal, the *extreme* limit of the number of places in the *period* of the recurring decimal is one less than the denominator.

Thus $\frac{1}{7}$ produces a recurring decimal of 6 places.

$\frac{2}{19}$ produces a recurring decimal of 18 places.

$\frac{3}{29}$ produces a recurring decimal of 28 places.

103. When a Vulgar Fraction is in its lowest terms it can only be expressed as an Exact Decimal when the denominator is composed of factors each of which is one of the numbers 2 and 5.

Thus $\frac{5}{8}$ can be expressed as an exact decimal because $8 = 2 \times 2 \times 2$.

$\frac{3}{20}$ can be expressed as an exact decimal because $20 = 2 \times 2 \times 5$.

$\frac{4}{125}$ can be expressed as an exact decimal because $125 = 5 \times 5 \times 5$.

The reason for this is, that no vulgar fraction can be expressed as an Exact Decimal, unless it can be transformed to one which has 10, or some power of 10, for its denominator. Now no number can by multiplication be made a power of 10 unless it be composed of factors each of which is 2 or 5.

Thus 8 can be made into a power of 10 by multiplying it by $5 \times 5 \times 5$.

125 can be made into a power of 10 by multiplying it by $2 \times 2 \times 2$.

40 can be made into a power of 10 by multiplying it by 5×5.

Hence $\frac{3}{8} = \frac{3}{2\times2\times2} = \frac{3\times5\times5\times5}{2\times2\times2\times5\times5\times5} = \frac{375}{1000} = \cdot375$

$$\frac{7}{125} = \frac{7}{5\times5\times5} = \frac{7\times2\times2\times2}{5\times5\times5\times2\times2\times2} = \frac{56}{1000} = \cdot056$$

$$\frac{9}{40} = \frac{9}{2\times2\times10} = \frac{9\times5\times5}{2\times2\times10\times5\times5} = \frac{225}{1000} = \cdot225$$

But such numbers as 7, 12, 30, cannot be made into powers of 10 by multiplication, and hence $\frac{3}{7}$ $\frac{5}{12}$ $\frac{11}{30}$ cannot be reduced to exact decimals.

It may also be remarked that, when a Vulgar Fraction in its lowest terms is reduced to an exact decimal, the order of that decimal is expressed by the greatest number of times that either of the factors 2 or 5 occurs in the denominator.

Examples. (xlviii)

Convert into decimals the following vulgar fractions:

(1) $\frac{7}{20}$ (2) $\frac{11}{25}$ (3) $\frac{6}{7}$ (4) $\frac{1}{80}$

(5) $\frac{1}{800}$ (6) $\frac{1}{41}$ (7) $\frac{11}{21}$ (8) $\frac{13}{80}$

(9) $\frac{17}{1875}$ (10) $\frac{129}{25}$

RECURRING DECIMALS

104. *Pure* recurring decimal fractions are those, in which the period commences immediately after the decimal point.

thus $\cdot\dot{3}$, $\cdot\dot{2}\dot{7}$, $\cdot\dot{0}42\dot{9}$ are pure recurring decimals.

Mixed recurring decimal fractions are those, in which one or more figures precede the period:

thus $\cdot 2\dot{3}$, $\cdot 24\dot{2}\dot{7}$, $\cdot 35\dot{0}42\dot{9}$ are mixed recurring decimals.

105. *To find the Vulgar Fraction which is equivalent to a given pure recurring Decimal.*

Ex. (1). Find the Vulgar Fraction equivalent to $\cdot\dot{3}$.

The decimal = $\cdot 3333 \ldots\ldots$

From 10 times the decimal, or $3\cdot 333 \ldots\ldots$

Take the decimal, or, $\cdot 333 \ldots\ldots$

Then 9 times the decimal = $3\cdot 000 \ldots\ldots$

$\therefore$ the decimal = $\frac{3}{9} = \frac{1}{3}$.

Ex. (2). Find the Vulgar Fraction equivalent to $\cdot\dot{2}4\dot{7}$.

The decimal = $\cdot 247247 \ldots\ldots$

From 1000 times the decimal, or, $247\cdot 247 \ldots\ldots$

Take the decimal, or, $\cdot 247 \ldots\ldots$

Then 999 times the decimal = $247\cdot 000 \ldots\ldots$

$\therefore$ the decimal = $\frac{247}{999}$

Ex. (3). Find the Vulgar Fraction equivalent to $\cdot\dot{0}42\dot{3}$.

The decimal = $\cdot 04230423 \ldots\ldots$

From 10000 times the decimal, or, $423\cdot 0423 \ldots\ldots$

Take the decimal, or, $\cdot 0423 \ldots\ldots$

Then 9999 times the decimal = $423\cdot 0000 \ldots\ldots$

$\therefore$ the decimal = $\frac{423}{9999} = \frac{47}{1111}$

Examples. (xlix)

Convert into Vulgar Fractions in their lowest terms:

(1) $\cdot\dot{6}$ (2) $\cdot\dot{2}\dot{7}$ (3) $\cdot\dot{0}4\dot{5}$ (4) $\cdot\dot{3}12\dot{3}$

(5) $\cdot\dot{0}07\dot{2}$ (6) $\cdot\dot{4}02\dot{3}$ (7) $\cdot\dot{0}005\dot{4}$ (8) $\cdot\dot{0}000\dot{9}$

106. Hence we deduce the following rule for reducing a pure recurring decimal to a Vulgar Fraction:

Take one of the periods to form the numerator, and for the denominator the number formed by repeating 9 *as many times as there are figures in the period.*

$$\text{Thus}\quad \cdot\dot{7} = \tfrac{7}{9}$$
$$\cdot\dot{0}\dot{5} = \tfrac{5}{99}$$
$$\cdot\dot{4}32\dot{7} = \tfrac{4327}{9999}$$

107. *To find the Vulgar Fraction, which is equivalent to a given mixed recurring decimal.*

Ex. (1). Find the Vulgar Fraction equivalent to $\cdot 2\dot{3}\dot{7}$.

The decimal $= \cdot 23737\ldots\ldots$

From 1000 times the decimal, or, $237\cdot 37\ldots\ldots$

Take 10 times the decimal, or, $2\cdot 37\ldots\ldots$

Then 990 times the decimal $= 235$,

$\therefore$ the decimal $= \frac{235}{990} = \frac{47}{198}$

Ex. (2). Find the Vulgar Fraction equivalent to $\cdot 04\dot{7}2\dot{6}$.

The decimal $= \cdot 04726726\ldots\ldots$

From 100000 times the decimal, or, $4726\cdot 726\ldots\ldots$

Take 100 times the decimal, or, $4\cdot 726\ldots\ldots$

Then 99900 times the decimal $= 4722$,

$\therefore$ the decimal $= \frac{4722}{99900} = \frac{787}{16650}$

Ex. (3). Find the Vulgar Fraction equivalent to $3 \cdot 1\dot{4}$.

$$\text{The decimal} = 3 \cdot 1444 \ldots\ldots$$

$$\text{From 100 times the decimal, or,} \quad 314 \cdot 44 \ldots\ldots$$

$$\text{Take 10 times the decimal, or,} \quad \underline{31 \cdot 44 \ldots\ldots}$$

$$\text{Then 90 times the decimal} = 283,$$

$$\therefore \text{the decimal} = \tfrac{283}{90}$$

Examples. (1)

Convert into Vulgar Fractions in their lowest terms:

(1) $\cdot 4\dot{2}\dot{5}$ (2) $\cdot 47\dot{5}\dot{9}$ (3) $4 \cdot 25\dot{3}$ (4) $\cdot 004\dot{2}\dot{6}$

(5) $53 \cdot 00\dot{2}4\dot{3}$ (6) $7 \cdot 2\dot{0}1\dot{1}$ (7) $2 \cdot 53\dot{0}\dot{6}$

108. Hence we deduce the following rule for reducing a Mixed recurring decimal to a vulgar fraction.

Form the Numerator by taking from the figures up to the end of the first period the figures that precede the first period; and form the Denominator by setting down 9 as many times as there are figures in the period, and affixing 0 as many times as there are figures between the decimal point and the first period.

Thus
$$\cdot 2\dot{4}\dot{5} = \tfrac{245-2}{990} = \tfrac{243}{990}$$

$$\cdot 004\dot{7}\dot{3} = \tfrac{473-4}{99000} = \tfrac{469}{99000}$$

$$4 \cdot \dot{5} = \tfrac{45-4}{9} = \tfrac{41}{9}$$

$$7 \cdot 34\dot{5} = \tfrac{7345-734}{900} = \tfrac{6611}{900}$$

109. The method of performing arithmetical operations with recurring decimals will be best explained by taking the operations separately.

I. Addition.

Find the sum of $3 \cdot 4\dot{9}$, $4 \cdot \dot{0}4\dot{7}$, and $\cdot 14\dot{6}\dot{3}$.

First make the decimals all of the same order, thus

$$3 \cdot 499\dot{9}, \quad 4 \cdot \dot{0}4\dot{7}0, \quad \cdot 14\dot{6}\dot{3}.$$

Then, since the periods consist of 1, 3, 2 figures respectively, and the L. C. M. of 1, 3 and 2 is 6, carry on all the decimals *six* places farther, thus

$$\begin{array}{r} 3{\cdot}4999999999 \\ 4{\cdot}0470470470 \\ {\cdot}1463636363 \\ \hline 7{\cdot}69\dot{3}4106\dot{8} \end{array}$$

II. Subtraction.

Here we proceed on the same principle as in Addition.

Thus to subtract $5{\cdot}24\dot{7}$ from $8{\cdot}\dot{0}5\dot{9}$,

$$\begin{array}{r} 8{\cdot}059059 \\ 5{\cdot}247777 \\ \hline 2{\cdot}81\dot{1}2\dot{8} \end{array}$$

In both operations some care is requisite in observing what figure would be carried on, if the columns omitted were taken into account.

III. In Multiplication and Division the recurring decimals should be converted into vulgar fractions, and when the Product or Quotient of these fractions has been found, it may be converted into a decimal.

Thus $4{\cdot}\dot{5} \times 3{\cdot}\dot{7} = \frac{45-4}{9} \times \frac{37-3}{9} = \frac{41}{9} \times \frac{34}{9} = \frac{1394}{81}$,

and ${\cdot}0\dot{5} \div {\cdot}04\dot{2} = \frac{5}{90} \div \frac{38}{900} = \frac{5}{90} \times \frac{900}{38} = \frac{5 \times 10}{38} = \frac{25}{19}$.

We may then, if it be required, convert $\frac{1394}{81}$ and $\frac{25}{19}$ into decimals, by the process explained in Art. 101.

Examples. (li)

Find the value of the following expressions:

(1) $2{\cdot}5\dot{7} + {\cdot}\dot{0}4\dot{3} + 13{\cdot}\dot{2}$.

(2) $14{\cdot}7\dot{6}\dot{2} + 3{\cdot}\dot{5}4\dot{9} + 2{\cdot}20\dot{4}$.

(3) $15{\cdot}0\dot{2}\dot{5} - 13{\cdot}\dot{2}4\dot{7}$.

(4) ${\cdot}02\dot{4}\dot{6} - {\cdot}00\dot{3}9\dot{7}$.

(5) $3{\cdot}\dot{7} \times 5{\cdot}\dot{4}\dot{9}$.

(6) ${\cdot}007\dot{2} \times {\cdot}\dot{4}\dot{5}$.

(7) $3{\cdot}\dot{4} \div 4{\cdot}0\dot{9}$.

(8) ${\cdot}0\dot{7}\dot{4} \div {\cdot}5\dot{9}$.

110. When vulgar and decimal fractions are combined in the same expression, it may *usually* be simplified in the neatest and easiest way by reducing the vulgar fractions to a decimal form.

Thus, if we have to find the sum of $476\frac{1}{4}$, $13\frac{3}{8}$, and $10\cdot375$, we should proceed thus,

$$\begin{array}{rr} 476\frac{1}{4} = & 476\cdot25 \\ 13\frac{3}{8} = & 13\cdot375 \\ & 10\cdot375 \\ \hline \text{Sum} & 500\cdot000 \end{array}$$

111. The following Miscellaneous Examples in decimals will give the student practice in the principles laid down in this Chapter.

Examples. (lii)

(1) Multiply $\cdot0204$ by $40\cdot2$; and divide $99\cdot9666$ by $\cdot037$.

(2) Multiply $\cdot0701$ by $700\cdot01$; and divide $8886\cdot66$ by 0037.

(3) Reduce to recurring decimals $\frac{13}{150}$ $\frac{17}{495}$ and $\frac{61}{33}$

(4) Reduce to recurring decimals $\frac{7}{450}$ $\frac{31}{165}$ and $\frac{53}{66}$

(5) Divide 454206 by $3\cdot05 \times \cdot073$; and by $6\cdot1 \times 2\cdot04$.

(6) Divide 3833336 by $\cdot031 \times 2\cdot05$; and by $8\cdot99 \times 20\cdot8$.

(7) Multiply $35\cdot603$ by $27\cdot61$, and $43\cdot\dot{2}9\dot{1}$ by $6\cdot2\dot{4}$.

(8) Divide $186\cdot4302$ by $31\cdot02$; $18643\cdot02$ by $\cdot3102$; and 1864302 by $3\cdot102$.

(9) Multiply $421\cdot619$ by $\cdot547$; and divide $\cdot2107206$ by $\cdot0004206$.

(10) Divide $\cdot0863547$ by $\cdot000713$ to four places of decimals.

(11) Divide 104 by 7, and shew why the decimal recurs.

(12) Divide $\cdot8607535$ by $\cdot000974$ to four places of decimals.

(13) Multiply $76\cdot371$ by $8\cdot54$; and divide $2\cdot9\dot{8}0\dot{1}$ by $7\cdot\dot{4}5\dot{0}$.

(14) Divide ·37848 by ·456; 3·7848 by ·0456; and 3784·8 by ·00456.

(15) Multiply 457·61 by ·527; and divide $477\cdot5\dot{5}8\dot{5}$ by $21\cdot\dot{3}5\dot{1}$.

(16) Divide 25·567 by 3·7; 255·67 by ·0037; and ·25567 by ·00037.

(17) Divide 207·861 by 5790; and 160 by ·00004.

(18) Reduce $5\frac{81}{625}$ and $3\frac{37}{54}$ to decimals; and ·109375 and $\cdot5\dot{7}4\dot{0}$ to vulgar fractions.

(19) Divide 1·9517 by 673000 and 64000 by ·0008.

(20) Reduce $4\frac{173}{3125}$ and $5\frac{43}{540}$ to decimals; and ·9375 and $\cdot4\dot{9}2\dot{5}$ to vulgar fractions.

(21) Subtract 3·64 from 12·064; and divide 1·0191 by 79000.

(22) Subtract 3·05 from 5·015; and divide 10·24 by ·0128.

(23) Divide $\frac{1\cdot44}{\cdot18}$ of ·36 by $\frac{36}{\cdot05}$ of 6.

(24) Divide $\frac{\cdot052}{1\cdot3}$ of 1·56 by $\frac{\cdot0624}{14\cdot4}$ of 25·92.

(25) Divide the difference of ·04 and ·404 by the sum of $3\frac{23}{125}$ and $4\frac{23}{50}$

(26) Express the sum of $\frac{1}{2}$ $\frac{1}{3}$ $\cdot0\dot{3}\dot{1}$ and $\cdot\dot{5}\dot{7}$ as a recurring decimal.

(27) Divide the difference of ·4607 and ·00809 by the difference of $6\frac{51}{80}$ and $5\frac{125}{128}$

(28) Express the sum of $\frac{1}{5}$ $\frac{1}{6}$ $1\cdot2\dot{7}\dot{1}$ and $1\cdot\dot{3}\dot{5}$ as a recurring decimal.

(29) Find the value to five places of decimals of

$$\frac{24}{17} - \frac{14}{11 \times 31}$$

(30) Find the value to five places of decimals of

$$\frac{27}{14} - \frac{21}{17 \times 31}$$

(31) Add together $23\cdot07\dot{6}$, $19\cdot\dot{2}4\dot{5}$, $31\cdot2\dot{0}\dot{3}$.

(32) Add together $23\cdot\dot{0}7\dot{6}$, $19\cdot2\dot{4}\dot{5}$, $31\cdot20\dot{3}$.

(33) Subtract $25\cdot\dot{0}4\dot{7}$ from $29\cdot25\dot{9}$.

(34) Multiply $3\cdot\dot{7}$ by $4\cdot0\dot{5}$.

(35) Divide $4\cdot2\dot{7}$ by $\cdot\dot{4}\dot{2}$.

XII. Square Root.

112. When a Number is multiplied by itself, the result is called the SQUARE of the number. Thus 144 is the square of 12, and 225 is the square of 15.

The symbol ^{2} placed over a number expresses the square of the number: thus 5^2 denotes the square of 5.

113. The SQUARE ROOT of a given number is that number, whose square is equal to the given number.

Thus the square root of 144 is 12, because the square of 12 is 144.

The symbol $\sqrt{}$, placed before a number, denotes that the square root of that number is to be taken: thus $\sqrt{25}$ is read "the square root of 25."

114. A number which has an integer for its square root is called a PERFECT SQUARE.

115. For Perfect Squares not greater than 100 we know the square roots, thus we know that the square root of 81 is 9; and for many Perfect Squares greater than 100 we know the square roots by experience, as, for instance, we know that the square root of 169 is 13, and the square root of 400 is 20, and the square root of 10000 is 100. But we have rules for finding the Square Root of any Number, as we shall now explain.

First, suppose we have to find the Square Root of 1225.

We draw a line separating the two figures on the right from the other two; thus

12¦25.

The figures 12 make what is called the *first period*,

The figures 25 make what is called the *second period.*

We then take the nearest perfect square not greater than 12, that is 9, and place it under the 12 and put its square root, that is 3, as the first figure of the square root we have to find, thus

$$\begin{array}{l} 12|25\ (3 \\ \ 9 \end{array}$$

We subtract 9 from 12, and annex to the remainder 3 *the second period* 25, to make a dividend, and we double the first figure of the root, and set down the result as the first term of a divisor; thus our process up to this point will stand thus:

$$\begin{array}{r|l} & 12|25\ (3 \\ & \ 9 \\ \hline 6 & 325 \end{array}$$

Now we shall have to annex another figure to the 6, and we must therefore reckon the 6 as *six tens*, or 60, and then we seek the number of times 60 is contained in 325, and this being *five* times, we set down 5 as the second figure of the root, and also annex 5 to the 6, so that our process up to this point will stand thus:

$$\begin{array}{r|l} & 12|25\ (35 \\ & \ 9 \\ \hline 65 & 325 \end{array}$$

We then multiply 65 by 5, and set the product down under the 325; and subtracting the product from the 325, we have no remainder, and we conclude that 35 is the square root of 1225; the full process being

$$\begin{array}{r|l} & 12\,25\ (35 \\ & \ 9 \\ \hline 65 & 325 \\ & 325 \\ \hline \end{array}$$

Next, to find the square root of 622521.

Drawing a line to mark off the two figures on the right, and another line to mark off the next two figures,

our process for finding the first two figures of the root will be the same as that explained in the first example, and it will stand thus:

```
     62|25|21 (78
     49
    ------
148 | 1325
    | 1184
    --------
    |  14121
```

We now annex to the remainder the *third* period 21, and we double the part of the root already found, 78, and set down the result 156 as a partial divisor, and proceed, as before, to divide 14121 by 1560, and annex the quotient 9 to the root and to the divisor; and multiplying 1569 by 9 we set the product under the 14121; thus our process in full will be

```
      62|25|21 (789
      49
     ------
 148 | 1325
     | 1184
     --------
1569 |  14121
     |  14121
     --------
```

∴ 789 is the root required.

NOTE.—In practice, instead of dividing 1325 by 140, it is usual to divide 132 by 14, and instead of dividing 14121 by 1560, to divide 1412 by 156. The quotient thus obtained is, however, sometimes too great, as will be seen in the next Examples.

We now give two Examples in which the first period has only *one* figure, which must always be the case when the proposed square has an *odd* number of figures in it.

To find the square root of 189475225.

Marking off the figures by pairs, commencing from the right, we have

```
          1|89|47|52|25 ( 13765
          1
      23 | 89
         | 69
     267 | 2047
         | 1869
    2746 |  17852
         |  16476
   27525 |   137625
         |   137625
```

NOTE.—In dividing 89 by 20 the quotient is 4, but if we added this to complete the divisor, it would become 24, which being multiplied by 4, would give 96, a number larger than 89.

To find the square root of 39601.

```
        3,96,01 ( 199
        1
   29 | 296
      | 261
  389 |  3501
      |  3501
```

NOTE I.—The division of 296 by 20 illustrates the remarks made on the last example.

NOTE II.—The second remainder, 35, is greater than the divisor, 29, a result not uncommon in this operation.

Examples. (liii)

Find the square roots of

(1) 196.	(2) 529.
(3) 1024.	(4) 5625.
(5) 88209.	(6) 119025.

(7) 106929.	(8) 751689.
(9) 193600.	(10) 697225.
(11) 36372961.	(12) 22071204.
(13) 550183936.	(14) 5256250000.
(15) 4124961.	(16) 546121000000.
(17) 32239684.	(18) 1918107 13444.

116. *To find the square root of a Decimal Fraction.*

When the given number has an *even* number of decimal places, we proceed to find the square root as if the number were an integer, and mark off in the root a number of decimal places equal to *half the number in the square.*

Thus, if the square be a decimal of the *sixth* order,

the root will be a decimal of the *third* order,

For example, to find the square root of $5 \cdot 322249$.

```
    5·|32|22|49 ( 2·307
    4
    ------
43 | 132
   | 129
   -------
46 |  322
```

Since 46 is not contained in 32, we annex an 0 to the divisor, and also to the root, and bring down the next period, thus,

```
4607 | 32249
     | 32249
     -------
```

Examples. (liv)

Find the square roots of

(1) $16 \cdot 81$.	(2) $281 \cdot 9041$.	(3) $\cdot 9025$.
(4) $\cdot 2601$.	(5) $\cdot 0625$.	(6) $\cdot 000729$.
(7) $17242 \cdot 3161$.	(8) $1 \cdot 002001$.	(9) $44415 \cdot 5625$.
(10) $18947 \cdot 5225$.		

117. In finding the square root of a decimal fraction we must be careful to make the decimal such, that the index of its order is an *even* number.

Thus, if we have to find the square root of ·4, we change the decimal into an equivalent decimal of the *second*, *fourth*, *sixth* ... order, thus, ·40, ·4000, ·400000,...

This is done in order that the denominator of the equivalent fraction may be a perfect square, which is the case in the fractions

$$\frac{40}{100}\quad\frac{4000}{10000}\quad\frac{400000}{1000000}\ldots\text{.}$$

but not in the fractions

$$\frac{4}{10}\quad\frac{400}{1000}\quad\frac{40000}{100000}\ldots\ldots\ldots$$

Also, since for every *pair* of figures in the square we have *one* figure in the root, we shall have to take a number of figures in the decimal part of the square double the number of decimal places we are to have in the root.

Suppose, for example, we have to find the square root of ·144 to *four* places of decimals.

We must have *eight* decimal places in the square, thus, ·14400000, and we mark off these and proceed as in the extraction of the root of whole numbers, the root being a decimal of the *fourth* order: thus,

```
       ·14|40|00|00 ( ·3794....
        9
  67  | 540
      | 469
 749  |  7100
      |  6741
7584  |  35900
      |  30336
         5564
```

Note.—The square root of a decimal of an *odd* order is a non-terminating decimal.

Examples. (lv)

Extract to four places of decimals the square roots of

(1) 20.	(2) 30.	(3) ·9.	(4) ·121.
(5) ·169.	(6) ·016.	(7) ·00064.	(8) ·00121.
(9) 16·245.	(10) $\cdot\dot{9}$.	(11) $\cdot\dot{2}\dot{5}$.	(12) $42\cdot\dot{0}\dot{3}$.

118. If we have to find the square root of a vulgar fraction, we can always by multiplication make the denominator a perfect square, if it be not already so, multiplying the numerator by the same number.

We then find the square root of the denominator, and find, exactly or approximately, the square root of the numerator, and make the results respectively the denominator and numerator of a fraction, which is the root required, exactly or approximately.

Ex. (1). $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$

Ex. (2). $\sqrt{\frac{2}{3}} = \sqrt{\frac{2\times 3}{3\times 3}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{3}$

We can now extract the square root of 6 to, say, *three* places of decimals, thus:

$$
\begin{array}{r|l}
 & 6\cdot|00|00|00\ (2\cdot449\ldots\ldots \\
 & 4 \\ \hline
44 & 200 \\
 & 176 \\ \hline
484 & 2400 \\
 & 1936 \\ \hline
4889 & 46400 \\
 & 44001 \\ \hline
 & 2399
\end{array}
$$

$$\therefore \sqrt{\frac{2}{3}} = \frac{2\cdot449\ldots}{3} = \cdot816\ldots\ldots$$

Or we might have reduced $\frac{2}{3}$ to a decimal, thus: $.666666\ldots$ and then have extracted the square root of his decimal.

Ex. (3). $\sqrt{8\frac{17}{64}} = \sqrt{\frac{529}{64}} = \frac{\sqrt{529}}{\sqrt{64}} = \frac{23}{8} = 2\frac{7}{8}$

Ex. (4). To find the square root of $\frac{1 \cdot 28}{12 \cdot 5}$

Here we can reduce the fraction to lower terms.

$$\text{Thus, } \sqrt{\frac{1\cdot28}{12\cdot5}} = \sqrt{\frac{\cdot64}{6\cdot25}} = \frac{\cdot8}{2\cdot5} = \cdot32$$

119. An integer can always be changed into a perfect square by multiplying it by itself, and sometimes by multiplying it by a number less than itself.

For example,

7 is changed into a perfect square if multiplied by 7

18 is changed into a perfect square if multiplied by 2.

Examples. (lvi)

Find the square roots of

(1) $\frac{36}{49}$	(2) $\frac{64}{121}$	(3) $\frac{289}{625}$
(4) $\frac{1369}{7569}$	(5) $\frac{15120}{182329}$	(6) $5\frac{1}{16}$
(7) $5\frac{19}{25}$	(8) $3\frac{22}{169}$	(9) $65\frac{64}{81}$
(10) $38\frac{11}{25}$	(11) $17\frac{16}{25}$	(12) $11\frac{37}{49}$

and find to four places of decimals the square roots of

(13) $\frac{5}{8}$	(14) $\frac{10}{24}$	(15) $6\frac{2}{5}$
(16) $9\frac{1}{2}$	(17) $76\frac{14}{17}$	

XIII. Cube Root.

120. When a number is multiplied by itself twice, the result is called the CUBE of the number. Thus 27 is the cube of 3, and 216 is the cube of 6.

121. The Cube Root of a given number is that number, whose cube is equal to the given number.

Thus the cube root of 343 is 7, because the cube of 7 is 343.

The symbol $\sqrt[3]{\ }$, placed before a number, denotes that the cube root of that number is to be taken : thus $\sqrt[3]{125}$ is read, "the cube root of 125."

122. A number, which has an integer for its Cube Root, is called a Perfect Cube.

The numbers, less than 1000, which are perfect cubes, should be committed to memory : they are

1, 8, 27, 64, 125, 216, 343, 512, 729,

and the cube roots of these numbers are respectively

1, 2, 3, 4, 5, 6, 7, 8, 9.

123. To find the Cube Root of a perfect cube, greater than 1000, we proceed by a rule, which we shall now explain.

Ex. (1). To find the cube root of 91125.

	4		91 | 125
			64
12	5	4800	27125
		625	
		5425	27125

First divide the number 91125 into two periods by drawing a line marking off *three* figures on the right.

Then take the nearest perfect cube not greater than 91, which is 64, and set down its cube root, which is 4, in a line with 91125, and some way to the left. This is the first figure of the root.

Then subtract 64 from 91, and to the remainder attach the second period, 125.

Now place three times the first figure of the root, 12, to

the extreme left, and three times the square of the first figure of the root, 48, with two zeros annexed to it, just on the left of the 27125.

Divide 27125 by 4800, and set the quotient, 5, midway between 12 and 4800. Then read 12 5 as 125; multiply this by 5; put the result, 625, under the 4800; add it to the 4800; this gives 5425; multiply this by 5; put the result, which is 27125, under the first remainder; subtract, and as there is no remainder, the process is complete, and the root is 45.

Examples. (lvii)

Find the cube roots of

(1) 4096.	(2) 32768.	(3) 74088.
(4) 493039.	(5) 614125.	(6) 262144.
(7) 39304.	(8) 389017.	(9) 195112.
(10) 970299.	(11) 59319.	(12) 250047.

Next, let us take the case in which the cube root has three figures, and extract the cube root of 428661064.

```
          7                  428 | 661 | 064
                             343
                             ---
   21     5       14700       85661
                   1075 )
                  ----- |
                  15775 |     78875
                     25 )     -----
                  -------
  225     4     1687500     6786064
                   9016
                -------
                1696516     6786064
                            -------
```

We separate the number 428661064 into three periods, and take the nearest perfect cube not greater than 428, which is 343, and we set down its cube root, which is 7. We then subtract 343 from 428, and annex to the remainder 661, the second period.

Then we set down three times 7, which is 21,
and three times the square of 7, which is 147, and add two zeros to it.

Then we divide 85661 by 14700, which gives the quotient 5, and this we put down midway between 21 and 14700.

Then we multiply 215 by 5, which gives 1075; we add this to 14700; we multiply the result, 15775, by 5; and subtract the product, 78875, from 85661; and to the remainder we annex the third period, 064.

We then set down three times 75, which is 225,
and three times the square of 75, which is 16875.

N.B.—This last result can be obtained by setting the square of 5, the second figure of the root, under the second divisor, and adding the three numbers coupled by the bracket.

We then annex two zeros to 16875 and repeat the process explained above, to find 4, the third figure of the cube root, which is in this case 754.

Next, take the case in which the root has *four* figures and find the cube root of 14832537993.

	2		14 | 832 | 537 | 993
			8
6	4	1200	6832
		256	
		1456	5824
		16	
72	5	172800	1008537
		3625	
		176425	882125
		25	
735	7	18007500	126412993
		51499	
		18058999	126412993

Hence the root required is 2457.

NOTE.—In dividing 6832 by 1200 the quotient is 5, but if we took this for the second figure of the root we should find

that the addition of 5 times 65, or 325, to 1200, would give 1525, and this multiplied by 5 would give 7625, a number too large to be subtracted from 6832.

Examples. (lviii)

Find the cube roots of

(1) 14706125. (2) 149721291. (3) 28934443.

(4) 300763000. (5) 2097152. (6) 5735339.

(7) 99252847. (8) 1092727. (9) 16777216.

(10) 194104539. (11) 84027672. (12) 130323843.

(13) 322828856. (14) 354894912. (15) 700227072.

(16) 134217728. (17) 122615327232.

(18) 673373097125.

124. *To extract the Cube Root of a Decimal Fraction.*

In order that a Decimal Fraction may be a Perfect Cube, it must be of the 3rd, 6th, 9th..... order, the Index of the order being some multiple of 3.

We may then proceed in the following way:

Ex. (1). To find the cube root of ·343.

$$\sqrt[3]{\cdot 343} = \sqrt[3]{\frac{343}{1000}} = \frac{7}{10} = \cdot 7.$$

Ex. (2). To find the cube root of ·039304.

$$\sqrt[3]{\cdot 039304} = \sqrt[3]{\frac{39304}{1000000}} = \frac{34}{100} = \cdot 34$$

Ex. (3). To find the cube root of ·012812904.

$$\sqrt[3]{\cdot 012812904} = \sqrt[3]{\frac{12812904}{1000000000}} = \frac{234}{1000} = \cdot 234.$$

125. To extract the cube root of an integer or decimal expression to a particular place of decimals, we must

take *three times the number* of decimal places in the expression.

Thus, to find the cube root of 4·23 accurately to *three* places of decimals we extract the cube root of 4·230000000, making the given expression a decimal of the *ninth* order. In working this example we find the cube root of 4230000000, *regarded as a whole number*, and mark off three decimal places in the result.

126. The cube root of a *Vulgar fraction* may be found by taking the cube roots of the numerator and denominator, or by reducing the fraction to a decimal of the 3rd, 6th, 9th...... order, and proceeding as in Art. 125.

Examples. (lix)

Find the cube roots of

(1) ·389017 (2) ·048228544 (3) 27054·036008

(4) $\frac{1331}{1728}$ (5) $\frac{250}{686}$ (6) $5\frac{13}{343}$ (7) $405\frac{28}{125}$;

and find to three places of decimals the cube roots of

(8) 5 (9) 576 (10) ·121861281

(11) 15·926972504 (12) $\frac{5}{9}$ (13) $\frac{3}{4}$

(14) $\frac{1}{3}$ (15) $7\frac{3}{5}$ (16) $3\frac{1}{5}$

127. The *fourth* root of a number is found by taking the square root of the square root of the number.

Thus $\sqrt[4]{4096} = \sqrt{64} = 8.$

The *sixth* root of a number is found by taking the cube root of the square root of the number.

Thus $\sqrt[6]{64} = \sqrt[3]{8} = 2.$

Examples. (lx)

Find the fourth roots of

(1) 531441 (2) 4100625 (3) 1575·2961;

and the sixth roots of

(4) 4826809 (5) 2472049011296 (6) 282429·536481.

128. We conclude this Part with a set of Miscellaneous Examples, which differ from those already given, inasmuch as they do not fall strictly under any Rule. They may be solved more neatly and more quickly by the application of Algebraic methods, but they must be regarded as Exercises on the general principles laid down in this work, and therefore the student should endeavour to adapt them to those principles, stating each step of the operation clearly, and showing that it is in accordance with the methods of Pure Arithmetic.

Examples. (lxi)

(1) The sum of two numbers is 72, and one of them is 40; what is the other?

(2) The difference of two numbers is 34, and the smaller is 56; what is the other?

(3) The sum of two numbers is 126, and one of them exceeds the other by 10; what are the numbers?

(4) What is the difference between six dozen dozen and half a dozen dozen?

(5) To the double of a certain number I add 12, and obtain as a result 86. What is the number?

(6) From five times a certain number I take 17, and obtain as a result 98. What is the number?

(7) What two numbers are those, whose difference is 14, and their sum 48?

(8) The sum of two numbers is 378, and their difference is 172. What are they?

(9) A man died in 1868 aged 93; his son died in 1822 at the age of 16. How old was the father when the son was born?

(10) The thirty-seventh part of a number is 41. What is the number?

(11) Forty-seven times a number is 1457. What is the number?

(12) Five-sevenths of a number is 35. What is the number?

(13) The double and the third part of a number, added together, give as a result 140. What is the number?

(14) What is the number, of which the half, the third, and the fourth parts, added together, give as a result 104?

(15) What is the number, of which the twelfth, twentieth, and fortieth parts, added together, give as a result 38?

(16) What is the number, of which the fourth part exceeds the seventh part by 30?

(17) What is the number, of which the twenty-fifth part exceeds the thirty-fifth part by 8?

(18) What is the number, which exceeds the sum of its third, tenth and twelfth parts by 58?

(19) When I have taken away from 33 the fourth, fifth and tenth parts of a certain number, the remainder is zero. What is the number?

(20) What is the number, of which the fourth, fifth and sixth parts exceed the half of the number by 112?

(21) The seventh part of a number is equal to the whole number diminished by 1626. What is the number?

(22) The sum of two numbers is 5760, and their difference is equal to one-third of the greater. What are the numbers?

(23) Divide 1800 into two parts, such that one is two-sevenths of the other.

(24) On multiplying a certain number by 4, and dividing the product by 3, I obtain as a quotient 24. What is the number?

(25) Divide 60 into two such parts, that one part is greater than the other by 24.

(26) Divide 129 into two such parts, that three-sevenths of one is less than the other by 59.

(27) I divide 2·49 by a certain number; the quotient is 4 and the remainder ·37. What is the divisor?

(28) Dividing a certain number by ·027, the quotient is 6116 and the remainder ·003. What is the dividend?

(29) What is the difference between twice the square of 12 and the square of twice twelve?

(30) In an orchard of fruit trees one-half of them bear apples, one-fourth pears, one-sixth plums, and the rest, 50 in number, cherries. How many trees are there in the orchard?

(31) What part of 25 units is $\frac{5}{8}$ of a unit?

(32) Three-fourths of the square root of a certain number is 21. What is the number?

(33) Five-sevenths of the square of a certain number is 140. What is the number?

(34) Four-fifths of the cube root of a certain number is 12. What is the number?

(35) Eight-thirteenths of the cube of a certain number is 36504. What is the number?

(36) Two-thirds of the sum of a certain mixed number and 21 is 23. What is the number?

(37) Three-fifths of the difference between a certain number and 37 is 18. What is the number?

(38) From the square root of a certain mixed number I take away $1\frac{2}{7}$, and five-sixths of the result is $\frac{10}{21}$. What is the mixed number?

(39) To certain number I add 2, I multiply the sum by 4, I divide the product by 3, and I take 3 from the quotient. The remainder is 17. What is the number?

(40) Shew that the seventh power of 8 is the cube of the seventh power of 2.

(41) What number is that, whose half multiplied by its third part gives 864?

(42) What is the number, of which the seventh and eighth parts being multiplied together, and the product divided by 3, the quotient is $298\frac{2}{3}$?

COMMERCIAL ARITHMETIC.

XIV. On English Money.

129. Having explained the principles and processes of Pure Arithmetic, we proceed to show how they are applied to Commercial affairs.

MEASURES OF MONEY.

4 farthings are equivalent to 1 penny,
12 pence are equivalent to . 1 shilling,
20 shillings are equivalent to 1 pound.

The symbol £ placed before or over a number denotes *pounds*,
...... *s.* ..... after .... *shillings*,
...... *d.* ..... after .... *pence*.

Thus £14. 5*s.* 7*d.*, or $\overset{\pounds}{14} . \overset{s.}{5} . \overset{d.}{7}$ stands for fourteen pounds, five shillings and seven pence.

Since 1 farthing is one-fourth of a penny,
2 farthings are one-half of a penny,
3 farthings are three-fourths of a penny.

Hence the symbol $\frac{1}{4}d.$ is placed for 1 farthing,
.... $\frac{1}{2}d.$ .... 2 farthings, or a halfpenny,
.... $\frac{3}{4}d.$ .... 3 farthings.

The symbol *q.*, placed after a number, is sometimes used to denote farthings: thus, 3*q.* stands for three farthings.

130. We call £14 a *simple* quantity, and £14. 5*s.* 7*d.* a *compound* quantity, because the former is expressed with reference to a *single* unit, while the latter is expressed with reference to *three different* units.

REDUCTION OF MONEY.

131. The expression 5*s*. 7*d*. stands for a sum of money, which is made up of five shillings and seven pence. Now, since one shilling is equivalent to twelve pence, five shillings are equivalent to sixty pence; and therefore five shillings and seven pence are equivalent to sixty-seven pence.

The process, by which we change the *compound* expression 5*s*. 7*d*. into the equivalent *simple* expression 67*d*., is arranged thus:

$$\begin{array}{l} s.\ \ d. \\ 5 \cdot 7 \\ 12 \\ \hline 67d. \end{array}$$

and we describe the process thus: *We change the 5 shillings into pence by multiplying by 12, and add to the product the 7 pence.*

Again, to change the compound expression £4. 7*s*. 10½*d*. into an equivalent number of farthings, we proceed thus:

$$\begin{array}{l} £\ \ \ s.\ \ \ d. \\ 4 \cdot 7 \cdot 10\tfrac{1}{2} \\ 20 \\ \hline 87s. \\ \ \ 12 \\ \hline 1054d. \\ \ \ \ \ 4 \\ \hline 4218q. \end{array}$$

First we change £4 to shillings and add 7*s*., making 87*s*.;
then .... 87*s*. to pence .... 10*d*., making 1054*d*.
then ...... 1054*d*. to farthings ...... 2*q*., making 4218*q*.

Examples. (lxii)

Reduce to farthings

(1) $3\frac{1}{4}d.$; $7\frac{1}{2}d.$; $9d.$; $11\frac{3}{4}d.$

(2) $2s.\ 3d.$; $5s.\ 7\frac{1}{2}d.$; $12s.\ 9\frac{3}{4}d.$; $17s.\ 7\frac{1}{4}d.$

(3) £3. 12s.; £5; £2. 17s. $6\frac{1}{2}d.$; £17. 4s. $5\frac{3}{4}d.$

Reduce to pence

(4) 6s.; 4s. 10d.; 7s. 10d.; 8s. 9d.; 13s. 7d.

(5) £4; £5. 2s. 4d.; £17. 14s. 5d.; £58. 13s. 11d.

(6) £174. 10s.; £432. 15s. 10d.; £1274. 17s. 9d.

132. The converse operation, by which we express a *simple* quantity in terms of an equivalent *compound* quantity, will be best explained by the following Examples.

Ex. (1). Nine farthings will be expressed as pence and farthings, if we divide 9 by 4 (since 4 farthings = 1 penny), set down the quotient as pence, and the remainder as farthings, thus: 9 farthings = $\frac{9}{4}d.$ = $2\frac{1}{4}d.$

Ex. (2). Again, 33 pence will be expressed as shillings and pence, if we divide 33 by 12 (since 12 pence = 1 shilling), set down the quotient as shillings, and the remainder as pence, thus: 33 pence = $\frac{33}{12}$ shillings = 2s. 9d.

Ex. (3). Also, 75 shillings = $\frac{75}{20}$ pounds = £3. 15s.

Ex. (4). To express 4275639 farthings in terms of £ s. d.

farthings.

4 | 4275639

12 | 1068909d. and 3 farthings over.

2,0 | 8907,5s. and 9 pence over.

£4453 and 15s. over.

∴ 4275639 farthings = £4453 15s. $9\frac{3}{4}d.$

These methods of expressing a given sum of money in another, but equivalent form are included in the word *Reduction.*

Examples. (lxiii)

Reduce to pence and farthings the following numbers of farthings:

(1) 57. (2) 173. (3) 197.

Reduce to shillings, pence and farthings the following numbers of farthings:

(4) 357. (5) 479. (6) 747.

Reduce to £ *s.* *d.* the following numbers of farthings:

(7) 4238. (8) 376289. (9) 542380.

133. The copper coins in use in Great Britain are the Farthing, the Halfpenny, and the Penny.

The silver coins in use are the Crown, (5*s.*), the Half-crown (2*s.* 6*d.*), the Florin (2*s.*), the Shilling, the Sixpence, the Fourpenny piece (or Groat), and the Threepenny piece.

The gold coins in use are the Sovereign or Pound, and the Half-sovereign. The Guinea (21*s.*) and the Half-guinea (10*s.* 6*d.*) are not in use, but reference is frequently made to them.

The following facts should be remembered:

4 Threepenny pieces make 1 shilling.
3 Fourpenny .... 1 shilling.
5 Sixpences .... 1 half-crown.
40 Sixpences .... 1 pound.
8 Half-crowns .... 1 pound.
21 Sixpences .... 1 half-guinea.
42 Sixpences .... 1 guinea.

Ex. (1). How many fourpenny pieces are there in £2. 12*s.*?

£ *s.*
2 . 12
20
———
52 shillings.
3
———
156 fourpenny pieces.

Ex. (2). How many half-crowns are there in £26. 17*s*. 6*d*.?

£26. = (8 × 26) half-crowns = 208 half-crowns,
17*s*. 6*d*. = 7 half-crowns;
∴ £26. 17*s*. 6*d*. = 208 + 7, or 215 half-crowns.

Ex. (3). In 24381 half-crowns how many half-guineas are there?

$$\begin{array}{r|l} & \text{half-crowns.} \\ & 24381 \\ & \quad\;\; 5 \\ \hline 21 \begin{cases} 7 \\ 3 \end{cases} & \begin{array}{l} 121905 \text{ sixpences} \\ \hline 17415 \\ \hline 5805 \text{ half-guineas.} \end{array} \end{array}$$

Examples. (lxiv)

(1) How many half-crowns are there in £27. 12*s*. 6*d*.?
(2) How many guineas are there in £2352?
(3) How many pounds are there in 4360 guineas?
(4) How many half-guineas are there in 15225 half-crowns?
(5) How many fourpenny pieces are there in 627 florins?
(6) How many florins are there in 246 guineas?
(7) How many half-crowns are there in 42375 half-guineas?

134. The following tables should be committed to memory, and the effect tested by writing down at sight the results in the set of Examples that follow the Tables.

Pence Table.

	s.	*d.*		*s.*	*d.*
12 pence are	1	0	84 pence are	7	0
20	1	8	90	7	6
24	2	0	96	8	0
30	2	6	100	8	4
36	3	0	108	9	0
40	3	4	110	9	2
48	4	0	120	10	0
50	4	2	130	10	10
60	5	0	132	11	0
70	5	10	140	11	8
72	6	0	144	12	0
80	6	8	150	12	6

Shillings Table.

	£	s.		£	s.
20 shillings are	1	0	130 shillings are	6	10
30	1	10	140	7	0
40	2	0	150	7	10
50	2	10	160	8	0
60	3	0	170	8	10
70	3	10	180	9	0
80	4	0	190	9	10
90	4	10	200	10	0
100	5	0	300	15	0
110	5	10	400	20	0
120	6	0	500	25	0

Examples. (lxv)

Express in terms of pence and farthings the following number of farthings:

(1) 5.	(2) 7.	(3) 11.	(4) 15.
(5) 17.	(6) 19.	(7) 21.	(8) 27.
(9) 30.	(10) 35.	(11) 36.	(12) 39.
(13) 42.	(14) 47.	(15) 59.	(16) 63.
(17) 69.	(18) 75.	(19) 87.	(20) 94.

Express in terms of shillings and pence the following number of pence:

(21) 19.	(22) 23.	(23) 27.	(24) 33.
(25) 39.	(26) 43.	(27) 57.	(28) 68.
(29) 74.	(30) 86.	(31) 99.	(32) 105.
(33) 117.	(34) 126.	(35) 134.	(36) 145.
(37) 163.	(38) 179.	(39) 195.	(40) 247.

Express in terms of pounds and shillings the following number of shillings:

(41) 27.	(42) 39.	(43) 57.	(44) 79.
(45) 93.	(46) 107.	(47) 129.	(48) 145.
(49) 176.	(50) 198.	(51) 235.	(52) 247.
(53) 258.	(54) 273.	(55) 297.	(56) 345.
(57) 373.	(58) 412.	(59) 437.	(60) 459.

COMPOUND ADDITION.

135. In adding compound expressions together, we follow the principles which regulate the process of Addition in the case of pure numbers.

Thus in adding sums of money we arrange them so that the pounds stand under pounds in vertical columns, shillings under shillings, pence under pence, and farthings under farthings. For example, if we have to add together 4*s.* $3\frac{1}{4}$*d.*, 3*s.* $3\frac{1}{2}$*d.*, 5*s.* 4*d.*, and 17*s.* $9\frac{3}{4}$*d.*, we arrange them thus:

$$\begin{array}{rr} s. & d. \\ 4\ . & 3\frac{1}{4} \\ 3\ . & 3\frac{1}{2} \\ 5\ . & 4 \\ 17\ . & 9\frac{3}{4} \\ \hline \pounds 1\ .\ 10\ . & 8\frac{1}{2} \end{array}$$

Adding the column of farthings, we find its sum to be 6 farthings, and this being equivalent to 1 penny and 2 farthings, we place $\frac{1}{2}$ under the column of farthings, and carry on 1 for addition to the column of pence.

The sum of the column of pence, increased by 1, we find to be 20 pence, and this being equivalent to 1 shilling and 8 pence, we place 8 under the column of pence, and carry on 1 for addition to the column of shillings.

The sum of the columns of shillings, increased by 1, we find to be 30 shillings, and this being equivalent to 1 pound and 10 shillings, we place 10 under the columns of shillings, and set down £1 by itself on the left hand.

Again, if we have to add together £26. 4*s.* $9\frac{3}{4}$*d.*, £32. 12*s.* $7\frac{1}{4}$*d.*, £245. 0*s.* 2*d.*, £7. 15*s.* $8\frac{1}{2}$*d.*, and 4*s.* $8\frac{3}{4}$*d.*, we arrange them thus:

$$\begin{array}{rrr} \pounds & s. & d. \\ 26\ . & 4\ . & 9\frac{3}{4} \\ 32\ . & 12\ . & 7\frac{1}{4} \\ 245\ . & 0\ . & 2 \\ 7\ . & 15\ . & 8\frac{1}{2} \\ 0\ . & 4\ . & 8\frac{3}{4} \\ \hline \pounds 311\ . & 18\ . & 0\frac{1}{4} \end{array}$$

Adding the column of farthings, we find its sum to be 9 farthings, and this being equivalent to 2 pence and 1 farthing, we place $\frac{1}{4}$ under the column of farthings, and carry on 2 for addition to the column of pence.

The sum of the column of pence, increased by 2. we find to be 36 pence, and this being equivalent to 3 shillings, we place 0 under the column of pence, and carry on 3 for addition to the column of shillings.

The sum of the columns of shillings, increased by 3, we find to be 38 shillings, and this being equivalent to 1 pound and 18 shillings, we place 18 under the columns of shillings, and carry on 1 for addition to the columns of pounds.

The sum of the columns of pounds, increased by 1, we find to be 311, which we place under those columns, and the sum is complete.

Examples. (lxvi)

Perform the operation of addition on the following sums of money.

(1) *d.*	(2) *d.*	(3) *d.*	(4) *d.*
$3\frac{1}{4}$	$5\frac{1}{2}$	$4\frac{3}{4}$	2
$2\frac{1}{2}$	$1\frac{1}{4}$	$\frac{1}{2}$	$3\frac{3}{4}$
$2\frac{3}{4}$	$2\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{2}$
$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{3}{4}$	$3\frac{3}{4}$

(5) *s.* *d.*	(6) *s.* *d.*	(7) *s.* *d.*	(8) *s.* *d.*
4 . 7	6 . 8	5 . 9	7 . 4
3 . 2	1 . 9	4 . 2	4 . 0
4 . 6	2 . 5	2 . 11	3 . 11
4 . 9	3 . 10	3 . 8	1 . 9
2 . 10	4 . 7	1 . 10	1 . 5

(9) *s.* *d.*	(10) *s.* *d.*	(11) *s.* *d.*	(12) *s.* *d.*
3 . $2\frac{1}{2}$	2 . $4\frac{1}{2}$	5 . $6\frac{3}{4}$	6 . $0\frac{1}{2}$
4 . $3\frac{3}{4}$	7 . $9\frac{1}{2}$	2 . $9\frac{1}{2}$	1 . $5\frac{3}{4}$
1 . $5\frac{3}{4}$	1 . $10\frac{3}{4}$	3 . $10\frac{3}{4}$	3 . $8\frac{3}{4}$
2 . $6\frac{1}{2}$	3 . $4\frac{1}{2}$	1 . $8\frac{1}{4}$	4 . $11\frac{1}{2}$
2 . $4\frac{1}{4}$	2 . $8\frac{1}{4}$	4 . $7\frac{1}{2}$	2 . $7\frac{3}{4}$

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(13)	3	5	2	(14)	5	8	3	(15)	6	8	7	(16)	7	6	8
	4	6	8		7	9	6		4	6	3		5	8	4
	7	9	3		3	4	9		8	9	10		9	6	0
	2	4	10		6	5	2		9	7	6		7	4	11
	4	9	2		9	0	4		4	3	0		2	6	10

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(17)	3	4	$3\frac{1}{2}$	(18)	4	7	$5\frac{3}{4}$	(19)	7	8	$4\frac{3}{4}$	(20)	6	2	$5\frac{1}{4}$
	2	5	$4\frac{1}{4}$		6	8	$9\frac{1}{2}$		6	9	2		5	3	$2\frac{3}{4}$
	7	6	$8\frac{1}{2}$		9	5	$2\frac{1}{4}$		5	2	$7\frac{1}{2}$		7	8	$4\frac{1}{2}$
	6	9	$6\frac{1}{4}$		8	7	$4\frac{1}{2}$		6	3	9		8	9	1
	4	7	$9\frac{3}{4}$		5	9	$6\frac{3}{4}$		7	5	$4\frac{1}{2}$		5	3	$3\frac{1}{2}$

(21) £	s.	d.	(22) £	s.	d.	(23) £	s.	d.	(24) £	s.	d.
16	19	4	26	9	2	17	9	10	21	11	3
14	13	2	13	0	11	61	11	4	37	5	9
67	8	10	9	16	4	18	5	9	4	6	2
42	5	8	67	17	8	28	14	7	17	17	7
12	7	9	24	19	2	21	3	7	39	18	5
15	10	4	39	15	3	93	14	6	47	11	10

	£	s.	d.		£	s.	d.		£	s.	d.
(25)	16	3	$7\frac{1}{2}$	(26)	35	8	$9\frac{1}{2}$	(27)	143	17	$9\frac{1}{2}$
	78	13	$9\frac{1}{4}$		76	10	$4\frac{3}{4}$		876	11	$2\frac{1}{4}$
	21	17	$4\frac{3}{4}$		25	8	9		972	9	10
	5	3	7		4	0	$7\frac{1}{2}$		397	4	$3\frac{1}{2}$
	36	11	$4\frac{1}{4}$		36	12	$5\frac{1}{4}$		674	4	$10\frac{1}{2}$
	42	8	$2\frac{1}{2}$		42	5	$9\frac{3}{4}$		538	9	$6\frac{1}{4}$

	£	s.	d.		£	s.	d.		£	s.	d.
(28)	5729	14	$9\frac{1}{2}$	(29)	9087	15	$7\frac{1}{2}$	(30)	432	17	$9\frac{3}{4}$
	627	7	$5\frac{3}{4}$		609	4	8		25	6	$3\frac{1}{2}$
	906	2	$11\frac{1}{2}$		2430	7	$8\frac{3}{4}$		347	19	$11\frac{3}{4}$
	4312	17	$4\frac{1}{4}$		56	14	7		5429	8	$7\frac{1}{4}$
	127	12	$9\frac{1}{2}$		401	12	$9\frac{1}{2}$		276	15	$2\frac{1}{2}$
	3097	4	$10\frac{1}{4}$		9320	5	$8\frac{1}{4}$		4329	3	$8\frac{3}{4}$

	£	s.	d.		£	s.	d.		£	s.	d.
(31)	2	11	$5\frac{1}{2}$	(32)	27	14	$2\frac{1}{4}$	(33)	35	17	3
	5	9	$6\frac{1}{4}$		39	7	$5\frac{3}{4}$		59	14	$7\frac{1}{4}$
	7	3	$8\frac{3}{4}$		57	18	11		97	13	$5\frac{1}{4}$
	3	15	2		48	11	$8\frac{1}{2}$		47	12	$3\frac{1}{2}$
	6	19	$9\frac{1}{2}$		75	13	$9\frac{1}{2}$		84	9	$10\frac{3}{4}$
	8	17	$11\frac{3}{4}$		91	16	$2\frac{1}{4}$		78	8	4

	£	s.	d.		£	s.	d.		£	s.	d.
(34)	257	2	$3\frac{1}{4}$	(35)	527	18	$2\frac{3}{4}$	(36)	274	0	$4\frac{1}{2}$
	349	8	$11\frac{1}{2}$		821	7	$4\frac{1}{4}$		3	5	$3\frac{1}{4}$
	546	19	9		937	16	$9\frac{1}{2}$		46	9	$7\frac{3}{4}$
	847	18	$2\frac{3}{4}$		742	9	$10\frac{3}{4}$		4327	14	$9\frac{1}{2}$
	437	17	$5\frac{1}{2}$		497	13	$2\frac{1}{2}$		562	14	$0\frac{3}{4}$
	859	4	$10\frac{1}{4}$		865	17	$3\frac{3}{4}$		4	5	2

	£	s.	d.		£	s.	d.		£	s.	d.
(37)	243	14	9	(38)	2476	18	6	(39)	1846	12	9
	57	12	8		4329	4	8		47	13	8
	286	5	6		427	2	9		247	15	11
	1247	18	9		1004	13	6		3046	10	2
	15	4	2		17	9	11		5724	12	9
	9	13	6		427	18	10		47	9	10
	427	4	0		4306	7	9		537	18	8
	2357	10	0		742	19	8		2457	19	6
	140	15	2		407	12	7		863	13	8
	207	3	8		46	0	2		45	5	2
	49	15	2		17	4	10		304	15	9

	£	s.	d.		£	s.	d.		£	s.	d.
(40)	4237	7	10	(41)	6293	8	4	(42)	756	8	3
	5396	14	3		527	19	5		2479	13	1
	257	3	2		7249	12	7		3597	9	9
	4976	15	8		9627	6	9		627	0	5
	2749	0	5		357	13	8		493	12	7
	276	19	11		246	18	2		5	8	9
	7498	9	4		7538	7	10		14	19	2
	5683	12	3		6289	12	9		256	8	7
	562	18	9		47	8	4		4875	13	9
	427	9	8		56	9	2		5792	15	6
	64	12	5		7423	19	8		897	12	8
	4378	5	10		2	5	11		748	0	2
	8642	17	6		357	2	9		57	4	3
	957	14	2		6579	8	7		8672	16	11

	£	s.	d.		£	s.	d.		£	s.	d.
(43)	672	9	4	(44)	8746	2	7	(45)	569	18	3
	856	13	2		437	9	5		8423	9	11
	4397	11	10		724	8	9		527	0	8
	5628	4	9		56	7	11		427	6	9
	945	12	8		4329	19	2		7	19	6
	76	18	3		7	18	5		4263	11	4
	4397	9	10		5827	13	8		7423	4	9
	6258	7	7		659	8	7		467	18	3
	427	9	4		56	7	8		857	13	5
	4	13	6		648	12	6		75	16	4
	4723	7	9		8742	9	4		10	4	3
	607	4	8		1043	2	5		5016	2	9
	4000	4	9		406	3	8		578	3	8
	743	12	5		27	8	9		53	17	10

	£	s.	d.		£	s.	d.		£	s.	d.
(46)	6548	12	7	(47)	847	13	9	(48)	4269	18	2
	304	13	9		42	15	8		5007	3	11
	7240	6	11		7437	9	10		346	9	3
	4379	7	2		508	12	5		460	13	10
	5	10	7		473	8	9		757	9	11
	7612	13	8		1047	1	4		3591	5	7
	420	7	8		11	11	7		406	2	9
	6712	4	9		2374	15	8		4	2	4
	9125	0	7		476	13	4		58	3	10
	709	14	5		60	0	7		7258	14	2
	1230	5	9		4720	6	5		8423	17	9
	46	19	4		582	17	3		657	18	11
	725	8	11		68	12	5		728	3	5
	4036	0	9		7409	14	2		6004	4	8

	£	s.	d.		£	s.	d.		£	s.	d.
(49)	7245	18	6	(50)	287	9	11	(51)	8746	13	9
	327	0	4		463	9	2		4073	15	2
	407	9	9		4704	8	10		5209	3	8
	5806	11	2		5608	13	5		756	17	2
	13	12	11		75	9	7		8648	0	10
	5008	15	8		724	12	4		7932	12	5
	732	4	9		795	10	3		6740	3	9
	36	0	4		6618	15	2		4008	15	4
	597	3	10		4	0	9		67	13	8
	12	4	8		437	12	11		426	5	9
	8066	17	6		76	2	9		7328	4	6
	7403	13	2		7430	5	4		8007	15	2
	258	0	5		685	17	6		59	9	11
	94	6	8		11	14	7		720	0	8

COMPOUND SUBTRACTION.

136. The process of subtracting one compound quantity from another is founded on the principles explained in Art. 20, and the following Example will supply all that is necessary to make the method clear:

	£	s.	d.
From	27 .	5 .	$2\frac{1}{4}$
Take	13 .	17 .	$4\frac{1}{2}$
	£13 .	7 .	$9\frac{3}{4}$

Arranging the columns as in Addition, we reason thus we cannot take 2 farthings from 1 farthing, and we therefore add 4 farthings to the 1 farthing, making 5 farthings, and taking 2 farthings from 5 farthings we obtain as a remainder 3 farthings, which we set down under the column of farthings.

We then add, by way of compensation, 1 penny to the 4 pence in the lower line. We have then to take 5 pence from 2 pence, and as we cannot do this, we add 12 pence to the 2 pence, making 14 pence, and taking 5 pence from 14 pence, we obtain as a result 9 pence, which we place under the column of pence.

We then add, by way of compensation, 1 shilling to the 17 shillings in the lower line. We have then to take 18 shillings from 5 shillings, and as we cannot do this, we add 20 shillings to the 5 shillings, making 25 shillings, and taking 18 shillings from 25 shillings, we obtain as a result 7 shillings, which we place under the column of shillings.

Finally, we add, by way of compensation, 1 pound to the 13 pounds, and we take 14 pounds from 27 pounds, obtaining as a remainder 13 pounds, which we place under the column of pounds.

Examples. (lxvii)

		£	s.	d.		£	s.	d.
(1)	From	94 .	12 .	7	take	58 .	9 .	2.
(2)		75 .	9 .	6	,,	47 .	8 .	8.

		£ s. d.		£ s. d.
(3)	From	58 . 13 . 4	take	49 . 14 . 5
(4)	,,	276 . 17 . $5\frac{1}{2}$	,,	37 . 19 . $7\frac{1}{4}$
(5)	,,	1247 . 5 . $10\frac{1}{4}$	,,	1246 . 11 . $8\frac{1}{2}$
(6)	,,	3000 . 10 . $7\frac{1}{2}$	,,	2998 . 13 . $11\frac{3}{4}$
(7)	,,	199 . 0 . $0\frac{1}{4}$	,,	198 . 19 . $10\frac{1}{2}$
(8)	,,	80609 . 5 . $2\frac{3}{4}$	,,	79089 . 12 . $5\frac{1}{4}$
(9)	,,	44005 . 7 . $6\frac{1}{4}$	,,	7896 . 10 . $2\frac{1}{2}$
(10)	,,	30704 . 0 . 5	,,	29484 . 0 . $6\frac{1}{2}$

COMPOUND MULTIPLICATION.

137. To multiply a compound expression, as £4. 8*s.* $9\frac{3}{4}$*d.*, by a number, as 9, is equivalent to taking the sum of nine expressions, each equal to £4. 8*s.* $9\frac{3}{4}$*d.* Instead of writing these expressions one under the other, and finding their sum by the process of addition, we obtain the required result by multiplying each of the four quantities, composing the expression, separately by 9, calculating the value of each result as in addition, setting down part of those results under the several columns, and carrying on part, as in addition, thus:

$$\begin{array}{r} \text{£} \quad s. \quad d. \\ 4 \;.\; 8 \;.\; 9\tfrac{3}{4} \\ 9 \\ \hline \text{£}39 \;.\; 19 \;.\; 3\tfrac{3}{4} \end{array}$$

The process may be more fully explained thus:

9 times 3 farthings = 27 farthings = $6\frac{3}{4}$*d.*: set down $\frac{3}{4}$ under the column of farthings, and carry on 6 to the pence.

9 times 9 pence = 81 pence, and 6 pence added gives 87 pence = 7*s.* 3*d.*: set down 3 under the column of pence, and carry on 7 to the shillings.

9 times 8 shillings = 72 shillings, and 7 shillings added gives 79 shillings = £3. 19*s.*: set down 19 under the column of shillings, and carry on 3 to the pounds.

9 times 4 pounds = 36 pounds, and 3 pounds added gives 39 pounds, which is set down under the pounds.

138. When the multiplier can be split up into factors, each of which is not greater than 12, we multiply the compound expression first by one of the factors, and then multiply the product by another of the factors, as in the case of Simple Multiplication.

Thus if we have to multiply £12. 4*s*. $7\frac{1}{2}$*d*. by 15, we multiply first by 5, and the product by 3, thus:

£	*s*.	*d*.	
12	4	$7\frac{1}{2}$	
		5	
61	3	$1\frac{1}{2}$	Product by 5.
		3	
£183	9	$4\frac{1}{2}$	Product by 15.

Again, to multiply £17. 14*s*. 9*d*. by 180, we may proceed thus:

£	*s*.	*d*.	
17	14	9	
		10	
177	7	6	Product by 10.
		6	
1064	5	0	Product by 60.
		3	
£3192	15	0	Product by 180.

Examples. (lxviii)

Find the value of

(1) 4 things at 7*s*. 3*d*. each. (2) 5 at 14*d*. (3) 6 at $7\frac{1}{2}$*d*.
(4) 7 at 9*s*. 6*d*. (5) 8 at 2*s*. 4*d*. (6) 10 at 2*s*. $2\frac{1}{2}$*d*.
(7) 11 at £2. 1*s*. 4*d*. (8) 12 at £1. 4*s*. 3*d*.
(9) 14 at 17*s*. 6*d*. (10) 15 at 7*s*. $10\frac{1}{2}$*d*. (11) 16 at 27*s*.
(12) 18 at 17*s*. 6*d*. (13) 20 at £5. 11*s*. 4*d*.
(14) 21 at 5*s*. $7\frac{1}{2}$*d*. (15) 22 at £5. 11*s*. 4*d*.
(16) 24 at £4. 7*s*. 2*d*. (17) 25 at 4*s*. 6*d*.
(18) 27 at 5*s*. $11\frac{1}{2}$*d*. (19) 28 at 2*s*. 8*d*. (20) 30 at £1. 12*s*.
(21) 33 at £1. 2*s*. (22) 35 at £1. 2*s*. 6*d*.

(23) 36 at 6*s*. 2½*d*. (24) 42 at £1. 12*s*. 6*d*.
(25) 44 at 19*s*. 10*d*. (26) 45 at 19*s*. 4*d*. (27) 48 at 3*s*. 7*d*.
(28) 50 at 2*s*. 5½*d*. (29) 77 at 3*s*. 2¼*d*. (30) 224 at 3½*d*.
(31) 336 at 5½*d*. (32) 360 at 5*s*. 4*d*. (33) 560 at 1*s*. 4*d*.

139. When the multiplier cannot be split up into factors, we may proceed as in the following examples:

Ex. (1). To multiply £17. 12*s*. 9¼*d*. by 79.

	£	*s*.	*d*.	
	17	12	9¼	
			10	
	176	7	8½	Product by 10.
			7	
	1234	13	11½	Product by 70.
Multiplying first line by 9	158	14	11¼	Product by 9.
Adding the last two results	£1393	8	10¾	Product by 79.

Ex. (2). To multiply £3. 17*s*. 9½*d*. by 3296.

	£	*s*.	*d*.	
	3	17	9½	
			10	
	38	17	11	Product by 10.
			10	
	388	19	2	Product by 100.
			10	
	3889	11	8	Product by 1000.
			3	
	11668	15	0	Product by 3000.
Multiplying 5th line by 2	777	18	4	,, by 200.
Multiplying 3rd line by 9	350	1	3	,, by 90.
Multiplying 1st line by 6	23	6	9	,, by 6.
Adding last four results	£12820	1	4	Product by 3296.

140. The following is a method by which the process of multiplying a compound quantity by a number greater than 1000 is somewhat shortened. We take as an illus-

tration the example just worked. The process is so simple that no verbal explanation is necessary.

```
 2 | 3296
   |_____
 d. 1648
    29664  the result of multiplying the top line by 9.
   _______
12 | 31312
   |______
 s. 2609 and 4d.
    23072 } the result of multiplying the top line by 17.
    3296  }
   _______
2,0| 5864,1
   |_______
 £ 2932 and 1s.
    9888   the result of multiplying the top line by 3.
   _______
 £12820 . 1s. . 4d.
```

Examples. (lxix)

Find the value of

(1) 29 things at 4*s.* 6*d.* each. (2) 39 at 12*s.* $6\frac{1}{2}$*d.*

(3) 47 at 1*s.* $6\frac{1}{2}$*d.* (4) 71 at 1*s.* 8*d.*

(5) 89 at 6*s.* 8*d.* (6) 123 at 5*s.* $6\frac{1}{2}$*d.*

(7) 145 at £1. 3*s.* 2*d.* (8) 2154 at £7. 1*s.* 3*d.*

(9) 3210 at £1. 18*s.* $6\frac{3}{4}$*d.* (10) 2175 at £2. 15*s.* $4\frac{1}{2}$*d.*

(11) 3684 at £2. 6*s.* $9\frac{1}{4}$*d.*

141. In retail transactions quickness of calculation is attained by the following method:

To find the cost of 12 *articles at* $3\frac{1}{4}$*d. each.*

The cost of 12 articles at 1*d.* each is 1 shilling.
,, ,, at $\frac{1}{4}$*d.* each is 3 pence,
,, ,, at $3\frac{1}{4}$*d.* each is 3*s.* 3*d.*

Hence we deduce the following rule:

To find the cost of 12 articles when the price of each is given in pence and farthings, take as many *shillings* as

there are *pence* in the given price, and as many *threepences* as there are *farthings.*

Thus, 12 articles at $5\frac{3}{4}d.$ each will cost $5s.\ 9d.$
,, at $7\frac{1}{2}d.$,, ,, $7s.\ 6d.$
,, at $9\frac{1}{4}d.$,, ,, $9s.\ 3d.$
,, at $17\frac{1}{2}d.$,, ,, $17s.\ 6d.$

Hence, value of 24 articles at $5d. = 2 \times 5s. = 10s.$
,, ,, ,, at $4\frac{3}{4}d. = 2 \times (4s.\ 9d.) = 9s.\ 6d.$
,, 36 ,, at $7\frac{1}{2}d. = 3 \times (7s.\ 6d.) = 22s.\ 6d.$
,, ,, ,, at $16\frac{1}{4}d. = 3 \times (16s.\ 3d.) = 48s.\ 9d.$;
and so, for any *multiple* of 12.

Thus also we obtain an easy method of calculation in such cases as the following.

Cost of 15 articles at $2\frac{1}{2}d. =$ cost of 12 + cost of 3
$= 2s.\ 6d. + 7\frac{1}{2}d. = 3s.\ 1\frac{1}{2}d.$

,, 57 ,, at $4\frac{1}{2}d. =$ cost of 48 + cost of 9
$= 18s. + 3s.\ 4\frac{1}{2}d. = 21s.\ 4\frac{1}{2}d.$

,, 111 ,, at $7\frac{1}{4}d. =$ cost of 108 + cost of 3
$= 65s.\ 3d. + 1s.\ 9\frac{3}{4}d. = 67s.\ 0\frac{3}{4}d.$

Examples. (lxx)

Write down, without going through the process of multiplication, the cost of

(1) 3 things at $2\frac{1}{4}d.$	(2) 5 at $3\frac{1}{2}d.$	(3) 7 at $4\frac{3}{4}d.$
(4) 8 at $7\frac{1}{2}d.$	(5) 6 at $9\frac{3}{4}d.$	(6) 9 at $11\frac{1}{2}d.$
(7) 4 at $10\frac{3}{4}d.$	(8) 11 at $3\frac{1}{4}d.$	(9) 10 at $4\frac{1}{2}d.$
(10) 7 at $10\frac{3}{4}d.$	(11) 9 at $7\frac{1}{4}d.$	(12) 11 at $11\frac{1}{2}d.$
(13) 12 at $4\frac{1}{4}d.$	(14) 12 at $7\frac{1}{2}d.$	(15) 12 at $9\frac{3}{4}d.$
(16) 12 at $11\frac{1}{4}d.$	(17) 12 at $1s.\ 2\frac{1}{2}d.$	(18) 12 at $1s.\ 4\frac{3}{4}d.$
(19) 12 at $1s.\ 5\frac{1}{4}d.$	(20) 12 at $1s.\ 7\frac{3}{4}d.$	(21) 13 at $4\frac{1}{2}d.$
(22) 14 at $5\frac{1}{4}d.$	(23) 15 at $7\frac{1}{4}d.$	(24) 16 at $8\frac{1}{2}d.$
(25) 17 at $5\frac{1}{2}d.$	(26) 18 at $9\frac{3}{4}d.$	(27) 19 at $8\frac{1}{4}d.$
(28) 20 at $5\frac{3}{4}d.$	(29) 26 at $2\frac{1}{2}d.$	(30) 28 at $4\frac{1}{4}d.$
(31) 33 at $5\frac{1}{2}d.$	(32) 37 at $9\frac{3}{4}d.$	(33) 41 at $7\frac{3}{4}d.$
(34) 43 at $8\frac{1}{2}d.$	(35) 57 at $2\frac{1}{4}d.$	(36) 73 at $3\frac{1}{2}d.$
(37) 87 at $4\frac{1}{2}d.$	(38) 90 at $5\frac{1}{2}d.$	(39) 97 at $9\frac{1}{4}d.$

COMPOUND DIVISION.

142. The process of dividing a compound quantity by a number is based upon the principles explained in the case of Simple Division, as will be seen from the following Examples.

Ex. (1). To divide £13. 17*s.* $1\frac{1}{2}$*d.* by 9.

$$\begin{array}{r|l} & \quad \pounds \quad\;\; s. \quad\; d. \\ 9) & 13 \;.\; 17 \;.\; 1\frac{1}{2} \\ \hline & \pounds 1 \;.\; 10 \;.\; 9\frac{1}{2} \text{ Quotient.} \end{array}$$

We reason thus:

£13 divided by 9 gives £1 as quotient and £4 remainder;
£4 = 80 shillings, and 17 shillings added gives 97 shillings.
97*s.* divided by 9 gives 10*s.* as quotient and 7*s.* remainder;
7*s.* = 84 pence, and 1 penny added gives 85 pence.
85*d.* divided by 9 gives 9*d.* as quotient and 4*d.* remainder;
4*d.* = 16 farthings, and 2 farthings added gives 18 farthings.
18*q.* divided by 9 gives 2*q.* as quotient, and no remainder.

Ex. (2). To divide £51. 15*s.* 5*d.* by 35.

$$\text{The factors of 35 are} \left\{ \begin{array}{r|l} & \;\pounds \quad\; s. \quad d. \\ 5 & 51 \;.\; 15 \;.\; 5 \\ \hline 7 & 10 \;.\; 7 \;.\; 1 \\ \hline & \pounds 1 \;.\; 9 \;.\; 7 \text{ Quotient.} \end{array} \right.$$

Ex. (3). To divide £53. 15*s.* 8*d.* by 112.

$$\text{The factors of 112 are} \left\{ \begin{array}{r|l} & \;\pounds \quad\; s. \quad d. \\ 4 & 53 \;.\; 15 \;.\; 8 \\ \hline 4 & 13 \;.\; 8 \;.\; 11 \\ \hline 7 & 3 \;.\; 7 \;.\; 2\frac{3}{4} \\ \hline & \quad\;\; 9 \;.\; 7\frac{1}{4} \text{ Quotient.} \end{array} \right.$$

Ex. (4). Divide £119232. 1*s*. 10½*d*. by 3465.

```
          £       s.   d.
3465)119232 . 1 . 10½(£34
     10395
     -----
      15282
      13860
      -----
       1422
         20
      -----
  3465)28441(8s.
       27720
       -----
         721
          12
       -----
   3465)8662(2d.
        6930
        ----
        1732
           4
       -----
   3465)6930(2q.
        6930
        ----
```

∴ the Quotient is £34. 8*s*. 2½*d*.

Examples. (lxxi)

I. Divide

(1) £1. 3*s*. 7½*d*. by 3. (2) £39. 7*s*. 6*d*. by 7.

(3) £11. 3*s*. 6*d*. by 12. (4) £43. 12*s*. 8*d*. by 11.

(5) £6. 2*s*. 11*d*. by 10. (6) £22. 11*s*. 6*d*. by 12.

II. Divide

(1) £98. 11*s*. 9*d*. by 54. (2) £13. 7*s*. 9*d*. by 63.

(3) £29. 14*s*. 0*d*. by 108. (4) £15. 8*s*. by 132.

(5) £3. 9*s*. 4½*d*. by 45. (6) £43. 12*s*. 8*d*. by 44.

III. Divide

(1) £167. 19*s*. 2*d*. by 145.

(2) £40. 8*s*. $4\frac{1}{4}$*d*. by 241.

(3) £453. 11*s*. $9\frac{1}{4}$*d*. by 365.

(4) £40669. 2*s*. 1*d*. by 9652.

(5) £93. 1*s*. $2\frac{1}{4}$*d*. by 291.

(6) £139. 3*s*. 6*d*. by 117.

IV. Find the value of the following

(1) $\frac{£13.\ 17s.\ 1\frac{1}{2}d.}{9}$

(2) $\frac{£86.\ 16s.\ 4\frac{3}{4}d.}{11}$

(3) $\frac{£8.\ 4s.\ 3d.}{12}$

(4) $\frac{£1.\ 9s.}{48}$

(5) $\frac{£53.\ 15s.\ 8d.}{112}$

(6) $\frac{£14}{240}$

(7) $\frac{£155\ 7s.\ 6d.}{1320}$

(8) £122. 15*s*. 4*d*.÷58.

(9) £70. 3*s*. $2\frac{3}{4}$*d*.÷95.

(10) £167. 4*s*. 3*d*. ÷117.

(11) £71. 10*s*. 6*d*.÷125.

(12) £120. 10*s*. 7*d*.÷216.

(13) £2184. 17*s*. 8*d*.÷504.

143. One quantity is contained in another of the same kind as often as the measure of the first is contained in the measure of the second, the same unit of measurement being taken in both cases.

Ex. (1). How many times is 1*s*. 1*d*. contained in 16*s*. 3*d*.?

1*s*. 1*d*. = 13*d*.; and 16*s*. 3*d*. = 195*d*.

Now 13 is contained 15 times in 195;

∴ 13*d*. is contained 15 times in 195*d*.

Ex. (2). How many times is £4. 3*s*. 2*d*. contained in £87. 6*s*. 6*d*.?

£4. 3*s*. 2*d*. = 998*d*.; and £87. 6*s*. 6*d*. = 20958*d*.

Now 23258÷998 = 21;

∴ £4. 3*s*. 2*d*. is contained in £87. 6*s*. 6*d*. 21 times.

Examples. (lxxii)

(1) How many times is £346. 16*s.* contained in £34680?

(2) .... £5. 11*s.* 4*d.* .... £122. 9*s.* 4*d.*?

(3) .... £1. 12*s.* 6*d.* .... £68. 5*s.*?

(4) .... £17. 12*s.* $9\frac{1}{4}d.$ ... £1393. 8*s.* $10\frac{3}{4}d.$?

(5) Among how many persons must £641. 14*s.* $11\frac{1}{4}d.$ be divided, so that the share of each may be £2. 15*s.* $6\frac{1}{4}d.$?

(6) Divide £17 into an equal number of sovereigns, half-sovereigns, half-crowns, shillings, and sixpences.

FRACTIONAL MULTIPLICATION AND DIVISION OF MONEY.

144. Ex. (1). Find the value of $\frac{3}{4}$ of 14*s.* 8*d.*

$$\tfrac{1}{4} \text{ of } 14s.\ 8d. = \frac{14s.\ 8d.}{4} = 3s.\ 8d.;$$

$$\therefore \tfrac{3}{4} \text{ of } 14s.\ 8d. = 3 \times 3s.\ 8d. = 11s.$$

It is immaterial whether we divide by 4, and then multiply the quotient by 3, or first multiply by 3, and then divide the product by 4, thus:

$$\tfrac{3}{4} \text{ of } 14s.\ 8d. = \frac{3 \times 14s.\ 8d.}{4} = \frac{44s.}{4} = 11s.$$

Ex. (2). Find the value of $\frac{2}{3}$ of $\frac{5}{7}$ of £43. 4*s.* 6*d.*

$$\tfrac{2}{3} \text{ of } \tfrac{5}{7} \text{ of } £43.\ 4s.\ 6d. = \tfrac{10}{21} \text{ of } £43.\ 4s.\ 6d.$$
$$= \frac{10 \times £43.\ 4s.\ 6d.}{21}$$
$$= 10 \times £2.\ 1s.\ 2d. = £20.\ 11s.\ 8d.$$

Ex. (3). What is the value of $2\frac{3}{7}$ of 14*s.* 9*d.*?

$$2\tfrac{3}{7} \text{ of } 14s.\ 9d. = \tfrac{17}{7} \text{ of } 177d.$$
$$= \frac{17 \times 177d.}{7} = \frac{3009d.}{7} = 429\tfrac{6}{7}d. = £1.\ 15s.\ 9\tfrac{6}{7}d.$$

NOTE.—To find the value of $\frac{3}{5} \times$ 2*s.* 9*d.*, we extend the meaning of the sign × (as is explained in Art. 65), and replace it by the word *of.*

Thus $\frac{3}{5} \times 2s.\ 9d. = \frac{3}{5}$ of 2*s.* 9*d.* $= \frac{8s.\ 3d.}{5} =$ 1*s.* $7\frac{4}{5}d.$

Ex. (4). Divide 4*s*. 2*d*. by $\frac{5}{8}$;

$$4s.\ 2d. \div \tfrac{5}{8} = 4s.\ 2d. \times \tfrac{8}{5}$$
$$= \tfrac{8}{5} \text{ of } 4s.\ 2d. = 8 \times 10d. = 6s.\ 8d.$$

Ex. (5). Divide £4. 3*s*. 9*d*. by $2\frac{2}{3}$;

$$£4.\ 3s.\ 9d. \div 2\tfrac{2}{3} = £4.\ 3s.\ 9d. \div \tfrac{8}{3}$$
$$= \tfrac{3}{8} \text{ of } £4.\ 3s.\ 9d. = \frac{£12.\ 11s.\ 3d.}{8} = £1.\ 11s.\ 4\tfrac{7}{8}d.$$

Examples. (lxxiii)

Find the value of

(1) $\frac{3}{4}$ of 4*s*. 9*d*.	(2) $\frac{5}{8}$ of 7*s*. 2*d*.
(3) $\frac{5}{16}$ of a guinea.	(4) $\frac{2}{7}$ of 3*s*. 6*d*.
(5) $\frac{1}{2}$ of $\frac{3}{4}$ of 4*s*. 10*d*.	(6) $\frac{2}{3}$ of $\frac{4}{9}$ of £83. 16*s*. 3*d*.
(7) $9\frac{1}{3}$ of 1*s*. $1\frac{1}{2}$*d*.	(8) $5\frac{13}{15}$ of half-a-crown.
(9) $2\frac{3}{5}$ of £5. 2*s*. 6*d*.	(10) $\frac{27}{56}$ of £99. 14*s*.
(11) $2\frac{13}{45}$ of £32. 5*s*. 8*d*.	(12) $2\frac{2}{3}$ of $3\frac{2}{5}$ of £17. 7*s*. 6*d*.
(13) £257. 2*s*. 3*d*. $\times \frac{5}{12}$	(14) £425. 3*s*. 9*d*. $\times \frac{7}{15}$
(15) £101. 17*s*. 5*d*. $\times 3\frac{1}{23}$	(16) £17. 7*s*. $7\frac{1}{4}$*d*. $\times 4\frac{5}{47}$
(17) £60. 1*s*. 8*d*. $\div \frac{7}{9}$	(18) £2. 6*s*. 9*d*. $\div 1\frac{1}{3}$
(19) £53. 15*s*. 8*d*. $\div 6\frac{10}{17}$	(20) £36. 2*s*. 9*d*. $\div 4\frac{1}{5}$

NOTE.—If we have to multiply a compound expression by a mixed number, it is not always necessary to turn the mixed number into an improper fraction, as we did in Ex. (3), but we can frequently effect the multiplication more neatly by multiplying first by the fractional part, and then by the whole number, and adding the two results.

Thus, to multiply £427. 12*s*. 9*d*. by $5\frac{2}{3}$

	£	*s*.	*d*.	
	427	12	9	
			$5\frac{2}{3}$	
3	855	5	6	the result of multiplying the top line by 2.
	285	1	10	
	2138	3	9	the result of multiplying the top line by 5.
	£2423	5	7	

Examples. (lxxiv)

Multiply

(1) £245. 13*s*. 4*d*. by $5\frac{3}{4}$ (2) £439. 18*s*. 3*d*. by $7\frac{1}{3}$

(3) £4214. 15*s*. 2*d*. by $6\frac{5}{7}$ (4) £8629. 12*s*. 8*d*. by $3\frac{5}{8}$

(5) £7258. 17*s*. 6*d*. by $2\frac{3}{5}$ (6) £4372. 19*s*. 4*d*. by $6\frac{3}{8}$

XV. On Measures.

145. Measures of Time.

1 second is written 1 sec., or 1^s.

60 seconds make 1 minute, written 1 min., or 1^m.

60 minutes make 1 hour, written 1 hr., or 1^h.

24 hours make 1 day, written 1 da., or 1^d.

7 days make 1 week, written 1 wk.

In rough calculations a year is taken to consist of 365 days.

In rough calculations a month is taken to consist of 30 days.

A *Lunar* Month, or the time between two new moons, is rather more than $29\frac{1}{2}$ days.

The 12 months into which we divide the year are called *Calendar* Months; they are of variable length, for 7 of them contain 31 days, 4 contain 30 days, and February has 28 days (and in Leap-year 29).

The names of the 4 months which have 30 days are given in the old verse:

> Thirty days have September,
> April, June, and November.

To find whether a particular year is a Leap-year, we divide the number of the year by 4; if no remainder be left, the year is Leap-year, but to correct an error in our present Calendar, the *centuries* which are not exactly divisible by 400, as 1900, 2100 .. are to be taken as common years, and not as leap-years.

Examples. (lxxv)

Reduction.

(1) Reduce 6 hr. 17 min. 25 sec. to seconds; $17^h. 0^m. 43^s.$ to seconds.

(2) Reduce 3 yr. 143 d. 16 hr. to seconds; 1 yr. 13 d. 0 hr. 4 min. to minutes.

(3) Reduce 48567 min. to days; 23576 sec. to hours.

(4) Reduce 742392 sec. to days; 174296 sec. to days.

(5) Find the number of days, reckoning from noon of the one to noon of the other, between the following days in the year 1872:

1st February and 29th May; 4th July and 2nd December; 3rd January and 15th October; 24th February and 23rd June.

Also between 25th December, 1872, and 25th May, 1873.

Addition.

(6)

hr.	min.	sec.
14	21	37
17	13	32
9	47	43
12	53	54
22	17	50

(7)

da.	hr.	min.
23	15	16
57	12	38
13	17	43
24	22	7
16	5	58

(8)

wk.	da.	hr.
4	3	16
2	5	17
3	6	9
10	4	13
4	2	19

(9)

yr.	da.	hr.
3	137	15
4	243	6
1	56	7
6	135	12
7	85	9

(10)

hr.	min.	sec.
14	43	13
32	36	40
10	12	53
16	38	47
2	52	8

(11)

da.	hr.	min.	sec.
42	14	30	31
65	22	19	42
74	11	42	15
24	18	58	57
43	3	29	48

Subtraction.

(12)

hr.	min.	sec.
7	14	26
4	19	38

(13)

da.	hr.	min.
123	16	4
39	22	17

(14)

wk.	da.	hr.
4	6	18
3	6	20

(15)

yr.	da.	hr.
3	147	14
2	213	17

(16)

yr.	da.	hr.
4	45	16
2	78	19

(17)

da.	hr.	min.	sec.
14	1	0	13
8	15	23	27

(18) Multiply 13 hr. 14 min. 43 sec. by 35;
17 hr. 13 min. 39 sec. by 43.

(19) Divide 15 wk. 5 da. 17 h. 26 min. by 49;
14 hr. 56 min. 41 sec. by 73.

146. MEASURES OF LENGTH.

12 inches make 1 foot, usually written 1 ft.,
3 feet ..... 1 yard, ... 1 yd.,
$5\frac{1}{2}$ yards ..... 1 pole, ... 1 po.,
40 poles ..... 1 furlong, ... 1 fur.,
8 furlongs ... 1 mile, ... 1 mi.,
3 miles ..... 1 league............ 1 lea.

Hence 1 furlong = 220 yards, and 1 mile = 1760 yards.

NOTE.—Six feet make a fathom.

Cloth Measures.

$2\frac{1}{4}$ inches make 1 nail,
4 nails ..... 1 quarter,
4 quarters make 1 yard,
5 quarters ..... 1 ell.

Ex. (1). Reduce 3 mi. 5 fur. 17 po. 4 yd. 1 ft. 3 in. to inches.

mi.	fur.	po.	yd.	ft.	in.
3	5	17	4	1	3

8
―――
29 fur.
40
―――
1177 po.
$5\frac{1}{2}$
―――
$588\frac{1}{2}$ the result of dividing 1177 by 2
5889
―――
$6477\frac{1}{2}$ yd.
3
―――
$19433\frac{1}{2}$ ft.
12
―――
233205 inches.

Ex. (2). Reduce 47293 yards to poles.

$$47293 \text{ yd.} = (47293 \div 5\tfrac{1}{2}) \text{ poles}$$
$$= (47293 \div \tfrac{11}{2}) \text{ poles}$$
$$= (47293 \times \tfrac{2}{11}) \text{ poles.}$$

We may proceed thus:

$$\begin{array}{r|l} & 47293 \text{ yards} \\ & \quad\quad\; 2 \\ \hline 11 & 94586 \text{ half-yards,} \\ \hline & \;\; 8598 \text{ poles and 8 half-yards over.} \end{array}$$

$\therefore$ 47293 yd. = 8598 po. 4 yd.

Examples. (lxxvi)

Reduction.

(1) Reduce 3 yd. 2 ft. to inches; 4 mi. 3 fur. 4 po. to feet.

(2) Reduce 7 mi. 14 po. $3\frac{1}{2}$ yd. to inches; 27 po. $4\frac{1}{4}$ yd. to inches.

(3) Reduce 74325 yd. to poles; 2423694 in. to furlongs.

(4) Reduce 723964 ft. to miles; 82976432 in. to miles.

Addition.

	yd.	ft.	in.		mi.	fur.	po.		fur.	po.	yd.
(5)	4	2	7	(6)	13	4	20	(7)	2	19	2
	19	1	9		43	3	9		4	25	$2\frac{1}{2}$
	5	2	10		56	2	13		6	11	$3\frac{1}{2}$
	23	2	8		4	7	32		5	23	4
	35	1	6		16	3	15		3	0	$1\frac{1}{2}$
	17	2	4		19	5	11		1	21	$1\frac{3}{4}$

Subtraction.

	yd.	ft.	in.		mi.	fur.	po		fur.	po.	yd.
(8)	134	2	7	(9)	235	0	19	(10)	5	23	$1\frac{1}{2}$
	59	1	11		184	5	24		4	27	4

(11) Multiply 7 yd. 2 ft. 9 in. by 11; 16 mi. 5 fur. 7 po. by 56.

(12) Multiply 32 po. 3 yd. 1 ft. by 57; 36 mi. 3 fur. 6 po. $3\frac{1}{2}$ yd. by 49.

(13) Divide 25 yd. 1 ft. 8 in. by 4; 17 mi. 3 fur. 7 po. by 27.

(14) Divide 14 po. 2 yd. 1 ft. 8 in. by 32; 11 mi. 7 fur. 7 po. by 55.

147. MEASURES OF SURFACE.

144 square inches make	1 square foot, written	1 sq. ft.,
9 square feet	1 square yard, ...	1 sq. yd.,
$30\frac{1}{4}$ square yards	1 square pole, ...	1 sq. po.,
40 square poles	1 rood, ...	1 ro.
4 roods	1 acre, ...	1 ac.

Hence 1 acre = 4840 square yards.

640 acres = 1 square mile.

Land surveyors make use of a Chain 22 yards in length, divided into 100 equal parts, called Links.

The square of 22 is 484, and therefore 10 Square Chains make an Acre.

NOTE.—The Square Inch is a square whose side is an inch in length.

Ex. (1). How many square inches are there in 3 ac. 2 ro. 27 po. 27 sq. yd. 7 sq. ft. 23 sq. in.?

ac.	ro.	po.	sq. yd.	sq. ft.	sq. in.
3	2	27	27	7	23

```
      3 . 2 . 27 . 27 . 7 . 23
      4
     ---
     14 ro.
     40
    ----
    587 po.
     30¼
   -----
    146¾  the result of the division of 587 by 4.
   17637
   -----
   17783¾ sq. yd.
       9
  -------
  160054
       6¾  the result of multiplying ¾ by 9.
  -------
  160060¾ sq. ft.
     144
 --------
   640263
  640240
 160060
     108  the result of multiplying ¾ by 144.
 --------
 23048771 sq. in.
```

Ex. (2). Reduce 74237 sq. yards to poles.

$$74237 \text{ sq. yd.} = (74237 \div 30\tfrac{1}{4}) \text{ poles}$$
$$= (74237 \div \tfrac{121}{4}) \text{ poles}$$
$$= (74237 \times \tfrac{4}{121}) \text{ poles.}$$

We may proceed thus:

```
                74237 yards
                    4
          |  --------
121 { 11  |   296948 quarter-yards
    { 11  |    26995 and 3 quarter-yards over
          |  --------
            poles 2454 and 1 parcel of 11 quarter-yards over.
```

The remainder is (11 + 3) quarter-yards, or 14 quarter-yards, or $3\frac{1}{2}$ yd.

$\therefore$ 74237 sq. yd. = 2454 po. $3\frac{1}{2}$ sq. yd.

Examples. (lxxvii)

Reduction.

(1) Reduce 5 ac. 3 ro. 17 po. 13 sq. yd. 6 sq. ft. 15 sq. in. to square inches.

(2) Reduce 7 ac. 13 po. 5 sq. yd. 3 sq. ft. to square inches.

(3) Reduce 250 acres to square yards, and 73 sq. yd. to square inches.

(4) Reduce 5239 sq. in. to sq. yd., and 15376 sq. yd. to acres.

(5) Reduce 34729 sq. yd. to sq. poles, and 562934 sq. in. to square poles.

Addition.

(6)

ac.	ro.	po.
47	2	13
72	1	24
89	2	32
4	2	23
27	3	8
42	2	5

(7)

sq.yd.	sq.ft.	sq.in.
19	7	42
27	5	52
32	8	124
5	2	72
21	6	98
56	3	135

(8)

ac.	ro.	po.	sq.yd.
46	2	16	22
17	3	14	13
7	1	39	14
24	2	15	19
12	0	17	22
4	1	9	16

(9)

ac.	ro.	po.
156	2	17
423	0	15
37	2	33
149	1	28
27	1	14
364	3	0

(10)

ac.	ro.	po.
476	1	39
527	2	13
348	0	15
49	3	4
37	0	28
247	1	11

(11)

ac.	ro.	po.
1423	3	38
1274	0	14
396	2	23
157	1	36
258	0	12
1249	3	25

Subtraction.

(12)

ac.	ro.	po.
57	2	30
29	3	34

(13)

sq.yd.	sq.ft.	sq.in.
42	8	124
36	8	139

(14)

ac.	ro.	po.
16	2	0
14	3	24

(15)

ac.	ro.	po.
247	1	14
243	3	24

(16)

sq.yd.	sq.ft.	sq.in.
39	7	12
32	8	134

(17)

ac.	ro.	po.
245	3	19
178	3	23

(18) Multiply 5 ac. 3 ro. 24 po. by 15; 17 ac. 2 ro. 13 po. by 53.

(19) Divide 7 ac. 2 ro. 18 po. by 21; 29 ac. 2 ro. 37 po. by 71.

148. Measures of Solidity.

1728 cubic inches make 1 cubic foot, written 1 cub. ft.
27 cubic feet make 1 cubic yard, written 1 cub. yd.

A Cube is a solid figure contained by six equal squares. Hence a cubic inch is a six-sided figure, each of whose sides is a square inch. The lines that form the boundaries of the sides are called the Edges of the Cube.

Examples. (lxxviii)

Reduction.

(1) Reduce 7 cub. yd. 13 cub. ft. to cubic feet; 25 cub. yd. 5 cub. ft. 143 cub. in. to cubic inches; 14 cub. yd. 1374 cub. in. to cubic inches.

(2) Reduce 74325 cub. in. to cubic feet; 439284 cub. in. to cubic yards.

(3) Reduce $5\frac{1}{4}$ cub. yd. to cubic inches; 3 cub. yd. $5\frac{3}{4}$ cub. ft. to cubic inches.

Addition.

	cub.yd.	cub.ft.	cub.in.		cub.yd.	cub.ft.	cub.in.		cub.yd.	cub.ft.	cub.in.
(4)	57	13	572	(5)	43	7	1638	(6)	528	16	432
	32	25	493		26	22	472		237	19	583
	46	19	374		19	16	1384		764	10	1359
	76	8	587		45	13	427		446	0	1275
	4	26	1249		26	5	1286		729	11	346
	52	14	1324		33	18	275		852	5	1473

Subtraction.

	cub.yd.	cub.ft.	cub.in.		cub.yd.	cub.ft.	cub.in.		cub.yd.	cub.ft.	cub.in.
(7)	47	17	543	(8)	247	19	1274	(9)	527	0	0
	38	23	726		239	18	1368		499	19	256

(10) Multiply 26 cub. yd. 5 cub. ft. 49 cub. in. by 27; 472 cub. yd. 17 cub. ft. 238 cub. in. by 53.

(11) Divide 78 cub. yd. 13 cub. ft. 252 cub. in. by 12; 472 cub. yd. 0 cub. ft. 1416 cub. in. by 59.

149. MEASURES OF CAPACITY.

2 pints make	1 quart,	written	1 qt.,
4 quarts	1 gallon,	......	1 gall.,
2 gallons ...	1 peck,	......	1 pk.,
4 pecks	1 bushel	......	1 bus.,
8 bushels ...	1 quarter	......	1 qr.

NOTE.—A barrel of beer contains 36 gallons.

Examples. (lxxix)

Reduction.

(1) Reduce 3 pk. 1 gall. 3 pt. to pints, and 214 qr. $3\frac{1}{2}$ bus. to pints.

(2) Reduce 4234 pt. to quarters, and 3047 gall. to quarters.

Addition.

	gall.	qt.	pt.		bus.	pk.	gall.		qr.	bus.	pk.
(3)	4	3	1	(4)	4	3	1	(5)	42	5	3
	3	2	$1\frac{1}{2}$		5	2	$1\frac{1}{2}$		27	7	2
	12	3	0		1	3	1		64	3	1
	14	0	$1\frac{1}{2}$		4	2	$1\frac{1}{2}$		49	6	2
	5	2	1		3	1	0		12	4	0

Subtraction.

	gall.	qt.	pt.		bus.	pk.	gall.		qr.	bus.	pk.
(6)	5	2	0	(7)	6	3	0	(8)	36	7	2
	4	3	1		5	3	1		29	7	3

(9) Multiply 5 qr. 3 bus. 2 pk. by 63, and 15 qr. 2 bus. 1 pk. by 73.

(10) Divide 13 gall. 1 pt. by 15, and 348 qr. 0 bus. 1 pk. by 43.

150. Troy Weight.

24 grains	make 1 pennyweight,	written 1 dwt.
20 pennyweights	make 1 ounce,	written 1 oz.
12 ounces	make 1 pound,	written 1 lb.

Chiefly used for weighing gold, silver, and jewels.

Examples. (lxxx)

Reduction.

(1) In 27 ounces of gold how many grains are there?

(2) Reduce 7 lb.; 14 lb. 3 oz.; 25 lb. 9 oz. 5 dwt. to pennyweights.

(3) Reduce 3 lb. 10 oz. 7 dwt. 5 gr.; 7 lb. 4 oz. 17 dwt. 15 gr. to grains.

(4) Reduce 3145 gr. to ounces; 42672 gr. to lb.

(5) Reduce 72469 gr. to lb.; 3246 dwt. to lb.

Addition.

(6)

lb.	oz.	dwt.
21	2	12
27	9	4
3	8	17
14	3	19
7	6	8

(7)

oz.	dwt.	gr.
7	13	21
4	6	19
6	17	23
2	9	5
8	16	13

(8)

lb.	oz.	dwt.	gr.
13	8	6	14
12	4	17	8
5	10	13	0
42	7	15	21
12	11	19	23

Subtraction.

(9)

oz.	dwt.	gr.
6	19	13
3	14	16

(10)

lb.	oz.	dwt.
37	8	6
29	10	13

(11)

lb.	oz.	dwt.	gr.
35	9	8	22
34	11	15	23

(12) Multiply 7 lb. 5 oz. 9 dwt. by 12; 6 lb. 8 oz. 19 dwt. by 21.

(13) Multiply 10 oz. 16 dwt. 23 gr. by 37; 3 lb. 7 oz. 10 dwt. 21 gr. by 41.

(14) Divide 16 lb. 4 oz. 16 dwt. by 8; 7 lb. 10 oz. 17 dwt. 7 gr. by 15.

(15) Divide 9 oz. 17 dwt. 8 gr. by 37; 15 lb. 8 oz. 9 dwt. 12 gr. by 63.

151. Avoirdupois Weight.

16 drachms	make 1 ounce,	written 1 oz.
16 ounces	.... 1 pound,	...... 1 lb.
14 pounds	.... 1 stone,	...... 1 st.
28 pounds	.... 1 quarter,	...... 1 qr.
4 quarters	.... 1 hundredweight,	...... 1 cwt.
20 hundredweight	.... 1 ton.	

The pound Avoirdupois contains 7000 grains Troy.
The pound Troy contains 5760 grains Troy.

Examples. (lxxxi)

Reduction.

(1) Reduce 11 cwt. to oz.; 17 lb. to dr.; 5 tons to lb.

(2) Reduce 6 tons 7 cwt. to oz.; 15 tons 2 qr. to lb.

(3) Reduce 3 cwt. 6 lb. 5 oz. to dr.; 3 tons 15 cwt. 7 lb. to lb.

(4) Reduce 4763 oz. to cwt.; 3749 lb. to tons.

(5) Reduce 7432 oz. to cwt.; 247294 dr. to cwt.

Addition.

(6)

lb.	oz.	dr.
3	3	9
19	8	6
7	10	13
14	5	7
8	15	14

(7)

qr.	lb.	oz.
3	16	8
4	7	12
16	19	5
8	20	13
12	5	9

(8)

cwt.	qr.	lb.
13	2	24
11	3	5
29	1	19
16	2	9
17	0	7

(9)

tons.	cwt.	qr.
34	17	2
12	6	3
4	11	1
18	6	0
4	15	2
75	13	1

(10)

cwt.	qr.	lb.
12	2	13
35	0	20
53	2	25
15	3	17
14	1	12
54	2	23

(11)

tons.	cwt.	qr.	lb
347	19	2	10
724	11	1	13
25	4	1	22
419	14	0	9
37	19	3	14
4	0	1	3

Subtraction.

(12)	lb. oz. dr.	(13)	qr. lb. oz.
	16 . 13 . 5		17 . 13 . 3
	14 . 11 . 12		14 . 15 . 11

(14)	cwt. qr. lb.
	19 . 1 . 4
	17 . 3 . 18

(15)	tons. cwt. qr.	(16)	cwt. qr. lb.	(17)	tons. cwt. qr. lb.
	37 . 19 . 2		16 . 0 . 3		74 . 15 . 1 . 13
	29 . 19 . 3		15 . 3 . 25		39 . 16 . 3 . 25

(18) Multiply 17 cwt. 23 lb. 14 oz. by 7; 4 cwt. 17 lb. by 45.

(19) Multiply 6 cwt. 3 qr. 5 lb. by 23; 10 oz. 9 dr. by 37.

(20) Divide 14 cwt. 2 qr. 8 lb. by 12; 32 tons 15 cwt. 1 qr. by 40.

(21) Divide 16 cwt. 3 qr. 9 lb. by 65; 37 tons 4 cwt. 3 qr. 7 lb. by 17.

152. To find the cost of 1 oz. Avoirdupois when the cost of 1 lb. is given.

Suppose the cost of 1 lb. to be 5 shillings.

Then the cost of 1 lb. is (5×48) farthings.

of 1 oz. is $\frac{5 \times 48}{16}$ farthings,

or 5×3 farthings.

Hence we deduce the following rule:

To find the cost of 1 *oz. when the value of* 1 *lb. is given, multiply the number of shillings in the price per lb. by* 3, *and take the result as farthings.*

Thus, if the cost per lb. be 4*s.* the cost per oz. is 12 farthings.

6*s.* 18

2*s.* 4*d.* 7

(for 2*s.* 4*d.* = $2\frac{1}{3}$*s.*, and $3 \times 2\frac{1}{3} = 7$).

153. To find the cost of 1 lb. Avoirdupois when the cost of 1 oz. is given.

This being exactly the opposite of the calculation just given, we have the following rule:

Divide the number of farthings in the price per oz. by 3, and the result will be the number of shillings in the price per lb.

Thus, if the cost per oz. be		$4\frac{1}{2}d.$,	the cost per lb. is		6 shillings.
,,	,,	$3\frac{3}{4}d.$	,,	,,	5
,,	,,	$5d.$	,,	,,	$\frac{20}{3}s.$ or $6s. 8d.$

Examples. (lxxxii)

What is the cost per oz. when the cost of a pound is

(1) $7s.$ (2) $5s. 4d.$ (3) $13s.$ (4) $7s. 8d.$ (5) $4s. 4d.$?

What is the cost per lb. when the cost of an ounce is

(6) $3d.$ (7) $5\frac{1}{4}d.$ (8) $9d.$ (9) $10\frac{1}{2}d.$ (10) $6\frac{3}{4}d.$?

154. APOTHECARIES' WEIGHT.

1. *Measures of Weight.*

$437\frac{1}{2}$ grains make 1 ounce,
16 ounces make 1 pound.

The grain is the same as the grain Troy.
The ounce is the same as the ounce Avoirdupois.

This is the table given in the British Pharmacopœia. The Avoirdupois ounce and pound are taken, in preference to the ounce and pound Troy of the old table, because the former are used by wholesale dealers in drugs and medicines. In prescribing, many physicians still employ the scruple ℈, of 20 grains, and the drachm ʒ, of 60 grains.

155. 2. *Measures of Capacity.*

60 Minims	make 1 fluid drachm,	written	fl. dr.,
8 Fluid Drachms	1 fluid ounce,		fl. oz.
20 Fluid Ounces	1 pint,		O,
8 Pints	1 gallon,		C.

NOTE.—O is a contraction for *Octavus, the eighth*, and C for *Congius*, a Roman liquid measure.

The relation of the measures of capacity to those of weight in these tables is given by the definition

that 1 Minim is the measure of ·91 Grain of Water.

The connection may be better remembered by the old rhyme

A Pint of Water
Weighs a Pound and a Quarter.

156. Multiplication of Compound Quantities when the multiplier contains a fraction. (See page 129.)

Examples. (lxxxiii)

Multiply

(1) 3 cwt. 2 qr. 12 lb. by $3\frac{2}{3}$ (2) 6 lb. 5 oz. 4 dr. by $2\frac{1}{5}$

(3) 4 mi. 3 fur. 10 po. by $18\frac{1}{5}$ (4) 15 yd. 2 ft. 3 in. by $43\frac{1}{7}$

(5) 37 ac. 3 ro. 8 po. by $4\frac{5}{12}$ (6) 25 ac. 2 ro. 15 po. by $29\frac{3}{5}$

(7) 27 sq. yd. 7 sq. ft. 36 sq. in. by $2\frac{5}{8}$

157. Division of Compound Quantities when the divisor contains a fraction. (See page 129.)

Examples. (lxxxiv)

Divide

(1) 5 cwt. 2 qr. 11 lb. by $2\frac{3}{4}$ (2) 7 lb. 4 oz. 14 dr. by $11\frac{1}{3}$

(3) 7 mi. 2 fur. 12 po. by $4\frac{5}{12}$ (4) 17 yd. 1 ft. 3 in. by $5\frac{1}{2}$

(5) 25 ac. 2 ro. 12 po. by $4\frac{3}{7}$ (6) 14 ac. 3 ro. 8 po. by $8\frac{2}{9}$

(7) 107 sq. yd. 4 sq. ft. 132 sq. in. by $18\frac{2}{5}$

158. XVI. Fractional Measures.

Ex. (1). How many shillings and pence are there in $\frac{5}{8}$ of a pound?

$$\frac{5}{8} \text{ of a pound} = \frac{5}{8} \text{ of 20 shillings.}$$
$$= \frac{5 \times 20}{8} \text{ shillings.}$$
$$= \frac{100}{8}s.$$
$$= 12s.\ 6d.$$

Ex. (2). Find the value of $\frac{3}{7}$ of £15. 5*s.* 8*d.*

$\frac{3}{7}$ of £15. 5*s.* 8*d.* = 3 times $\frac{1}{7}$ of £15. 5*s.* 8*d.*
= 3 times £2. 3*s.* 8*d.*
= £6. 11*s*

Or thus :

$$\begin{array}{r|rrr}
 & £ & s. & d. \\
 & 15\ . & 5\ . & 8 \\
 & & & 3 \\
\hline
7 & 45\ . & 17\ . & 0 \\
\hline
 & £6\ . & 11\ . & 0
\end{array}$$

Ex. (3). Find the value of $2\frac{3}{4}$ of $\frac{3}{22}$ of 5 acres.

$2\frac{3}{4}$ of $\frac{3}{22}$ of 5 acres = $\frac{11}{4}$ of $\frac{3}{22}$ of 5 acres.
= $\frac{11 \times 3}{4 \times 22}$ of 5 acres.
= $\frac{3}{8}$ of 5 ac.
= $\frac{15}{8}$ ac.
= 1 ac. 3 ro. 20 po.

Examples. (lxxxv)

Find the value of the following :

(1) $\frac{2}{3}$ of £1 ; $\frac{5}{8}$ of £2. 10*s.* ; $\frac{3}{7}$ of £5. 18*s.* 5*d.*

(2) $\frac{4}{5}$ of a mile ; $\frac{3}{16}$ of an acre ; $\frac{5}{8}$ of a cwt.

(3) $2\frac{4}{5}$ of £54. 9*s.* 8*d.* ; $3\frac{3}{14}$ of half-a-guinea ; $\frac{2}{3}$ of $3\frac{3}{8}$ of a mile.

(4) $\frac{4}{5}$ of $\frac{1}{7}$ of $1\frac{5}{19}$ of $1\frac{3}{8}$ of 2470 guineas ; $\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ guineas.

(5) $\frac{2}{5}$ of £1 + $\frac{2}{3}$ of 1*s.* + $\frac{5}{8}$ of 16*s.* 4*d.*

(6) $\frac{3}{24}$ of £1 + $\frac{4}{5}$ of 2*s.* 6*d.* + $\frac{3}{7}$ of a guinea.

(7) $\frac{3}{4}$ of 5 ac. 3 ro. + $\frac{5}{8}$ of 7 ac. 2 ro. 20 po. + $\frac{3}{5}$ of 3 ro. 15 po.

(8) $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.

(9) $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of a furlong + $\frac{3}{8}$ of a yard.

(10) $\frac{3}{4}$ of 2 cwt. 3 qr. + $\frac{3}{7}$ of 5 cwt. 3 qr. 14 lb. + $\frac{2}{5}$ of $7\frac{1}{2}$ lb.

159. The following are examples of an operation which is the *converse* of that just explained.

Ex. (1). Express 14*s.* 7*d.* as the fraction of £5.

14*s.* 7*d.* = 175*d.*, and £5 = 1200*d.*

Now 1*d.* = $\frac{1}{1200}$ of 1200*d.*;

$\therefore$ 175*d.* is $\frac{175}{1200}$ of 1200*d.*

Hence the fraction required is $\frac{175}{1200}$, or $\frac{35}{240}$, or $\frac{7}{48}$

Ex. (2). Express 6 lb. 5 oz. avoird. as the fraction of 3 lb. 12 oz.

6 lb. 5 oz. = 101 oz., and 3 lb. 12 oz. = 60 oz.;

$\therefore$ the fraction required is $\frac{101}{60}$

Ex. (3). Express $\frac{2}{3}$ of 5*s.* 9*d.* as the fraction of 4*s.* 7*d.*

5*s.* 9*d.* = 69*d.*, and 4*s.* 7*d.* = 55*d.*

$\therefore$ 5*s.* 9*d.* is $\frac{69}{55}$ of 4*s.* 7*d.*;

$\therefore$ $\frac{2}{3}$ of 5*s.* 9*d.* is $\frac{2}{3}$ of $\frac{69}{55}$ of 4*s.* 7*d.*

$\therefore$ the fraction required is $\frac{2 \times 69}{3 \times 55}$ or $\frac{46}{55}$

Ex. (4). Express $\frac{3}{7}$ of $2\frac{4}{5}$ of 5 ac. 3 ro. as the fraction of $\frac{3}{5}$ of 14 ac. 2 ro.

5 ac. 3 ro. = 23 roods, and 14 ac. 2 ro. = 58 roods;

$\therefore$ fraction required is ($\frac{3}{7}$ of $\frac{14}{5}$ of 23) $\div$ ($\frac{3}{5}$ of 58);

or $\frac{3 \times 14 \times 23 \times 5}{7 \times 5 \times 3 \times 58}$, or $\frac{2 \times 23}{58}$ or $\frac{23}{29}$

Note.—There are several modes of demanding the operation explained in the foregoing examples. Thus the demand

Express 3 *shillings as the fraction of* 6 *shillings*, may be put in the following terms:

(1) Reduce 3 shillings to the fraction of 6 shillings :

(2) What part of 6 shillings is 3 shillings?

(3) What fraction of 6 shillings is 3 shillings?

(4) If 6 shillings be the unit, what is the measure of 3 shillings?

Examples. (lxxxvi)

(1) Express $1\frac{3}{4}d.$ as the fraction of $6s.\ 8\frac{1}{2}d.$

(2) Express £10. 5*s.* 4*d.* as the fraction of £11. 6*s.* 5*d.*

(3) Express 5*s.* 6*d.* as the fraction of a guinea.

(4) Reduce $9s.\ 10\frac{1}{2}d.$ to the fraction of $13s.\ 2\frac{1}{4}d.$

(5) Reduce 2 days 3 hrs. 5 min. to the fraction of a week.

(6) Reduce 2 roods 20 poles to the fraction of an acre.

(7) What fraction is 8 lb. troy 1 oz. 19 dwt. 9 gr. of 13 lb. 7 oz. 5 dwt. 15 gr.?

(8) What part of 2 qr. 10 lb. 7 oz. 9 dr. is 1 qr. 7 oz. 13 dr.?

(9) What fraction of 4 lb. 1 oz. 8 dwt. 15 gr. is 1 lb. 1 oz. 9 dwt. 15 gr.?

(10) If the unit of measurement be $2\frac{1}{2}$ yd., what is the measure of $2\frac{1}{2}$ ft.?

(11) If the unit of measurement be 5 inches, what is the measure of $\frac{5}{374}$ of a mile?

(12) What fraction of 2 ac. 37 po. is 3 ac. 2 ro. 1 po.?

(13) Express $\frac{3}{8}$ of 1*s.* as the fraction of a guinea.

(14) Express $2\frac{4}{9}$ of 4 cwt. as the fraction of 3 qr. 4 lb.

(15) Reduce $\frac{1}{24}$ of a yard to the fraction of an inch.

(16) What part of a guinea is $\frac{2}{3}$ of a pound?

(17) What part of a gallon is $\frac{2}{5}$ of a pint?

(18) What fraction of 2 miles is $\frac{2}{3}$ of 6 poles 3 yd. 2 in.?

(19) What part of £3. 7*s.* is $2\frac{4}{5}$ of $4\frac{2}{7}$ of half-a-crown?

(20) Reduce $\frac{3}{5}$ of 7 lb. 2 dwt. to the fraction of $\frac{2}{3}$ of 5 lb. 6 oz.

(21) What fraction of $\frac{12}{5}$ of a crown is $\frac{5}{12}$ of a guinea?

XVII. Decimal Measures.

160. Reduction of Decimals.

Ex. (1). How many shillings and pence are there in 375 of a pound?

$$\begin{aligned} \cdot 375 \text{ of } £1 &= (20 \times \cdot 375)s. \\ &= 7\cdot 5s. \\ \text{and } \cdot 5 \text{ of } 1s. &= (12 \times \cdot 5)d. \\ &= 6d. \\ \therefore \cdot 375 \text{ of } £1 &= 7s.\ 6d. \end{aligned}$$

The operation is performed more briefly thus:

$$\begin{array}{rr} & £ \\ & \cdot 375 \\ & 20 \\ \hline s. & 7\cdot 500 \\ & 12 \\ \hline d. & 6\cdot 000 \end{array}$$

Ex. (2). Find the value of $3\cdot 16875$ of £1.

$$\begin{array}{rr} £ & 3\cdot 16875 \\ & 20 \\ \hline s. & 3\cdot 37500 \\ & 12 \\ \hline d. & 4\cdot 50000 \\ & 4 \\ \hline q. & 2\cdot 00000 \end{array}$$

$$\therefore £3\cdot 16875 = £3.\ 3s.\ 4\tfrac{1}{2}d.$$

Ex. (3). Find the value of $\cdot 4256$ of 12*s.* 8*d.*

$$\cdot 4256 \text{ of } 12s.\ 8d. = \cdot 4256 \text{ of } 152d. = (152 \times \cdot 4256)d.$$

$$\begin{array}{r} \cdot 4256 \\ 152 \\ \hline 8512 \\ 21280 \\ 4256 \\ \hline 64\cdot 6912 \end{array}$$

$\therefore$ Value required is $64\cdot 6912d.$

Ex. (4). Multiply 27 ac. 3 ro. 14 po. by ·235.

$$
\begin{array}{l}
\text{ac.}\quad \text{ro.}\quad \text{po.}\\
27 \cdot 3 \cdot 14\\
\underline{\;\;4\;\;}\\
111 \text{ ro.}\\
\underline{\;\;40\;\;}\\
4454 \text{ po.}\\
\underline{\;\;\cdot 235}\\
\;\;22270\\
\;13362\\
8908\\
40 \,\big|\, 1046\cdot690\\
\;\;4 \,\big|\, \;\;26 \cdot 6\cdot69 \text{ po.}\\
\text{ac. } 6 \cdot 2 \text{ ro. } 6\cdot69 \text{ po.}
\end{array}
$$

Ex. (5). Find the value of $\cdot 2\dot{5}$ of £1.

$\cdot 2\dot{5}$ of £1 $= \frac{25-2}{90}$ of £1 $= \frac{23}{90}$ of £1 $= \frac{460}{90}s. = 5s.\ 1\frac{1}{3}d.$

Or thus:

$$
\begin{array}{l}
£\ \cdot 2555\dot{5}\ \ldots\ldots\\
\underline{\qquad\quad 20}\\
s.\ 5\cdot 111\dot{1}\ \ldots\ldots\\
\underline{\qquad\quad 12}\\
d.\ 1\cdot 333\dot{3}
\end{array}
$$

∴ Value required is $5s.\ 1\frac{1}{3}d.$

Examples. (lxxxvii)

Find the value of

(1) ·625 of £1.

(2) £15·275.

(3) £·009765.

(4) ·9375 of a cwt.

(5) ·046875 of 1 lb. avoir.

(6) 2·003125 of £8.

(7) ·425 of 3*s*. 4*d*.

(8) 2·46875 of £1. 3*s*.

(9) $\cdot 8\dot{3}$ of 5*s*. (10) $4\cdot 1\dot{3}$ of 12*s*. 3*d*.

(11) ·35 of 2 qr. 14 lb. (12) 2·125 of 3½ guineas.

(13) 2·1372 of 2 tons 5 cwt. (14) $5\cdot 24\dot{7}$ of £5. 2*s*. 6*d*.

(15) ·45 of £3. 10*s*. + ·75 of 4*s*. 8*d*. + 3·245 of 3*s*. 4*d*.

(16) ·7 of £1 + $\cdot\dot{8}$ of 7*s*. 6*d*. − 2·45 of 1*s*. 8*d*.

(17) $\cdot\dot{2}8571\dot{4}$ of £3. 3*s*. + $\cdot\dot{1}4285\dot{7}$ of £3. 17*s*. + $\cdot 3\dot{4}$ of 16*s*. 6*d*.

161. The following examples illustrate the operation, which is the *converse* of that already explained.

Ex. (1). Express 5*s*. 6*d*. as the decimal of £1.

5*s*. 6*d*. = 66*d*., and £1 = 240*d*.;

∴ 5*s*. 6*d*. = $\frac{66}{240}$ of £1.

Now $\frac{66}{240} = \frac{11}{40} = \cdot 275$;

∴ 5*s*. 6*d*. = ·275 of £1.

Or, more briefly, thus :

$$\begin{array}{r|l}
12 & 6\cdot 0 \ d. \\ \hline
20 & 5\cdot 5 \ s. \\ \hline
 & \cdot 275 \ £.
\end{array}$$

Where we first express 6*d*. as the decimal of a shilling, i.e. ·5, and then express 5·5*s*. as the decimal of a pound, i.e. ·275.

Ex. (2). Express £7. 15*s*. 10½*d*. as the decimal of £1.

$$\begin{array}{r|l}
4 & 2\cdot 0 \\ \hline
12 & 10\cdot 5 \\ \hline
20 & 15\cdot 875 \\ \hline
£ & 7\cdot 79375
\end{array}$$

Ex. (3). Express £3. 5*s*. 9*d*. as the decimal of £5. 7*s*. 6*d*.

£	s.	d.		£	s.	d.
3 ·	5 ·	9		5 ·	7 ·	6
20				20		
65				107		
12				12		
789				1290		

Now $\frac{789}{1290} = \frac{263}{430} = \frac{26 \cdot 3}{43} = \cdot 611 \ldots\ldots$

∴ £3. 5*s*. 6*d*. is ·611 .... of £5. 7*s*. 6*d*.

Ex. (4). Express $\frac{2}{3}$ of 5*s*. $9\frac{1}{4}d$. as the decimal of $\frac{3}{5}$ of 6*s*. 2*d*.

5*s*. $9\frac{1}{4}d$. = 277*q*.; and 6*s*. 2*d*. = 296*q*.

∴ $\frac{2}{3}$ of 5*s*. $9\frac{1}{4}d$. is $\dfrac{\frac{2}{3} \times 277}{\frac{3}{5} \times 296}$ of $\frac{3}{5}$ of 6*s*. 2*d*.

Now $\dfrac{\frac{2}{3} \times 277}{\frac{3}{5} \times 296} = \frac{2 \times 277 \times 5}{3 \times 3 \times 296} = \frac{1385}{1332} = 1 \cdot 039 \ldots$

Examples. (lxxxviii)

(1) Reduce 16*s*. $3\frac{3}{4}d$. to the decimal of a pound.

(2) Reduce £1. 15*s*. 4*d*. to the decimal of 2 guineas.

(3) Reduce 8*s*. $0\frac{1}{4}d$. to the decimal of £3.

(4) Express £1. 2*s*. $3\frac{1}{4}d$. as the decimal of £17. 16*s*. 4*d*.

(5) What decimal of £2 is 11*s*. $9\frac{3}{4}d$.?

(6) Reduce $\frac{2}{3}$ of £4. 6*s*. 9*d*. to the decimal of £2. 10*s*.

(7) Reduce $\frac{2}{9}$ of a square pole to the decimal of an acre.

(8) Reduce 5 po. 4 yd. $2\frac{1}{2}$ ft. to the decimal of a furlong.

(9) Express 3 qr. 3 lb. 1 oz. $12\frac{4}{9}$ dr. as the decimal of a cwt.

(10) Express 15 hr. 14 min. 6 sec. as the decimal of 2 days.

(11) Express 6 cwt. 2 qr. 7 lb. as the decimal of a ton.

(12) Express 12 grains as the decimal of a lb. troy.

(13) What decimal of 10 guineas is £1. 19*s*. $4\frac{1}{2}$*d.*?

(14) Express $\frac{3}{5}$ of 14*s*. 4*d*. as the decimal of £1.

(15) Reduce 3·45 of half-a-guinea to the decimal of 2*s*. 6*d*.

(16) Express $\frac{3}{8}$ of 2 qr. 14 lb. as the decimal of a cwt.

(17) Express $4\frac{2}{5}$ of 7 oz. 4 dwt. as the decimal of a pound troy.

(18) Reduce $3\frac{7}{8}$ of $1\frac{1}{4}$ of 5 cwt. 2 qr. 21 lb. to the decimal of a ton.

(19) What decimal of a pound troy is $\frac{2}{3}$ of a dwt.?

(20) Reduce $3\frac{3}{4}$ guineas to the decimal of £2. 15*s*.

(21) Reduce 2*s*. 6*d*. to the decimal of $\frac{5}{12}$ of £1.

(22) Express 18*s*. $4\frac{1}{2}$*d*. as the decimal of £1000.

(23) Reduce £24·25 + 3·4125*s*. + 9·25*d*. to the decimal of £10.

(24) Express $\cdot 4\dot{3}$ of 8*s*. 3*d*. as the decimal of $\cdot 0\dot{1}$ of £9.

(25) Express $\cdot 0\dot{4}$ of £2. 5*s*. + $\cdot\dot{2}\dot{3}$ of 3*s*. 9*d*. as the decimal of $\cdot\dot{2}4\dot{5}$ of £4. 3*s*. 3*d*. to four places.

The following are *Miscellaneous Examples on Fractional and Decimal Measures.*

Examples. (lxxxix)

(1) Add together £15·125, 17·3125 shillings and 9·75 pence, and reduce the result to the decimal of £25.

(2) If $1\frac{2}{3}$ of a sum of money be equal to $\frac{3}{7}$ of 5*s*. 10*d*., find the sum.

(3) If a rupee be worth 2*s*. 4*d*., what decimal fraction is it of 9*s*. 4*d*.? Express £6·944 in rupees and decimal parts of a rupee.

(4) What is the sum of money of which $\frac{13}{23}$ is £5. 2*s*. 11*d*.?

(5) Find the value of ·55 of £3 + ·34875 of £10 + 5·46875 of £3.

(6) What fraction of a crown is $\frac{4}{5}$ of £12 − $\frac{5}{6}$ of 3 guineas?

(7) Reduce 5 oz. $2\frac{3}{8}$ dr. to the decimal of a lb. avoirdupois.

(8) What is the value of $\dfrac{3 + \frac{1}{4}}{4 + \frac{1}{5}}$ of £26. 5*s.*?

(9) What fraction of a mile is $\frac{3}{7}$ of 15 yd. 1 ft. 4 in.?

(10) Find the value of $\frac{1}{4}$ of 17*s.* 8*d.* + 2·625 of 1*s.* − $\frac{3}{4}$ of $\frac{5}{8}$ of 5*s.* 4*d.* + $\cdot 26\dot{3}$ of 25*s.*, and reduce the result to the decimal of £5.

(11) Express £1. 4*s.* $10\frac{3}{4}$*d.* as the decimal of £3. 19*s.* 8*d.*

(12) Add $\cdot 4097\dot{2}$ of 3*s.* to $\cdot 2\dot{7}$ of 8*s.*, and express the result as the decimal of £1.

(13) Subtract $1\cdot 3\dot{2}\dot{7}$ of 308 days from $1\cdot\dot{2}$ of a year, and express the result as the decimal of half a year.

(14) Subtract $\cdot 42708\dot{3}$ of £1 from $\cdot 23\dot{4}\dot{5}$ of £6. 17*s.* 6*d.*, and reduce the result to the decimal of £5.

(15) Add together $\frac{3}{28}$ of 17*s.* 6*d.* and $\cdot 99791\dot{6}$ of £1, and reduce the result to the decimal of £5.

(16) Find the value of $\frac{1}{5}$ of $\frac{4}{7}$ of $\frac{8}{19}$ of $2\frac{3}{4}$ of 247 guineas.

(17) Find the value of $\frac{1}{9} \times \cdot\dot{4}\dot{7}$ of £360. 2*s.* 3*d.*

(18) Find the value of $\cdot 0\dot{1} \times \cdot\dot{1}0\dot{1}$ of £74. 18*s.* 6*d.*

(19) Find the value of 500 times the difference between an eighty-fourth part of $2\frac{1}{2}$ cwt. and a thirtieth part of 1 cwt. 0 qr. 3 lb.

(20) What fraction is 4 lb. 3 oz. 10 dwt. 20 gr. of 6 lb. 5 oz. 6 dwt. 6 gr.?

(21) How many ounces of gold, at £3·89375 per ounce, are contained in a lump worth £113·6975?

(22) Find the value of ·375 of a guinea + $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot 0\dot{2}\dot{7}$ of £2. 15*s.*

(23) Add together $\frac{5}{18}$ of a guinea, $\frac{3}{32}$ of a pound, $\frac{7}{20}$ of a crown and $\frac{5}{16}$ of a shilling, and reduce the result to the decimal of a pound.

(24) Add $\frac{5}{7}$ of $\frac{1}{4}$ of 6*s.* 5*d.* to $\frac{1}{6}$ of $\frac{2}{5}$ of half-a-crown, and express the result as the decimal of a pound.

(25) Express $\frac{3}{16}$ of ·375 of ten shillings + $\frac{4}{5}$ of half-a-crown − $\frac{3}{4}$ of a shilling as the decimal of a pound.

(26) Express $\frac{4}{9}$ of 21*s.* + $\frac{3}{5}$ of $\frac{1}{2}$ of £1 − $\frac{1}{12}$ of $\frac{3}{4}$ of 5*s.* − $\frac{1}{6}$ of $\frac{13}{18}$ of 1*s.* as the decimal of a pound.

(27) Express the difference between $\frac{11}{14}$ of a guinea and $\frac{1}{3}$ of 15*s.* 9*d.* as the fraction of £100.

(28) Express $\frac{2}{9}$ of a guinea + $\frac{3}{8}$ of a pound + $\frac{1}{8}$ of 5*s.* − 11*s.* 9*d.* as the fraction of £1. 5*s.*

XVIII. Decimal Coinage.

162. By a *decimal system* of measures we mean a system, in which each unit of measurement contains *ten* units of the denomination next below it in the scale of measurement.

Various schemes have been devised for decimalising the coinage in this country, of which the following seems to be most in favour with the advocates of the proposed change:

The Pound to remain as it now is.

The Florin, which is the *tenth* part of £1, to be as it now is.

The Cent, a silver coin, to be the *tenth* part of the florin.

The Mil, a copper coin, to be the *tenth* part of the cent.

Thus £1 = 10 florins = 100 cents = 1000 mils.

The great advantage of such a system would be, that we could reduce a *compound* quantity to a *simple* quantity, without going through the process of multiplication.

Thus £2. 5 f. 6 c. 7 m. = 2567 mils,

£7. 0 f. 6 c. 3 m. = 7063 mils,

£14. 2 m. = 14002 mils.

Hence the *commercial* rules for Addition, Subtraction, Multiplication and Division of money would be replaced by the corresponding rules of *pure* Arithmetic.

Thus, to find the sum of £725. 5 f. 6 c. 3 m., £327. 4 f. 3 c., £25. 4 c. 2 m., £74. 2 f. and £56, we should write the operation thus,

$$\begin{array}{r} \text{mils} \\ 725563 \\ 327430 \\ 25042 \\ 74200 \\ 56000 \\ \hline \text{mils } 1208235 \end{array}$$

and this sum is equivalent to £1208. 2 f. 3 c. 5 m.

Again, to multiply £42. 3 f. 6 c. 4 m. by 378, we should proceed thus.

$$\begin{array}{r} \text{mils} \\ 42364 \\ 378 \\ \hline 338912 \\ 296548 \\ 127092 \\ \hline \text{mils } 16013592 \end{array}$$

and the product is £16013. 5 f. 9 c. 2 m.

Further, we could express any compound quantity as the *decimal* of any particular unit of the system, by inspection.

Thus £42. 4 f. 3 c. 2 m. = £42·432
= f. 424·32
= c. 4243·2

Examples. (xc)

Reduce to mils the following amounts in decimal coinage:

(1) £72. 6 f. (2) £6. 4 f. 5 c. 6 m. (3) £45. 3 f. $2\frac{1}{2}$ c.
(4) £7. 3 c. (5) £18. 4 m. (6) £19. $3\frac{1}{2}$ c.

Find the sum of

(7) £64. 8 f. 4 c. 2 m., £32. 3 c. 2 m., £14, £15. 2 f.

(8) £374. 3½ f., £432. 3 f. 4½ c., 5 f. 2 c. 3 m., £16. 4 m.

Find the difference between

(9) £17. 3 f. 4 c. 5 m. and £15. 5 f. 9 c. 8 m.

(10) £15. 4 m. and £14. 9 f. 9 c. 9 m.

Multiply

(11) £16. 7 f. 3 c. 8 m. by 43. (12) £143. 3 c. 4 m. by 73.

Divide

(13) £175. 4 f. 2 c. 5 m. by 15.

(14) £1674. 9 f. 1 c. 8 m. by 189.

163. To decimalise a sum expressed in our present system, we have to express the sum as the decimal of £1, according to the method of Art. 161.

Thus, to express £25. 7*s.* 10½*d.* in decimal coinage, we proceed thus:

$$\begin{array}{r|l} 4 & 2 \cdot 0 \\ \hline 12 & 10 \cdot 5 \\ \hline 20 & 7 \cdot 875 \\ \hline \end{array}$$

£ 25·39375, or £25. 3 f. 9 c. 3·75 m.

The converse operation, of changing decimal coinage into our present system, is performed in accordance with the method of Art. 160.

Thus, to express £27. 5 f. 6 c. 7 m. in our present coinage, we proceed thus:

$$\begin{array}{r} £\ 27 \cdot 567 \\ 20 \\ \hline s.\ 11 \cdot 340 \\ 12 \\ \hline d.\ 4 \cdot 080 \end{array}$$

∴ £27. 5 f. 6 c. 7 m. = £27. 11*s.* 4·08*d.*

Examples. (xci)

Express in decimal coinage

(1) 10*s*. (2) 5*s*. (3) 2*s*. 6*d*. (4) 1*s*.
(5) 6*d*. (6) 1*d*. (7) £3. 8*s*. (8) £4. 13*s*.
(9) £7. 14*s*. 6*d*. (10) £13. 15*s*. $3\frac{1}{2}$*d*.
(11) £42. 0*s*. 5*d*. (12) £54. 3*s*. $1\frac{1}{2}$*d*.

Express in our ordinary coinage

(13) A cent. (14) A mil.
(15) £14. 5 f. (16) £3. 4 f. 5 c.
(17) £7. 8 f. 6 c. 4 m. (18) 423400 mils.
(19) 43725 cents. (20) 4 f. 6 c. 7·325 m.
(21) 543·25 cents. (22) 7423·425 mils.

XIX. Practice.

164. Practice is the name given to a method by which we find the cost of any number of articles of the *same* kind when the price of one is given, or the cost of any quantity of goods of *mixed* denominations, when the cost of a single unit of any denomination is given.

I. Simple Practice.

When the articles are of the *same* kind or denomination.

Ex. (1). Suppose I have to find the cost of 2478 articles at 3*s*. 4*d*. each.

Knowing that 3*s*. 4*d*. is one-sixth part of £1, I reason

thus: if the articles had cost £1 each, the total cost would have been £2478;

∴ as they cost $\frac{1}{6}$ of £1 each, the cost will be £$\frac{2478}{6}$, or £413.

The process may be written thus:

3*s*. 4*d*. is $\frac{1}{6}$ of £1	£2478	= cost of the articles at £1 each.
	£413	= cost of the articles at 3*s*. 4*d*. each.

Ex. (2). Find the cost of 2897 articles at £2. 12*s*. 9*d*. each.

	£	*s*.	*d*.	
£2 is 2 × £1	2897	0	0	= cost at £1 each.
10*s*. is $\frac{1}{2}$ of £1	5794	0	0	= ...... £2 ....
2*s*. is $\frac{1}{5}$ of 10*s*.	1448	10	0	= ...... 10*s*.....
8*d*. is $\frac{1}{3}$ of 2*s*.	289	14	0	= ...... 2*s*. ....
1*d*. is $\frac{1}{8}$ of 8*d*.	96	11	4	= ...... 8*d*. ....
	12	1	5	= ...... 1*d*. ....
	£7640	16	9	= cost at £2.12*s*.9*d*. each.

Note.—A shorter method would be to take the parts thus

10*s*. = $\frac{1}{2}$ of £1; 2*s*. 6*d*. = $\frac{1}{4}$ of 10*s*.; 3*d*. = $\frac{1}{10}$ of 2*s*. 6*d*.

Ex. (3). Find the cost of 425 articles at £2. 18*s*. 4*d*. each.

Since £2. 18*s*. 4*d*. is the difference between £3 and 1*s*. 8*d*. (which is $\frac{1}{12}$ of £1), the shortest course is to find the cost at £3 each, and to *subtract from it* the cost at 1*s*. 8*d*. each, thus:

	£	*s*.	*d*.	
£3 is 3 × £1	425	0	0	= cost at £1 each.
1*s*. 8*d*. is $\frac{1}{12}$ of £1	1275	0	0	= ...... £3 ....
	35	8	4	= ...... 1*s*. 8*d*. each.
	£1239	11	8	= cost at £2. 18*s*. 4*d*. each.

Ex. (4). A bankrupt pays 6*s.* $7\frac{1}{2}$*d.* in the pound: what is the dividend on a debt of £362. 15*s.*?

	£ . s . d.	
5*s.* is $\frac{1}{4}$ of £1	362 . 15 . 0	= amount of debt.
1*s.* is $\frac{1}{5}$ of 5*s.*	90 . 13 . 9	= amount at 5*s.* in the £.
6*d.* is $\frac{1}{2}$ of 1*s.*	18 . 2 . 9	= .... 1*s.* ....
$1\frac{1}{2}$*d.* is $\frac{1}{4}$ of 6*d.*	9 . 1 . 4·5	= .... 6*d.* ....
	2 . 5 . 4·125	= .... $1\frac{1}{2}$*d.* ......
	£120 . 3 . 2·625	= amount at 6*s.* $7\frac{1}{2}$*d.* in £.

NOTE.—Shorter thus: 4*s.* = $\frac{1}{5}$ of £1; 2*s.* 6*d.* = $\frac{1}{8}$ of £1; $1\frac{1}{2}$*d.* = $\frac{1}{20}$ of 2*s.* 6*d.*

Ex. (5). Find the cost of $784\frac{1}{2}$ articles at £2. 12*s.* 10*d.* each.

The cost at £1 each is £784. 10*s.*

	£ . s . d.	
	784 . 10 . 0	= cost at £1 each.
	2	
10*s.* is $\frac{1}{2}$ of £1	1569 . 0 . 0	= ...... £2 each.
2*s.* is $\frac{1}{5}$ of 10*s*	392 . 5 . 0	= ...... 10*s.* ...
10*d.* is $\frac{1}{12}$ of 10*s.*	78 . 9 . 0	= ...... 2*s.* ....
	32 . 13 . 9	= ...... 10*d.* ...
	£2072 . 7 . 9	= cost at £2. 12*s.* 10*d.* each.

165. The fractions of a Unit, which have for their numerator *Unity*, are called ALIQUOT PARTS of the unit. Thus 5*s.*, being $\frac{1}{4}$ of £1, is an aliquot part of a pound; and 4 lb. being $\frac{1}{7}$ of 1 qr., is an aliquot part of a quarter.

The following tables of Aliquot Parts may be studied with advantage:

Of a Pound Sterling.		*Of a Shilling.*	
10*s*. is $\frac{1}{2}$	2*s*. is $\frac{1}{10}$	6*d*. is $\frac{1}{2}$	2*d*. is $\frac{1}{6}$
6*s*. 8*d*. is $\frac{1}{3}$	1*s*. 8*d*. is $\frac{1}{12}$	4*d*. is $\frac{1}{3}$	$1\frac{1}{2}$*d*. is $\frac{1}{8}$
5*s*. is $\frac{1}{4}$	1*s*. is $\frac{1}{20}$	3*d*. is $\frac{1}{4}$	1*d*. is $\frac{1}{12}$
4*s*. is $\frac{1}{5}$	6*d*. is $\frac{1}{40}$		
3*s*. 4*d*. is $\frac{1}{6}$	4*d*. is $\frac{1}{60}$		
2*s*. 6*d*. is $\frac{1}{8}$	3*d*. is $\frac{1}{80}$		

Troy Weight.

Of a Pound Troy.	*Of an Ounce Troy.*
6 oz. is $\frac{1}{2}$	10 dwt. is $\frac{1}{2}$
4 oz. is $\frac{1}{3}$	5 dwt. is $\frac{1}{4}$
3 oz. is $\frac{1}{4}$	4 dwt. is $\frac{1}{5}$
$1\frac{1}{2}$ oz. is $\frac{1}{8}$	2 dwt. is $\frac{1}{10}$
1 oz. is $\frac{1}{12}$	1 dwt. is $\frac{1}{20}$

Avoirdupois Weight.

Of a Ton.	*Of a Cwt.*	*Of a Quarter.*	*Of a Pound.*
10 cwt. is $\frac{1}{2}$	2 qr. is $\frac{1}{2}$	14 lb. is $\frac{1}{2}$	8 oz. is $\frac{1}{2}$
5 cwt. is $\frac{1}{4}$	1 qr. is $\frac{1}{4}$	7 lb. is $\frac{1}{4}$	4 oz. is $\frac{1}{4}$
4 cwt. is $\frac{1}{5}$		4 lb. is $\frac{1}{7}$	2 oz. is $\frac{1}{8}$
2 cwt. is $\frac{1}{10}$			

Examples. (xcii)

Find the cost of the following articles:

(1) 6325 at $9\frac{1}{4}$*d*. (2) 3254 at $10\frac{1}{2}$*d*.

(3) 5271 at 1*s*. $3\frac{1}{2}$*d*. (4) 1007 at 1*s*. $10\frac{3}{4}$*d*.

(5) $7104\frac{1}{2}$ at 1*s*. $8\frac{1}{2}$*d*. (6) 4000 at 1*s*. $11\frac{3}{4}$*d*.

(7) 2510 at 14*s*. $7\frac{1}{4}$*d*. (8) 7251 at 14*s*. $8\frac{1}{4}$*d*.

(9) $2154\frac{1}{4}$ at £7. 1*s*. 3*d*. (10) 3684 at £2. 6*s*. $9\frac{1}{4}$*d*.

(11) $2017\frac{3}{4}$ at £1. 13*s*. (12) 3142 at £1. 7*s*. $10\frac{1}{4}$*d*.

(13) 3210 at £1. 18*s*. $6\frac{3}{4}$*d*. (14) 4321 at £1. 17*s*. $3\frac{3}{4}$*d*.

(15) 2175 at £2. 15*s*. $4\frac{1}{2}$*d*. (16) $3768\frac{1}{2}$ at £1. 7*s*. $4\frac{1}{2}$*d*.

(17) 4276 at £12. 11*s*. 5½*d*. (18) 5783 at £14. 9*s*. 6½*d*.

(19) 3689⅔ at £16. 12*s*. 9*d*. (20) 7483 at £22. 13*s*. 2½*d*.

(21) What is the dividend on £4234. 10*s*. at 5*s*. 6*d*. in the £?

(22) What is the dividend on £4975 at 3*s*. 4½*d*. in the £?

(23) What is the dividend on £3729. 18*s*. 6*d*. at 7*s*. 9¾*d*. in the £?

II. Compound Practice.

Ex. (1). When we have to find the cost of a quantity of goods of *mixed* denominations (as 14 cwt. 3 qr. 17 lb.), the cost of a single unit of one of the denominations being given (as £3. 7*s*. 6*d*. per cwt.) we proceed thus:

	£.	*s*.	*d*.	
14 cwt. is 14 × 1 cwt.	3 .	7 .	6	= cost of 1 cwt.
			2	
	6 .	15 .	0	
			7	
2 qr. is ½ of 1 cwt.	47 .	5 .	0	= cost of 14 cwt.
1 qr. is ½ of 2 qr.	1 .	13 .	9	= ...... 2 qr.
14 lb. is ½ of 1 qr.		16 .	$10\frac{1}{2}$	= ...... 1 qr.
2 lb. is $\frac{1}{7}$ of 14 lb.		8 .	$5\frac{1}{4}$	= ...... 14 lb.
1 lb. is ½ of 2 lb.		1 .	$2\frac{13}{28}$	= ...... 2 lb.
			$7\frac{13}{56}$	= ...... 1 lb.
	£50 .	5 .	$10\frac{25}{56}$	= cost of 14cwt.3qr.17lb.

Ex. (2). What is the rent of 12 ac. 3 ro. 26 po. at £3. 5*s*. an acre?

	£	*s*.	*d*.	
12 ac. is 12 × 1 ac.	3 .	5 .	0	= the rent of 1 acre.
2 ro. is ½ of 1 ac.	39 .	0 .	0	= .... 12 ac.
1 ro. is ½ of 2 ro.	1 .	12 .	6	= .... 2 ro.
20 po. is ½ of 1 ro.		16 .	3	= .... 1 ro.
5 po. is ¼ of 20 po.		8 .	1·5	= .... 20 po.
1 po. is $\frac{1}{5}$ of 5 po.		2 .	0·375	= .... 5 po.
			4·875	= .... 1 po.
	£41 .	19 .	3·75	= the rent of 12a. 3r. 26p.

NOTE.—When the divisor is any number less than 12 (except 7) it is desirable to employ decimals, instead of vulgar fractions, to express the result of the division after the line of pence.*

Examples. (xciii)

(1) 25 cwt. 2 qr. 14 lb. at £3. 17*s*. 6*d*. per cwt.
(2) 72 cwt. 1 qr. 18lb. at £4. 5*s*. 8*d*. per cwt.
(3) 78 cwt. 3 qr. 12 lb. at £2. 17*s*. 9*d*. per cwt.
(4) 37 cwt. 2 qr. 13 lb. at £3. 17*s*. 10*d*. per cwt.
(5) 53 cwt. 0 qr. 17 lb. at £3. 11*s*. 6*d*. per cwt.
(6) 2 tons 7 cwt. 2 qr. 5 lb. at £32 per ton.
(7) 17 cwt. 1 qr. 11 lb. at £25. 7*s*. 4*d*. per cwt.
(8) 5 tons 0 cwt. 3 qr. 17 lb. at £7. 3*s*. 4*d*. per ton.
(9) 8 tons 3 cwt. 0 qr. 27 lb. at £5. 6*s*. 8*d*. per ton.
(10) 3 lb. 4 oz. 7 dwt. at £4. 2*s*. 6*d*. per lb.
(11) 5 ac. 3 ro. 4 po. $4\frac{1}{2}$ yd. at £10 per rood.
(12) 12 cwt. 3 qr. 22 lb. 12 oz. at £3. 18*s*. 2*d*. per cwt.
(13) 10 ac. 3 ro. 26 po. at £2. 18*s*. $10\frac{2}{3}$*d*. per acre.
(14) 6 tons 12 cwt. 3 qr. $10\frac{1}{2}$lb. at £3. 14*s*. $8\frac{1}{4}$*d*. per cwt.
(15) 63 cwt. 3 qr. $17\frac{1}{2}$ lb. at 12 guineas per cwt.
(16) 29 ac. 3 ro. 5 po. at 100 guineas per acre.
(17) 16 oz. 6 dwt. 20 gr. at £3. 17*s*. 6*d*. per oz.
(18) 25 ac. 1 ro. 10 po. at £42. 2*s*. 4*d*. per acre.
(19) 13 cwt. 3 qr. 17 lb. at £22. 8*s*. per cwt.
(20) 319 cwt. 3 qr. 16 lb. at £2. 12*s*. 6*d*. per cwt

Invoices and Accounts.

166. An INVOICE is a statement in detail, sent by a Seller to the Buyer at the time the goods are delivered to the Buyer, of the quantity, description, and price of the goods.

An ACCOUNT is a statement sent by the Seller to the Buyer at the end of a term of credit, shewing the totals and dates of each Invoice and the sum total of the whole.

Each separate article or amount in an Invoice or an Account is called an ITEM.

A DETAILED ACCOUNT is a full statement, sent by the Seller to the Buyer at the end of a term of credit, shewing the dates of delivery, the quantities, description, prices, and sum total of the goods delivered by the Seller to the Buyer during that term of credit.

When an account has been made out it is *rendered*, i.e., sent in to the Buyer.

NOTE.—The terms here defined include all that is taught in the old text-books on BILLS OF PARCELS, an expression not now in common use.

Specimen of an Invoice.

CAMBRIDGE, *May* 28, 1874.

Richard Holmes, Esq.,

Bought of J. Williams & Co.,

19, High Street.

		£	s.	d.
5 lb. of Black Tea	at 3s. 4d.		16	8
8 lb. of Loaf Sugar	at $6\frac{1}{2}d.$		4	4
$2\frac{1}{2}$ lb. of Butter	at 1s. 7d.		3	$11\frac{1}{2}$
	£	1	4	$11\frac{1}{2}$

Specimen of an Account.

CAMBRIDGE, *June* 30, 1874.

Richard Holmes, Esq.,

To J. Williams & Co., 19, High Street.

1874.		£	s.	d.
May 28,	To Goods, as per Invoice	1	4	$11\frac{1}{2}$
June 4,	To ditto	1	10	0
,, 15,	To ditto		15	0
,, 25,	To ditto		7	6
	£	3	17	$5\frac{1}{2}$

Specimen of a Detailed Account.

CAMBRIDGE, *June* 30, 1874.

Richard Holmes, Esq.,

To J. Williams & Co.,

19, High Street.

			£	s.	d.
1874.					
May 28,	5 lb. of Black Tea .	at 3*s*. 4*d*.		16	8
,,	8 lb. of Loaf Sugar .	at 6½*d*.		4	4
,,	2½ lb. of Butter . .	at 1*s*. 7*d*.		3	11½
June 4,	10 lb. of Cheese . .	at 1*s*. 0*d*.		10	0
,,	5 lb. of Black Tea .	at 4*s*. 0*d*.	1	0	0
June 15,	3 Pots of Marmalade	at 10*d*.		2	6
,,	11 lb. of Biscuit . .	at 1*s*. 0*d*.		11	0
,,	1 Jar of Pickles . .	at 1*s*. 6*d*.		1	6
June 25,	1 Packet of Candles .	at 5*s*. 6*d*.		5	6
,,	4 lb. of Raisins . .	at 6*d*.		2	0
		£	3	17	5½

Examples. (xciv)

Make out Invoices of the following Sales, supplying names and dates of your own selection.

(1) 10¼ lb. ribs of beef at 11*d*. per lb.; 7¾ lb. shoulder of mutton at 8*d*.; 2½ lb. rump steak at 1*s*.; 5¾ lb. neck of mutton at 9*d*.; 1½ lb. suet at 11*d*.; 3¼ lb. veal cutlets at 1*s*.

(2) 5 lb. of black tea at 2*s*. 8*d*.; 2¼ lb. of green tea at 4*s*. 6*d*.; 15½ lb. of lump sugar at 5½*d*.; 17 lb. of moist sugar at 4*d*.; 7½ lb. of raisins at 11*d*.; 4 lb. currants at 6½*d*.

(3) 15½ yd. of flannel at 2*s*. 3*d*.; 29 yd. of calico at 8½*d*.; 25 yd. of Irish linen at 2*s*. 4*d*.; 17 yd. of towelling at 1*s*. 2*d*.; 12½ yd. of brown holland at 11½*d*.; 3½ doz. handkerchiefs at 9*s*. 10*d*. a dozen.

Make out Accounts of the following Sales, supplying names and dates of your own selection.

(4) 23 yd. of black silk at 7*s*. 8*d*.; 17 yd. of ribbon at 10½*d*.; 13½ yd. of silk velvet at 9½*d*.; 1½ doz. pairs of stockings at 2*s*. 9*d*. a pair; 5 pairs of gloves at 3*s*. 3*d*.; 18 yd. of muslin at 6½*d*.

(5) 5 pairs of blankets at £1. 4*s.*; $12\frac{1}{2}$ yd. of merino at 2*s.* 11*d.*; $15\frac{3}{4}$ yd. of cloth at 9*s.* 6*d.*; $5\frac{1}{2}$ yd. of flannel at 1*s.* 9*d.*; 2 counterpanes at £1. 3*s.* 9*d.* each; $25\frac{1}{4}$ yd. of calico at 9d.

(6) $39\frac{1}{2}$ yd. of Brussels carpet at 5*s.* 4*d.*; $62\frac{3}{4}$ yd. of Kidderminster carpet at 3*s.* 6*d.*; 27 yd. of cocoa-nut matting at 1*s.* 2*d.*; $34\frac{1}{2}$ yd. of drugget at 2*s.* 3*d.*; $43\frac{1}{2}$ yd. of India matting at 1*s.* 1*d.*

Make a Detailed Account of the Sales in Questions 4, 5, 6, supplying names and dates of your own selection.

XX. Problems.

167. The *Unitary Method*, which is rapidly displacing the Rule of Three, will be gradually explained in this and the succeeding Sections.

Ex. (1). If 23 bullocks cost £483, what is the cost of 1 bullock?

Since 23 bullocks cost £483,

∴ 1 bullock will cost £$\frac{483}{23}$ or £21.

Ex. (2). If 7 men do a piece of work in 12 days, how long will it take 1 man to do it?

Since 7 men can do the work in 12 days,

1 man can do the work in (7 × 12) days, or 84 days.

Examples. (xcv)

(1) If 17 lb. of sugar cost 8*s.* 6*d.*, what is the price of 1 lb.?

(2) If a man walk 62 miles in 4 days, how far does he walk daily?

(3) If 4 men reap a field in 6 days, how long will 1 man be doing it?

(4) If 10 eggs cost 1*s.* 3*d.*, what is the price of an egg?

(5) A sack of flour lasts 9 people 34 days, how long will it last 1 person?

(6) If the income-tax on £265 is £6. 12*s.* 6*d.*, what is the tax on £1?

NOTE I.—Each of the questions above requires only Multiplication *or* Division for its solution. In some of those which follow, Multiplication *and* Division are required.

Ex. (3). If 75 men finish a piece of work in 12 days, how many men will finish it in 20 days?

In 12 days the work is done by 75 men,

In 1 day the work is done by (75×12) men,

In 20 days the work is done by $\frac{75 \times 12}{20}$ men, or 45 men.

Ex. (4). A bankrupt's debts are £2520, and his assets (that is the value of his property) are £1890; what can he pay in the pound?

In the place of £2520, he can pay £1890,

In the place of £1, he can pay £$\frac{1890}{2520}$, or £$\frac{3}{4}$, or 15*s.*;

$\therefore$ he pays 15 shillings in the pound.

Ex. (5). A bankrupt's debts are £4264, and he pays 12*s.* 6*d.* in the pound; what are his assets?

That which he has to meet a debt of £1 is $12\frac{1}{2}$*s.*

That which he has to meet a debt of £4264 is $(4264 \times 12\frac{1}{2})$*s.*;

$\therefore$ his assets are $\frac{4264 \times 25}{2}$*s.*, or £2665.

Ex. (6). If 27 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do the same in 45 days?

Since 27 men can do the work in (14×10) hours, or 140 hours,

1 man can do the work in (140×27) hr.

$\therefore$ 12 men can do the work in $\frac{140 \times 27}{12}$ hr., or 315 hr.

Now 315 hours have to be distributed equally over 45 days;

$\therefore$ the number of hours they work each day $= \frac{315}{45}$ or 7.

Ex. (7). If 7 lb. of tea cost 15*s.* 9*d.*, what will be the cost of 12 lb.?

Since 7 lb. of tea cost 15*s.* 9*d.*,

1 lb. of tea cost $\frac{15s.\ 9d.}{7}$ or 2*s.* 3*d.*

$\therefore$ 12 lb. of tea cost $12 \times$ (2*s.* 3*d.*), or £1. 7*s.*

Ex. (8). If 9 horses can plough 46 acres in a certain time, how many acres can 12 horses plough in the same time?

Since 9 horses can in the given time plough 46 ac.,

1 horse can in the given time plough $\frac{46}{9}$ ac.

$\therefore$ 12 horses can in the given time plough $\frac{46 \times 12}{9}$ ac.,

or $61\frac{1}{3}$ ac.

Ex. (9). If 15 horses can plough a certain quantity of land in five days, how many horses will be required to plough it in 3 days?

In 5 days the land can be ploughed by 15 horses;

In 1 day the land can be ploughed by (15×5) horses;

In 3 days the land can be ploughed by $\frac{15 \times 5}{3}$ or 25 horses.

NOTE II.—In simple questions of this kind we have a *supposition* and a *demand.* Each contains two kinds of things; in the *supposition* the magnitudes of *both* kinds are given; in the *demand* a magnitude of one kind is given, and the appropriate corresponding magnitude of the other kind has to be found. The first line of the solution contains the magnitudes of the supposition so arranged, that *at the end of the line we have that kind of thing, of which the magnitude is required in the demand.*

Thus in Ex. (9) the order of the supposition is changed and the magnitude, 15 horses, put at the end of the line, because we have to find how many horses will be required in the demand.

Examples. (xcvi)

(1) If a man walk 62 miles in 4 days, in how many days will he walk 93 miles?

(2) If 12 men reap a field in 4 days, in what time will 32 men reap it?

(3) If 14 lb. of coffee cost 15*s.* 2*d.*, what will be the cost of 129 lb.?

(4) If 350 acres of land cost £12250, what will 273 acres cost?

(5) If 3 lb. of mutton cost half-a-crown, how much will 14 lb. cost?

(6) When 8 eggs are bought for a shilling, how much must be given for three score?

(7) If 15 lb. of sugar cost 5*s.* $7\frac{1}{2}$*d.*, what is the cost of a cwt.?

(8) How many men can perform in 12 days a piece of work which 15 men can perform in 20 days?

(9) The rent of 17 acres is £59. 10*s.*, what is the rent of 86 acres?

(10) If a man walk 116 miles in 8 days, how far will he walk in 14 days?

(11) If 6 grains of silver be worth five farthings, what should be the weight of a crown?

(12) How much must be given for 25 lambs at the rate of £176 for 80?

(13) A farmer sells a flock of 270 sheep at £48 a score, what does he get for them?

(14) A servant's wages being £10. 8*s.* per annum, how much ought she to receive for 7 weeks?

(15) If 3 cwt. of sugar cost £5. 13*s.* 3*d.*, what is the value of 147 cwt.?

(16) If 4 yards of flannel be worth 7*s.* 8*d.*, what are 57 yards worth?

(17) A clerk's salary is £191. 12*s.* 6*d.* per annum; what ought he to receive for 60 days' service?

(18) A ship performs a voyage in 63 days, sailing at the rate of 6 knots an hour: how long would it take her, if she sailed at the rate of 7 knots an hour?

(19) A man's annual income is £408. 16*s.*: what does he receive for 15 days?

(20) A bankrupt's effects are worth £215, and his debts are £1075; what does he pay in the pound?

NOTE III.—To one of the magnitudes in a supposition there is a corresponding magnitude of the same kind in the demand, and these magnitudes must be expressed in units of the same denomination.

Ex. A man walks 1 m. 1 fur. 7 po. in 20 minutes: how long will he take to walk 41 m. 2 fur. 12 po.?

Here 1 m. 1 fur. 7 po. = 367 poles,

and 41 m. 2 fur. 12 po. = 13212 poles.

Then he walks 367 poles in 20 minutes;

he walks 1 pole in $\frac{20}{367}$ min.;

he walks 13212 poles in $\frac{13212 \times 20}{367}$ min., or 720 min.

Examples. (xcvi)—*continued.*

(21) If 3 bushels of wheat be worth 17*s.* 6*d.*, what is the worth of 43 qr. 6 bus.?

(22) If 15 yards of silk cost £1. 13*s.* 9*d.*, how much will 20 yd. 1 ft. cost?

(23) If 14 lb. of cheese can be bought for 9*s.* 11*d.*, what will be the price of 4 cwt. 32 lb.?

(24) If 3 cwt. 3 qr. cost £6. 15*s.*, what will be the cost of 2 cwt.?

(25) If 2 cwt. 3 qr. 7 lb. cost £5. 17*s.* 8½*d.*, what is the cost of 9 cwt.?

(26) If 3 cwt. cost £4. 7*s*. 6*d*., what is the cost of 7 cwt. 2 qr. 14 lb.?

(27) If 27 bus. $2\frac{1}{2}$ pk. cost £10. 7*s*. $2\frac{1}{4}$*d*., what is the cost of a bushel and a half?

(28) If 3 cwt. 69 lb. cost £14. 3*s*. 6*d*., how much may be bought for £23. 12*s*. 6*d*.?

(29) If 7 bus. 2 pk. cost £3. 5*s*. 5*d*., what will $4\frac{1}{2}$ bushels cost?

168. *Problems involving Fractions.*

Ex. If $\frac{3}{7}$ of an estate be worth £1500, what is the value of $\frac{4}{5}$ of the estate?

Since $\frac{3}{7}$ of the estate is worth £1500,

$\frac{1}{7}$ of the estate is worth £$\frac{1500}{3}$;

∴ the estate is worth £$\frac{7\times1500}{3}$ or £3500.

Hence $\frac{4}{5}$ of the estate is worth £$\frac{4\times3500}{5}$ or £2800.

Examples. (xcvii)

(1) If $\frac{3}{5}$ of an estate be worth £7520, what is the value of $\frac{5}{8}$ of the estate?

(2) A person owns $\frac{3}{8}$ of a ship and sells $\frac{2}{3}$ of his share for £1260; what is the value of the ship?

(3) If $3\frac{2}{5}$ lb. of tea cost 15*s*. 3*d*., how much can I buy for £4. 3*s*. $10\frac{1}{2}$*d*.?

(4) If $\frac{3}{14}$ of a ton of coal cost 4*s*. 6*d*., what is the price of $5\frac{1}{2}$ cwt.?

(5) If $\frac{2}{11}$ of a piece of work be done in 25 days, how much will be done in $11\frac{2}{3}$ days?

(6) A man walks 18 m. 2 fur. 26 po. $3\frac{2}{3}$ yd. in $5\frac{1}{2}$ hours. How long does he take to walk a mile and a half?

(7) A gentleman possessing $\frac{3}{14}$ of an estate sold $\frac{2}{7}$ of $\frac{1}{3\frac{1}{5}}$ of his share for £$120\frac{5}{8}$; what would $\frac{1}{5}$ of $\frac{3}{16}$ of the estate sell for at the same rate?

(8) If the carriage of 15·5 cwt. of goods for 60 miles cost 7*s*. 9*d*., how far ought 3·25 cwt. to be carried for the same money?

(9) What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sell $\frac{1}{9}$ of $\frac{7}{8}$ of his share for £350?

(10) When the ounce of gold is worth £3·89, what is the cost of ·04 lb.?

(11) If the price of candles $8\frac{1}{2}$ inches long be 9*d.* per half-dozen, and that of candles of the same thickness and quality $10\frac{1}{4}$ inches long be 11*d.* per half-dozen, which kind do you advise a person to buy?

(12) If the carriage of 60 cwt. for 20 miles cost £$14\frac{1}{2}$, what weight can be carried the same distance for £$5\frac{7}{16}$?

169. *Problems relating to Work done in a certain time.*

NOTE I.—If a man can do a piece of work in 7 hours, the part of the work which he can do in 1 hour will be represented by $\frac{1}{7}$.

Ex. (1). *A* can do a piece of work in 5 days, and *B* can do it in 12 days. How long will *A* and *B*, working together, take to do the work?

Here $\frac{1}{5}$ represents the part *A* does daily,

and $\frac{1}{12}$ represents the part *B* does daily;

$\therefore \frac{1}{5}+\frac{1}{12}$ represents the part *A* and *B* do daily:

$\therefore$ they do $\frac{17}{60}$ in 1 day:

$\therefore$ they do $\frac{1}{60}$ in $\frac{1}{17}$ day:

$\therefore$ they do the whole work in $\frac{60}{17}$ days, or $3\frac{9}{17}$ days.

Ex. (2). *A* can do a piece of work in 50 days, *B* in 60 days, and *C* in 75 days. In what time will they do it, all working together?

Here $\frac{1}{50}+\frac{1}{60}+\frac{1}{75}$ represents the part they do daily.

$\therefore$ they do $\frac{6+5+4}{300}$, or $\frac{15}{300}$, or $\frac{1}{20}$ daily.

$\therefore$ they do the work in 20 days.

Ex. (3). *A* can reap a field in $4\frac{1}{5}$ days, and *B* can reap it in $5\frac{2}{3}$ days. How long will they take to reap it, working together?

$$A \text{ does } \frac{1}{4\frac{1}{5}} \text{ or } \frac{5}{21} \text{ daily.}$$

$$B \text{ does } \frac{1}{5\frac{2}{3}} \text{ or } \frac{3}{17} \text{ daily.}$$

$\therefore$ together they do $\frac{5}{21} + \frac{3}{17}$, or $\frac{148}{357}$ daily.

$\therefore$ they do $\frac{1}{357}$ of the work in $\frac{1}{148}$ day :

$\therefore$ they do the work in $\frac{357}{148}$ days, or $2\frac{61}{148}$ days.

Ex. (4). *A* and *B* can do a piece of work in 4 hours; *A* and *C* in $3\frac{3}{5}$ hours; *B* and *C* in $5\frac{1}{7}$ hours. In what time can *A* do it alone?

A and *B* can do $\frac{1}{4}$ in an hour.

A and *C* can do $\frac{5}{18}$ in an hour.

$\therefore$ two men of *A*'s strength, assisted by *B* and *C*, can do $\frac{1}{4} + \frac{5}{18}$ in an hour.

Now *B* and *C* can do $\frac{7}{36}$ in an hour.

$\therefore$ two men of *A*'s strength can do $\frac{1}{4} + \frac{5}{18} - \frac{7}{36}$ in an hour, or $\frac{19}{36} - \frac{7}{36}$, or $\frac{12}{36}$, or $\frac{1}{3}$ in an hour.

$\therefore$ *A* can do $\frac{1}{6}$ in an hour :

$\therefore$ *A* can do the work in 6 hours.

NOTE II.—If a tap can fill a vessel in 5 hours, the part filled by it in 1 hour will be represented by $\frac{1}{5}$.

Ex. (1). A vessel can be filled by three taps, running separately, in 20, 30, and 40 minutes respectively. In what time will they fill it when they all run at the same time?

They fill $\frac{1}{20} + \frac{1}{30} + \frac{1}{40}$ of the vessel in 1 minute ;

$\therefore$ they fill $\frac{6+4+3}{120}$, or $\frac{13}{120}$ in 1 minute ;

$\therefore$ they fill $\frac{1}{120}$ in $\frac{1}{13}$ of a minute ;

$\therefore$ they fill the vessel in $\frac{120}{13}$ or $9\frac{3}{13}$ minutes.

Ex. (2). A bath is filled by a pipe in 40 minutes. It

is emptied by a waste pipe in an hour. In what time will the bath be full if both pipes be opened at once?

One pipe fills $\frac{1}{40}$ of the bath in a minute.

The other empties $\frac{1}{60}$ of the bath in a minute.

$\therefore$ when both are running, $\frac{1}{40} - \frac{1}{60}$, or $\frac{1}{120}$ of the bath is filled in a minute;

$\therefore$ the bath is filled in 120 minutes.

Examples. (xcviii)

(1) *A* can do a piece of work in 6 hours; *B* can do it in 9 hours. In what time will they do it if they work together?

(2) *A* can do a piece of work in 35 days; *B* can do it in 40 days; *C* can do it in 45 days. In what time will they do it, all working together?

(3) *A* and *B* can reap a field of wheat in 3 days; *A* and *C* in $3\frac{1}{2}$ days; *B* and *C* in 4 days. In what time could they reap it, all working together?

(4) If three pipes fill a vessel in 6, 8, and 12 minutes respectively, in what time will the vessel be filled when all three are opened at once?

(5) *A* does $\frac{7}{10}$ of a piece of work in 14 days. He then calls in *B*, and they finish the work in 2 days. How long would *B* take to do the whole work by himself?

(6) *A* does a piece of work in 3 hours, which is twice the time *B* and *C* together take to do it; *A* and *C* could together do it in $1\frac{1}{3}$ hours. How long would *B* alone take to do it?

(7) *A* can do a piece of work in 27 days, and *B* in 15 days; *A* works at it alone for 12 days, *B* then works alone 5 days, and then *C* finishes the work in 4 days. In what time could *C* have done the work by himself?

(8) A cistern is filled by 2 pipes in 18 and 20 minutes respectively, and emptied by a tap in 40 minutes; what part of it will be filled in 10 minutes when all are opened at the same instant?

170. *Problems relating to Clocks.*

The minute-hand moves 12 times as fast as the hour-hand, and therefore in 12 minutes the minute-hand gains 11 minute-divisions on the hour-hand.

Ex. (1). Find the time between 3 and 4 o'clock when the hands of a watch are together.

At 3 o'clock there are 15 minute-divisions between the hands; we have therefore to find how long it will take the minute-hand to gain 15 minute-divisions on the hour-hand.

The minute-hand gains 11 minute-divisions in 12 minutes;

1 minute-division in $\frac{12}{11}$ minutes;

15 minute-divisions in $\frac{15 \times 12}{11}$ minutes;

$\therefore$ the time required is $\frac{15 \times 12}{11}$ min., or $16\frac{4}{11}$ min. past 3.

Ex. (2). At what time between 2 and 3 are the hands of a clock at right angles to each other?

When the hands are at right angles there is a space of 15 minute-divisions between them.

Hence, since at 2 o'clock there are 10 minute-divisions between the hands, we have to find how long it will take the minute-hand to gain $10 + 15$, or 25 minute-divisions on the hour-hand.

The minute-hand gains 11 minute-divisions in 12 minutes;

1 minute-division in $\frac{12}{11}$ minutes;

25 minute-divisions in $\frac{25 \times 12}{11}$ minutes;

$\therefore$ the time required is $\frac{25 \times 12}{11}$ min., or $27\frac{3}{11}$ min. past 2.

Ex. (3). At what *times* between 6 and 7 are the hands of a clock at right angles to each other?

Twice between 6 and 7 this will occur: first, before the minute-hand has overtaken the hour-hand; secondly, after the minute-hand has passed the hour-hand.

Now, since at 6 o'clock there are 30 minute-divisions between the hands, we have to find:

First, how long it will take the minute-hand to gain $30 - 15$, or 15 minute-divisions on the hour-hand;

Secondly, how long it will take the minute-hand to gain $30 + 15$, or 45 minute-divisions on the hour-hand.

The process in each case will be similar to that in the preceding examples, and the results are $16\frac{4}{11}$ min. and $49\frac{1}{11}$ min. past 6.

Ex. (4). Find the time between 7 and 8 o'clock when the hands of a watch are opposite to each other.

When the hands are opposite there is a space of 30 minutes between them, and at 7 o'clock there is a space of 35 minutes between the hands.

Hence in this case we have to find how long it will take the minute-hand to gain a space of $35 - 30$, or 5 minutes on the hour-hand.

The process will be similar to that in the preceding examples, and the result is $5\frac{5}{11}$ min. past 7.

Examples. (xcix)

At what time are the hands of a watch together between the hours of

(1) 4 and 5. (2) 6 and 7. (3) 9 and 10?

At what time are the hands of a watch at right angles to each other between

(4) 4 and 5. (5) 7 and 8. (6) 11 and 12?

At what time are the hands of a watch opposite to each other between

(7) 1 and 2. (8) 4 and 5. (9) 8 and 9?

We now give a set of Miscellaneous Examples, which may be solved by the application of the principles laid down in this Section.

Examples. (c)

(1) If for a given sum I can have 1200 lb. carried 36 miles, how many pounds can I have carried 24 miles for the same sum?

(2) If a silver cup weighing 20 oz. 19 dwt. $2\frac{2}{11}$ gr. cost £5. 15*s.* 3*d.*, what is the price per oz.?

(3) If 3 cwt. 3 qr. 21 lb. $12\frac{4}{9}$ oz. cost £4. 8*s.* 9*d.*, what is the price per cwt.?

(4) If $\frac{4}{9}$ of a ship be worth £3264, what is the value of $\frac{5}{8}$ of the ship?

(5) A silver tankard weighs 1 lb. 10 oz.; what is its value, when a dozen spoons, weighing $3\frac{3}{4}$ oz. each, are worth £13. 10*s.*?

(6) A man spends £15. 8*s.* every 35 days, and saves £100 a year. What is his annual income?

(7) What is the income-tax on £675 at 5*d.* in the £?

(8) When the income-tax is 6*d.* in the £, a man pays £15. 7*s.* 6*d.*; what is his income?

(9) What is the dividend on a debt of £375. 15*s.* 9*d.*, when 13*s.* 4*d.* is paid in the £?

(10) A person after paying 5*d.* in the £ for income-tax has £724. 11*s.* 8*d.* left. What is his gross income?

(11) A man's income is reduced from £680 to £660. 3*s.* 4*d.*, when he has paid income-tax. What is the tax on £1?

(12) If a man travel 540 miles in 24 days, walking 6 hours a day, how many miles will he travel in 3 days, walking 8 hours a day?

(13) If 11 yards of cloth cost £3. 19*s.* $0\frac{3}{4}$*d.*, how many yards can be bought for £24. 15*s.* $11\frac{1}{4}$*d.*?

(14) A bankrupt's effects amount to 2548 guineas, and his debts to £3057. 12*s.* What will he pay in the pound?

(15) If 356 ac. 3 ro. $39\frac{1}{2}$ po. be let for £951. 19*s.* 10*d.*, what is the rent of a piece of ground which contains 17 ac. 2 ro.?

(16) If a cubic foot of ice weigh $57\frac{3}{8}$ lb., how many cubic feet of ice will weigh a ton?

(17) If 10 horses and 132 sheep can be kept 8 days for £50. 10*s.*, what sum will keep 15 horses and 148 sheep for the same time, supposing 5 horses to eat as much as 84 sheep?

(18) Find the cost of 4 tons 13 cwt. 2 qr. 3 lb., when 3 qr. 5 lb. cost £1. 13*s.* $4\frac{1}{2}$*d.*

(19) A man receives only 15*s.* in the £ of what is due to him, and thereby loses £150. 10*s.* 6*d.* What was due to him?

(20) If a fourpenny loaf weigh 3 lb. when wheat is 57*s.* a quarter, what will it weigh when wheat is 64*s.* 6*d.* a quarter?

(21) The income of a parish is £7591. 10*s.*; how much in the £ will produce a rate of £474. 16*s.* $4\frac{1}{2}$*d.*?

(22) What is the cost of $183\frac{1}{2}$ yards of cloth, when $2\frac{1}{2}$ yards cost a guinea?

(23) A man's wages are £1. 19*s.* 6*d.* for a month of 30 days: how much can he demand from the 10th of May to the 27th of October inclusive?

(24) If 15 men can perform a piece of work in 22 days, how many men will finish another piece of work 4 times as large in a fifth part of the time?

(25) If the rent of 39 ac. 2 ro. 20 po. be £148. 11*s.* $10\frac{1}{2}$*d.* what is the rent of 6 acres?

(26) A garrison of 2100 men has provisions for 9 months. but receives a reinforcement of 600 men; how long will the provisions last?

(27) If 72 men dig a trench in 63 days, in how many days will 42 men dig another trench three times as great?

(28) What is the income corresponding to an income-tax of £13. 2*s.* 6*d.*, at the rate of 7*d.* in the pound?

(29) If 2 cwt. 3 qr. 14 lb. of an article cost £6. 14*s.* 2*d.*, how much can be bought for £14. 17*s.* 6*d.*?

(30) If 42 men finish a work in 36 days, how many will finish twice as large a work in 27 days?

(31) *A* gives away in charity $\frac{1}{8}$ of his income, and pays $\frac{1}{10}$ of it in rates and taxes; with these deductions he has £473. 13*s.* 1*d.* left; what is his gross income?

(32) After paying income-tax at the rate of 4*d.* in the pound, a man has £476. 18*s.* 4*d.* left. What sum did his income-tax amount to?

COMPLEX PROBLEMS.

171. We now proceed to cases in which the supposition, expressed in the simplest form, contains *more than two magnitudes*, the demand containing the same number of magnitudes, all of which are given, except one, which has to be found.

Ex. (1). If 12 horses can plough 96 acres in 6 days, how many horses will plough 64 acres in 8 days?

In 6 days 96 acres can be ploughed by 12 horses.

In 1 day 96 acres can be ploughed by 6×12 horses.

In 1 day 1 acre can be ploughed by $\frac{6 \times 12}{96}$ horses.

In 8 days 1 acre can be ploughed by $\frac{6 \times 12}{8 \times 96}$ horses.

In 8 days 64 acres can be ploughed by $\frac{64 \times 6 \times 12}{8 \times 96}$ horses.

∴ the number of horses required is 6.

Ex. (2). If 35 bushels of oats last 7 horses for 20 days, how many days will 96 bushels last 18 horses?

35 bushels last 7 horses for 20 days.

1 bushel lasts 7 horses for $\frac{20}{35}$ days.

1 bushel lasts 1 horse for $\frac{20 \times 7}{35}$ days.

96 bushels last 1 horse for $\frac{20 \times 7 \times 96}{35}$ days.

96 bushels last 18 horses for $\frac{20 \times 7 \times 96}{35 \times 18}$ days.

∴ the number of days is $21\frac{1}{3}$

Examples. (ci)

(1) If 40 acres of grass be mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days?

(2) If £12 will pay 8 men for 5 days work, how much will pay 32 men for 24 days' work?

(3) If a regiment of 939 soldiers consume 351 quarters of wheat in 168 days, how many soldiers will consume 1404 quarters in 56 days?

(4) If two horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 quarters in 24 days?

(5) If a carrier receive two guineas for the carriage of 3 cwt. for 150 miles, how much ought he to receive for the carriage of 7 cwt. 3 qr. 14 lb. for 50 miles?

(6) If I pay 10*s.* for the carriage of 2 tons for 6 miles, what must I pay for the carriage of 12 tons 17 cwt. for 34 miles?

(7) If the carriage of 60 cwt. for 20 miles cost £14. 10*s.*, what weight can I have carried 30 miles for £5. 8*s.* 9*d.*?

(8) If 3 men earn 6*s.* 8*d.* in 4 days, what sum will 18 men earn in 16 days?

(9) How many bushels of wheat will serve 72 people 8 days, when 4 bushels serve 6 people 24 days?

(10) If a man travel 150 miles in 5 days when the days are 12 hours long, in how many days of 10 hours each will he travel 500 miles?

(11) If the carriage of goods weighing 5 cwt. 2 qr. 12 lb. for 150 miles come to £3. 5*s.* 5*d.*, what will be the charge for carrying four waggon-loads of the same, each weighing 7 cwt. 0 qr. 2 lb., the same distance?

(12) If £15. 12*s.* pay 16 labourers for 6 days, how many labourers at the same rate will £35. 2*s.* pay for 8 days?

(13) If the gas for 5 burners, 5 hours every day, for 10 days, cost 4*s.* 3*d.*, how many burners may be lighted 4 hours every evening for 15 days at a cost of £3. 16*s.* 6*d.*?

(14) If a travelling party of three spend £38 in 4 weeks, how long will £95 last a travelling party of five at the same rate?

(15) If it cost £26. 14*s.* 7*d.* to keep two horses for five months, what will it cost to keep three horses for eleven months?

(16) If it cost £29. 7*s.* 6*d.* to keep 5 horses for 6 weeks, how long may 3 horses be kept for £20. 11*s.* 3*d.*?

(17) If 5 men can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, working 16 hours a day, in what time can 7 men reap a field of 15 acres, working 12 hours a day?

(18) If 858 men in 6 months consume 234 quarters of wheat, how many quarters will be required for the consumption of 979 men for 3 months and a half?

(19) The wages of 5 men for 6 weeks being £14. 5*s.*, how many weeks will 4 men work for £19?

(20) If 7 men mow 22 acres in 8 days, working 11 hours a day, in how many days, working 10 hours a day, will 12 men mow 360 acres?

(21) If 10 horses consume 7 bus. 2 pk. of oats in 7 days, in what time will 28 horses consume 3 qr. 6 bus. at the same rate?

(22) If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

(23) If the wages of 29 men for 54 days amount to £80. 9*s.* 6*d.*, how many men must work 12 days to receive £407?

(24) What must I pay for the hire of 4 horses for 5 months, if I pay £18 for the hire of 3 horses for a month?

XXI. Simple Interest.

172. Interest is that which is paid by one, who borrows money, for the use of the money.

The money lent is called the Principal.

The Borrower agrees to pay at what is called a certain Rate of interest, which is usually reckoned by the sum paid. for the use of £100 for 1 year. Thus, if I borrow £500 for 1 year, and agree to pay £25 for the use of the money, I am said to borrow at the *Rate of* 5 *per cent. per annum*, that is, I agree to pay £5 for the use of every £100 in the loan at the end of the year.

The sum made up of the Principal and Interest added together, is called the AMOUNT at the end of the time for which the money is borrowed.

173. The solution of questions relating to Interest depends on precisely the same principles as those explained in the last Section, and it is only because of the necessity of explaining technical terms that there is any occasion to separate this or the succeeding Sections from Section XX.

For, just as we reason about the question

What must I pay for the hire of 4 horses for 5 months, if I pay £18 for the hire of 3 horses for a month?

so do we reason about the question

What must I pay for the use of £550 for 3 years, if I pay £5 for the use of £100 for a year?

Ex. (1). To find the Simple Interest on £2675 for 3 years at 4 per cent., we reason thus:

Interest on £100 for 1 year is £4;

on £1 for 1 year is £$\frac{4}{100}$

on £2675 for 1 year is £$\frac{4 \times 2675}{100}$

on £2675 for 3 years is £$\frac{4 \times 2675 \times 3}{100}$

$\therefore$ the interest is £$\frac{10700 \times 3}{100}$ or £(107×3), or £321.

Hence we derive the following Rule:

Multiply the Principal by the Rate per Cent. and by the Number of Years, and divide the product by 100.

The process stands thus:

$$\begin{array}{r} \pounds \\ 2675 \\ 4 \\ \hline 10700 \\ 3 \\ \hline \pounds 321\,00 \end{array}$$

$\therefore$ the interest is £321.

Ex. (2). Find the interest on £3749. 16*s*. 8*d*. for $5\frac{1}{2}$ years at $3\frac{1}{4}$ per cent.

£	*s*.	*d*.	
3749 .	16 .	8	Principal.
		$3\frac{1}{4}$	Rate.
937 .	9 .	2	
11249 .	10 .	0	
£12186 .	19 .	2	
		$5\frac{1}{2}$	Number of Years.
6093 .	9 .	7	
60934 .	15 .	10	
£670,28 .	5 .	5	
20			
s. 5,65			
12			
d. 7,85			

∴ the interest is £670. 5*s*. 7·85*d*.

Note I.—When we divide £67028 by 100, the quotient, obtained by cutting off the two figures on the right, is £670, and the remainder £28; this remainder, reduced to shillings and 5 shillings being added, is equivalent to 565 shillings; this, divided by 100, gives 5*s*. as quotient, and 65 shillings as remainder; this remainder, reduced to pence and 5 pence being added, is equivalent to 785*d*.; this, divided by 100, is equivalent to 7·85*d*.

Examples. (cii)

Find the Simple Interest

(1) On £2750 for 6 years at 5 per cent. per ann.

(2) On £3625 for 8 years at 4 per cent. per ann.

(3) On £2700 for 3 years at $4\frac{1}{2}$ per cent. per ann.

(4) On £8825 for $6\frac{1}{2}$ years at 4 per cent. per ann.

(5) On £3256. 15*s*. for 5 years at 4 per cent. per. ann.

(6) On £3265. 12*s*. for $5\frac{1}{2}$ years at 4 per cent. per ann.

(7) On £240 for $7\frac{3}{4}$ years at 5 per cent. per ann.

(8) On £586. 8*s*. 10*d*. for $3\frac{1}{2}$ years at $4\frac{1}{4}$ per cent. per ann.

Note II.—When the interest for parts of a year is required, they may be expressed as the fraction of a year: thus

Ex. (3). To find the interest on £3742. 16*s*. for 2 yr. 5 mo. at 4 per cent.

	£	*s*.
	3742 .	16
		4
	14971 .	4
		$2\frac{5}{12}$
12	74856 .	0
	6238 .	0
	29942 .	8
	£361,80 .	8
		20
		s. 16,08
		12
		d. ,96

∴ the interest is £361. 16*s*. 0·96*d*.

Note III.—In calculating Interest for a number of *days*, it is to be remembered that 73 days is $\frac{1}{5}$ of a common year.

Ex. (4). Find the Interest on £276 for 219 days at 4 per cent.

$$
\begin{array}{rl}
 & £ \\
 & 276 \\
 & \underline{\quad\ 4} \\
 & 1104 \\
 & \underline{\quad\ 3} \qquad \text{(for 219 days} = \tfrac{3}{5} \text{ of a year)} \\
5 & \underline{3312\qquad\quad} \\
 & £6{,}62 \ .\ 8s. \\
 & \underline{\quad 20} \\
 & s.\ 12{,}48 \\
 & \underline{\quad 12} \\
 & d.\ 5{,}76
\end{array}
$$

$\therefore$ the interest is £6. 12*s*. 5·76*d*.

Ex. (5). Find the Interest on £401. 10*s*. for 273 days at 3 per cent.

$$\text{Interest for 1 year} = £\frac{401\frac{1}{2} \times 3}{100} = £\frac{803 \times 3}{200}$$

$$\therefore \text{Interest for 1 day} = \tfrac{1}{365} \text{ of } £\tfrac{2409}{200}$$

$$\therefore \text{Interest for 273 days} = £\tfrac{273 \times 2409}{365 \times 200} = £\tfrac{9009}{1000} = £9{\cdot}009.$$

NOTE IV.—In calculating the number of days between two given days of the year, the rule is *to include one of them only* in the calculation. Thus from Jan. 4 to Jan. 9 will be 5 days.

Examples. (ciii)

Find the Simple Interest

(1) On £156 for 9 months at 4 per cent. per ann.

(2) On £280 for 7 months at $3\frac{1}{2}$ per cent. per ann.

(3) On £400 for 3 years and 35 days at $3\frac{3}{4}$ per cent. per ann.

(4) On £380 for 3 years and 45 days at $4\frac{3}{4}$ per cent. per ann.

(5) On £304. 4*s*. 3*d*. for 8 days at 4 per cent.

(6) On £584. 10*s*. for 42 days at 5 per cent.

(7) On £380. 4*s*. 2*d*. from Jan. 1 to April 30, 1872, at $4\frac{1}{2}$ per cent.

(8) On £554. 10*s*. for 3 months at 4 per cent.

(9) On £700 for 2 years and 31 days at $5\frac{3}{4}$ per cent.

(10) On £204. 17*s*. 7*d*. from Aug. 3 to Jan. 9 at 5 per cent.

174. In particular cases shorter methods may with advantage be employed in practical calculations, of which we will mention two.

I. If the rate of interest be 5 per cent.

To find the interest on £1276. 8*s*. 4*d*. for 1 year at 5 per cent.

Take a shilling for every 20 shillings,

and a penny for every 20 pence.

The interest required = 1276*s*. + 5*d*.

= £63. 16*s*. 5*d*.

Examples. (civ)

Find the yearly interest at 5 per cent. on the following sums:

(1) £27289; (2) £2645. 10*s*.; (3) £34569. 15*s*.;
(4) £4672. 4*s*. 2*d*.; (5) £863. 12*s*. 6*d*.; (6) £4587. 2*s*. 6*d*.

II. If the rate of interest be 3 per cent.

This is the case of most frequent occurrence in practical work, as the rate is that of the chief part of Government Securities.

The interest being payable *half-yearly*, the calculation is made for half a year's interest in the following way.

Find the interest on £2467. 12*s.* 10*d.* at 3 per cent. for half a year.

$$
\begin{array}{rlrlr}
\pounds & & s. & & d. \\
2467 & . & 12 & . & 10 \\
1233 & . & 16 & . & 5 \\
\hline
\pounds 37.01 & . & 9 & . & 3 \\
20 & & & & \\
\hline
s.\ 0\cdot 29 & & & & \\
12 & & & & \\
\hline
d.\ 3\cdot 51 & & & &
\end{array}
$$

∴ Interest required is £37. 0*s.* 3*d.*

Examples. (cv)

Find the half-yearly interest at 3 per cent. on the following sums, neglecting fractions of a penny.

(1) £3587. 6*s.* 8*d.* (2) £7523. 3*s.* 4*d.*

(3) £8642. 18*s.* 9*d.* (4) £6543. 12*s.* 6*d.*

(5) £37643. 9*s.* 4*d.* (6) £72987. 13*s.* 10*d.*

175. We have explained how to find the Interest (and Amount) when the Principal, Rate, and Time are given. We shall now explain how to find the Rate, or Time, or Principal, when the other two and also the Interest (or Amount) are given.

Ex. (1). At what Rate per cent. will £520 amount to £754 in 9 years?

Here Interest = £754 − £520 = £234.

Thus the interest on £520 for 9 years is £234.

∴ interest on £520 for 1 year is £$\frac{234}{9}$,

on £1 for 1 year is £$\frac{234}{520 \times 9}$,

on £100 for 1 year is £$\frac{100 \times 234}{520 \times 9}$, or £5.

∴ Rate required is 5 per cent.

Ex. (2). In what Time will the Interest on £360 amount to £126 at 5 per cent.?

Interest on £360 for 1 year is £$\frac{5 \times 360}{100}$, or £18.

Then, since £18 is the interest for 1 year,

£1 is the interest for $\frac{1}{18}$ yr.,

£126 is the interest for $\frac{126}{18}$ yr., or 7 years.

∴ Time required is 7 years.

Ex. (3). What Principal will amount to £840 in 3 years at 4 per cent.?

Interest on £100 for 3 years at 4 per cent. is £12.

∴ £112 is the amount which has for its Principal £100,

£1 is the amount which has for its Principal £$\frac{100}{112}$;

£840 is the amount which has for its Principal £$\frac{840 \times 100}{112}$ or £750.

∴ Principal required is £750.

Examples. (cvi)

(1) At what rate will the interest on £326 for 15 years amount to £220. 1*s*.

(2) At what rate will the interest on £372. 10*s*. for 18 years amount to £301. 14*s*. 6*d*.?

(3) At what rate will £245 amount to £252. 19*s*. 3*d*. in 9 months?

(4) In what time will £640 amount to £678. 8*s*. at $4\frac{1}{2}$ per cent.?

(5) In what time will £502. 13*s*. 4*d*. amount to £578. 1*s*. 4*d*. at $4\frac{1}{2}$ per cent.?

(6) In what time will £702. 13*s*. 4*d*. amount to £808. 1*s*. 4*d*. at 4 per cent.

(7) The interest on a sum of money amounts to £202. 10*s*. in 12 years at $4\frac{1}{2}$ per cent.; what is the principal?

(8) In what time will the interest on £537. 10*s.* amount to £80. 12*s.* 6*d.* at 4 per cent.?

(9) On what sum will the interest amount to £168. 15*s.* $6\frac{3}{4}$*d.* in 5 years at $4\frac{1}{2}$ per cent.?

XXII. Compound Interest.

176. Compound Interest is that which is paid, not only for the use of the original sum lent, but also for the *use of the interest* as it becomes due.

The interest on £500 for 1 year at 4 per cent. is £20.

If then £500 be lent at Compound Interest for 2 years at 4 per cent., the Interest for the *first* year is £20.

Now as the borrower has to pay for the use of this £20, the Interest for the *second* year must be calculated on £520.

Hence Interest for second year = £$\frac{520 \times 4}{100}$ = £20·8 = £20. 16*s.*

To put the matter in a more simple way, we have supposed *the borrower to retain* the interest due at the end of the first year, but the reasoning will be the same, if we suppose *the lender to receive* the interest at the end of the first year, and to put it out immediately at the same rate of interest.

177. We may calculate Compound Interest by the following rule:

Find the interest for the first year: add it to the original principal: call the result the Second Principal:

find the interest on this for the second year: add it to the second principal: call the result the Third Principal: find the interest on this for the third year, and so on:

Ex. (1). Find the Compound Interest on £7500 for 3 years at 4 per cent.

$$\begin{array}{rl} \pounds & \\ 7500 & \text{is the Principal for the } \textit{First} \text{ year.} \\ 4 & \\ \hline \pounds 300,00 & \end{array}$$

The interest for the *first* year is £300.

Add this to the Original Principal, £7500.

$$\begin{array}{lrl} & \pounds & \\ \text{Then} & 7800 & \text{is the Principal for the } \textit{Second} \text{ year.} \\ & 4 & \\ \hline & \pounds 312,00 & \end{array}$$

The interest for the *second* year is £312.

Add this to the Second year's principal, £7800.

$$\begin{array}{lrl} & \pounds & \\ \text{Then} & 8112 & \text{is the Principal for the } \textit{Third} \text{ year.} \\ & 4 & \\ \hline & \pounds\ 324,48 & \\ & 20 & \\ \hline & s.\ 9,60 & \\ & 12 & \\ \hline & d.\ 7,20 & \end{array}$$

The interest for the *third* year is £324. 9*s.* 7·2*d.*

∴ Compound interest reqd. is

£300 + £312 + £324. 9*s.* 7·2*d.* = £936. 9*s.* 7·2*d.*

If the *Amount* at Compound Interest be required, add the Original Principal, £7500, to the Compound Interest. £936. 9*s.* 7·2*d.*

Then Amount required = £8436. 9*s.* 7·2*d.*

Ex. (2). Find the Compound Interest on £540 for 3 years at 4 per cent.

£ 540
4
£ 21,60
20
s. 12,00

£	s.	
540 .	0	
21 .	12	First year's interest.
561 .	12	Second Principal.
	4	
£ 22,46 .	8	
20		
s. 9,28		

£	s.	
561 .	12	
22 .	9·28	Second year's interest
584 .	1·28	Third Principal.
	4	
£ 23,36 .	5·12	
20		
s. 7,2512		

£	s.	
21 .	12	= Interest for first year.
22 .	9·28	= Interest for second year.
23 .	7·2512	= Interest for third year.
£ 67 .	8·5312	
	12	
	d. 6·3744	

∴ Compound Interest required is £67. 8*s.* 6·3744*d*

Examples. (cvii)

Find the Compound Interest on

(1) £1250 for 2 years at 4 per cent.
(2) £2500 for 2 years at 3 per cent.
(3) £4000 for 3 years at 5 per cent.
(4) £5000 for 3 years at $4\frac{1}{2}$ per cent.
(5) £10000 for 3 years at $3\frac{1}{2}$ per cent.
(6) £3945 for 4 years at 4 per cent.
(7) £2746 for 3 years at 4 per cent.
(8) £4725 for 3 years at 4 per cent.

Find the amount of

(9) £250 in 2 years at $3\frac{3}{4}$ per cent. compound interest.
(10) £150 in 4 years at 4 per cent. compound interest.
(11) £375 in 2 years at $4\frac{1}{2}$ per cent. compound interest.
(12) £200 in 3 years at 3 per cent. compound interest.
(13) £550 in $3\frac{1}{2}$ years at 4 per cent. compound interest.
(14) £4790 in $2\frac{1}{3}$ years at $3\frac{1}{2}$ per cent. compound interest.

NOTE I.—When the Compound Interest is required for $3\frac{1}{2}$ years, find the compound interest for the whole of the 4th year, and take half the result as the compound interest for the half-year.

NOTE II.—The working of sums in Compound Interest is greatly shortened by the use of decimals, thus:

Ex. (3). Find the Compound Interest on £2465 for 3 years at 5 per cent.

£	
2465	*First* Principal.
5	
£12325	

Divide by 100, we get

£	
123·25	First interest.
2465	First principal.
2588·25	*Second* principal.
5	
£12941·25	

Divide by 100, we get

£	
129·4125	Second interest.
2588·25	Second principal.
2717·6625	*Third* principal.
5	
£13588·3125	

Divide by 100, we get

£	
135·883125	*Third* interest.
129·4125	*Second* interest.
123·25	*First* interest.

Comp.Int.reqd.£388·545625

Ex. (4). Find the Amount of £2758·25 in 3 years at 4 per cent. Compound Interest.

$$\begin{array}{rl} \pounds & \\ 2758{\cdot}25 & \text{First principal.} \\ 4 & \\ \hline \pounds 11033{\cdot}00 & \end{array}$$

Divide by 100, and we get £110·33 for First year's interest.

$$\begin{array}{rl} \pounds & \\ 2758{\cdot}25 & \\ 110{\cdot}33 & \\ \hline 2868{\cdot}58 & \text{Second principal.} \\ 4 & \\ \hline \pounds 11474{\cdot}32 & \end{array}$$

Divide by 100, and we get £114·7432 for Second year's interest.

$$\begin{array}{rl} \pounds & \\ 2868{\cdot}58 & \\ 114{\cdot}7432 & \\ \hline 2983{\cdot}3232 & \text{Third principal.} \\ 4 & \\ \hline \pounds 11933{\cdot}2928 & \end{array}$$

Divide by 100, and we get £119·332928 for Third year's interest.

$$\begin{array}{rl} \pounds & \\ 2983{\cdot}3232 & \\ 119{\cdot}332928 & \\ \hline \pounds 3102{\cdot}656128 & \text{Amount required.} \end{array}$$

178. The process for finding the Amount of a sum at Compound Interest may, in some cases, be presented in a very brief and neat form, as we proceed to show.

If the rate of interest be 4 per cent.,

∴ Amount of £100 at the end of 1 year is £104,

of £1 at the end of 1 year is $\frac{104}{100}$ of £1.

Hence it follows that

Amount of *any sum* at 4 per cent. in 1 year = $\frac{104}{100}$ of that sum.

Again,

Amount for the *second* year = $\frac{104}{100}$ of amount for the first year;

∴ Amount of *any sum* at 4 per cent. in 2 years

$= \frac{104}{100}$ of $\frac{104}{100}$ of that sum.

Suppose, then, we have to find the amount of £540 in 3 years at 4 per cent. compound interest.

The amount is $\frac{104}{100}$ of $\frac{104}{100}$ of $\frac{104}{100}$ of £540.

Thus we have to multiply £540 three times by 104, and to divide the result three times by 100, which latter operation will be effected by marking off 6 decimal places in the final product.

Now 540, or any other whole number, can be multiplied by 104 by a short process, thus:

```
540   × 104
 2160

56160
```

The second line being shifted *two* places to the right, and thus expressing the multiplication by 4, while the 540 may be regarded as representing 540 *hundreds*.

Hence the full process for finding the amount of £540 in 3 years at 4 per cent. compound interest will stand thus:

```
£
540     × 104
 2160
------
56160    × 104
 224640

5840640   × 104
 23362560
```

∴ Amount reqd. is £607·426560

Examples. (cviii)

Employ decimals to find the amount at Compound Interest of

(1) £3745 for 3 years at 5 per cent.

(2) £7400 for 3 years at $2\frac{1}{2}$ per cent.

(3) £34690 for 3 years at 4 per cent.

(4) £475. 15*s*. for 2 years at 3 per cent.

(5) £4264·35 for 3 years at 4 per cent.

179. Interest may be payable either yearly, half-yearly, or quarterly, or at some other stated period.

In finding the Compound Interest on £2000 in 2 years, when interest is payable *half-yearly*, at 5 per cent., we reason thus:

5 per cent. for a year = $2\frac{1}{2}$ per cent. half-yearly,

2 years = 4 half-years.

Hence we have to find the Compound Interest on £2000, *for 4 times of payment*, at $2\frac{1}{2}$ *per cent.*

For 1st payment the interest is £$\frac{2000 \times 2\frac{1}{2}}{100}$ = £50.

2nd payment the interest is £$\frac{2050 \times 2\frac{1}{2}}{100}$ = £51·25.

3rd payment the interest is £$\frac{2101 \cdot 25 \times 2\frac{1}{2}}{100}$ = £52·53125.

4th paymt., the int. is £$\frac{2153 \cdot 78125 \times 2\frac{1}{2}}{100}$ = £53·84453125.

∴ Compound Interest for the whole time = £207·62578125.

Examples. (clx.)

When Compound Interest is payable half-yearly, find the amount of

(1) £2500 in 2 years at 4 per cent.

(2) £6000 in 3 years at 5 per cent.

(3) £2745 in 2 years at 3 per cent.

XXIII. Present Worth and Discount.

180. Suppose A owes B £105, to be paid at the end of a year. If A be disposed to pay off the debt at once the sum which he ought to pay should be such that, if put out at interest by B, it will amount at the end of a year to £105. Suppose further that B can put out his money at 5 per cent. interest: then if he put out £100 at interest, this is the sum which will amount at the end of a year to £105.

Hence £100 is the sum, which A ought to pay at once, and this is called the PRESENT WORTH of the debt.

The difference between the Debt and the Present Worth, which is in the case under consideration £5, is called the DISCOUNT.

181. To find the Present Worth of a debt, due at the end of a given time, when interest is reckoned at a given rate, requires exactly the same process as that by which, Art. 175, Ex. (3), we found the Principal, when the Rate and Time and Amount were given.

Ex. (1). Thus, to find the Present Worth of £1781. 8*s*. due 4 years hence, reckoning interest at 5 per cent.

The Interest on £100 for 4 years at 5 per cent. is £20.

∴ £120 has for its Present Worth £100;

∴ £1 has for its Present Worth $\frac{100}{120}$ of £1.

∴ £1781. 8*s*. has for its Present Worth $\frac{100}{120} \times$ (£1781. 8*s*.)

∴ Present Worth reqd. $= \frac{100 \times 35628}{120}\,s. = (10 \times 2969)\,s.$
$= £1484.\ 10s.$

Ex. (2). To find the Discount on £1781. 8*s*. due 4 years hence, reckoning interest at 5 per cent.

The Present Worth is £1484. 10*s*., as we have just shewn:

∴ The Discount = £1781. 8*s*. − £1484. 10*s*.
= £296. 18*s*.

Examples. (cx)

Find the Present Worth of

(1) £5520, due 4 years hence at 5 per cent.

(2) £3171. 14*s.*, due 5 years hence at 3 per cent.

(3) £826. 10*s.*, due $3\frac{1}{2}$ years hence at 4 per cent.

(4) £416. 2*s.*, due 5 years hence at 4 per cent.

(5) £8949, due 4 years hence at $3\frac{1}{2}$ per cent.

Find the Discount on

(6) £8314. 10*s.*, due 5 years hence at 3 per cent.

(7) £930. 19*s.* 3*d.*, due $3\frac{1}{2}$ years hence at 3 per cent.

(8) £876. 10*s.* $8\frac{1}{4}$*d.*, due 3 years hence at 5 per cent.

(9) £5556, due $4\frac{1}{2}$ years hence at $3\frac{1}{2}$ per cent.

(10) £618. 2*s.* 6*d.*, due $3\frac{3}{4}$ years hence at 4 per cent.

Note.—When the discount for parts of a year is required, they may be expressed as the fraction of a year, thus:

Ex. (3). To find the discount on £512. 10*s.* due 2 years and 8 months hence, reckoning interest at $3\frac{1}{2}$ per cent.

The Interest on £100 for $2\frac{2}{3}$ yr. at $3\frac{1}{2}$ per cent. is £$(2\frac{2}{3} \times 3\frac{1}{2})$ or £$9\frac{1}{3}$

∴ £$109\frac{1}{3}$ has for its Present Worth £100;

∴ £1 has for its Present Worth $\frac{100}{109\frac{1}{3}}$ of £1;

∴ £$512\frac{1}{2}$ has for its Present Worth $\frac{100}{109\frac{1}{3}} \times$ £$512\frac{1}{2}$;

∴ Present Worth required = £$\frac{100 \times 1025 \times 3}{328 \times 2}$ = £$\frac{50 \times 25 \times 3}{8}$

= £468. 15*s.*

Hence Discount required

8. 15*s.* = £43. 15*s.*

Examples. (cxi)

(1) Find the discount on £388. 17*s.* 9*d.*, due 18 months hence, interest being reckoned at 4 per cent.

(2) What is the present worth of £5747, due 9 months hence, the rate of interest being $3\frac{1}{2}$ per cent.?

(3) What is the present worth of £10673, due 7 months hence, the rate of interest being [illegible]$\frac{1}{2}$ per cent.?

(4) A tradesman deducts 12*s.* $10\frac{1}{2}$*d.* from a bill of £16. 1*s.* $10\frac{1}{2}$*d.*, due 12 months hence, on being paid ready money. At what rate does he allow discount?

(5) Find the discount on £196. 4*s.* $4\frac{1}{2}$*d.*, due 6 months hence, at 8 per cent.

(6) Find the discount on £78. 9*s.* 9*d.*, due 8 months hence, at 6 per cent.

(7) Find the present worth of £252. 19*s.* 3*d.*, due 9 months hence, at $4\frac{1}{3}$ per cent.

(8) Find the discount on £151. 17*s.* 6*d.*, due $4\frac{4}{9}$ years hence, at $4\frac{1}{2}$ per cent.

(9) Find the present worth of £678. 8*s.*, due 16 months hence, at $4\frac{1}{2}$ per cent.

(10) The discount on a bill of £275. 6*s.* 8*d.*, due 15 months hence, is £13. 2*s.* $2\frac{2}{3}$*d.* At what rate per cent. is the simple interest calculated?

(11) Find the present value of £578. 1*s.* 4*d.*, due 3 years and 4 months hence, at $4\frac{1}{2}$ per cent.

(12) Find the discount on £574. 3*s.* 4*d.*, due 2 years and 3 months hence, at $2\frac{2}{3}$ per cent.

(13) Find the present value of £808. 1*s.* 4*d.*, due 3 years and 9 months hence, at 4 per cent.

(14) Find the difference between the simple interest and discount on £750. 16*s.* 8*d.*, due $2\frac{1}{2}$ years hence, at $2\frac{2}{5}$ per cent.

(15) A tradesman accepts £19. 6*s.* 3*d.* in payment of a debt of £20. 5*s.* $6\frac{3}{4}$*d.*, due 12 months hence, in consideration of being paid at once. What rate of discount does he allow?

(16) Find the difference between the discount and interest on £313. 19*s.* for 8 months, at 6 per cent.

Ex. (4). Find the present worth of £405. 3*s.* 4½*d.* due 3 years hence, reckoning Compound Interest at 5 per cent.

Compound Interest on £100 for 3 years at 5 per cent. = £15. 15*s.* 3*d.*

∴ of £115. 15*s.* 3*d.* the Present Worth is £100,

of £1 the Present Worth is $£\dfrac{100}{115\frac{61}{80}}$

of £405. 3*s.* 4½*d.* the Present Worth is $£\dfrac{100 \times 405\frac{27}{160}}{115\frac{61}{80}}$

∴ Present Worth $= £\dfrac{100 \times 64827 \times 80}{9261 \times 160} = £350.$

Examples. (cxii)

(1) Find the present worth of £6945. 15*s.*, due three years hence, reckoning compound interest at 5 per cent.

(2) Find the discount on £4774. 1*s.*, due 2 years hence, reckoning compound interest at 3 per cent.

(3) Find the discount on £100, due 2 years hence, at 4 per cent. compound interest.

(4) On what sum does the compound interest for 3 years at 5 per cent. amount to £165. 10*s.* 1½*d.*?

182. The Discount, of which we have been treating, is called Mathematical Discount or True Discount, to distinguish it from Practical Discount, of which there are two kinds:

(1) The deduction made by a trader, when an account is paid to him before the time when he proposes to demand payment. It is then calculated as interest on the account. Thus if a trader gives notice on his bill that he will allow 10 per cent. discount for immediate payment, and if the amount of the bill be £25. 10*s.*, he deducts 2 shillings for each pound, and 1 shilling for the odd 10*s.*, and the customer pays him £22. 19*s.*

(2) The deduction made by a lender of money from

the sum which he proposes to lend. Thus if a borrower binds himself by a bill to pay £100 a year hence, and a discounter advances money on the security of this bill, at the rate of 5 per cent., he gives to the holder of the bill £95, and takes the bill.

True Discount is the Interest on the Present Worth of a debt. Practical Discount is the Interest on the Debt itself. Hence Practical Discount is greater than True Discount.

The following are Examples of Practical Discount:

Examples. (cxli)—*continued.*

Find the Practical Discount on the following, omitting fractions of a penny:

(5) £4000, from October 18 to November 16, at 6 per cent.

(6) £3000, from December 23 to February 13, at 4 per cent.

(7) £2000, from May 10 to May 17, at 6 per cent.

(8) £1000, from June 12 to July 10, at 7 per cent.

(9) £1000, from July 17 to July 24, at $5\frac{1}{2}$ per cent.

(10) £2000, from March 31 to April 18, at $4\frac{1}{2}$ per cent.

(11) £2000, from January 15 to March 31, at $4\frac{1}{2}$ per cent. (February 28 days).

XXIV. Averages and Percentages.

183. The Average of two or more groups of numbers is found by adding the numbers together and dividing the sum by the number of groups.

Thus to find the average of 13, 15, 74, 23, 6, and 31, we find the sum of the numbers to be 162, and as the number of groups is 6, the average will be $162 \div 6$, or 27.

Note.—Express any remainder, which may occur, *decimally.*

Examples. (cxiii)

(1) Find the average of 14, 26, 9, 18, 13, 24, 27, 39.

(2) Find the average of 1600, 276, 974, 0, 236, 845, 1239.

(3) Find the average population of three towns, consisting respectively of 34729, 46238 and 87296 inhabitants.

(4) Find the average of $15\frac{1}{2}$, $36\frac{3}{4}$, $17\frac{5}{8}$, 0, $10\frac{3}{8}$, $74\frac{1}{5}$, $28\frac{1}{4}$, and 33.

(5) Find the average of $12\frac{12}{25}$, 21, $7\frac{3}{4}$, ·034,

PERCENTAGES.

184. When we speak of an agent getting 3 per cent. as a commission on the management of an estate, we mean that from every £100 collected he deducts £3 to remunerate himself for the trouble of collection.

When we read that the population of a town has increased 15 *per cent.* since the last census, we mean that if the number of inhabitants *then* had been divided into groups of 100, and the number of inhabitants *now* into groups of 115, the number of groups would be the same in both cases.

Ex. (1). How much is 3 per cent. on £1479?

Since £100 yields £3,

£1 yields £$\frac{3}{100}$

£1479 yields £$\frac{1479 \times 3}{100}$ or £44·37.

Ex. (2). The number of boys in a school increases in a certain period from 125 to 180, what is the increase per cent.?

On 125 the increase is 55.

On 1 the increase is $\frac{55}{125}$

On 100 the increase is $\frac{55 \times 100}{125}$ or $\frac{220}{5}$ or 44;

$\therefore$ the increase is 44 per cent.

NOTE.—In calculations with respect to percentage on sums of money it is to be remembered that

5 per cent. is equivalent to 1 shilling for every pound,

$2\frac{1}{2}$ per cent. is equivalent to sixpence for every pound,

10 per cent. is equivalent to 2 shillings for every pound,

$12\frac{1}{2}$ per cent. is equivalent to half-a-crown for every pound.

The percentage on an exact number of pounds at 10 per cent. may be very easily determined by cutting off the last figure on the right, taking the figures that remain as the pounds, and doubling the figure cut off for the number of shillings, in the percentage.

Thus 10 per cent. on £3495 is £349. 10*s.*

Examples. (cxiv)

(1) What will be the charge for commission on £133. 10*s.* at $2\frac{1}{2}$ per cent.?

(2) What will the insurance on the furniture of a house, valued at £625, amount to at $\frac{1}{8}$ per cent.?

(3) What sum must be paid to insure the cargo of a vessel, valued at £8350, at 4 guineas per cent.?

(4) What is the brokerage on £852. 10*s.* at $\frac{3}{8}$ per cent.?

(5) What is the insurance on a cargo, valued at £4270, at $3\frac{1}{2}$ per cent.?

(6) What is the brokerage on £1508. 17*s.* 10*d.* at $1\frac{1}{2}$ per cent.?

Write down, following the remarks of the Note in Art. 184, the following percentages:

5 per cent. on

(7) £2400. (8) £3475. (9) £4234. 10*s.* (10) £3275. 15*s.*

$2\frac{1}{2}$ per cent. on

(11) £4300. (12) £3479. (13) £5236. 10*s.* (14) £4278. 15*s.*

10 per cent. on

(15) £245. (16) £8276. (17) £324. 10*s.* (18) £728. 15*s.*

$12\frac{1}{2}$ per cent. on

(19) £40. (20) £76. (21) £92. 10*s.* (22) £103. 15*s.*

XXV. Profit and Loss.

185. If I sell for £105 that for which I gave £100, I gain £5 on an outlay of £100.

If I sell for £95 that for which I gave £100, I lose £5 on an outlay of £100.

The following Examples will shew the method of solving questions relating to Profit and Loss, the principles laid down in Section xx. being followed.

Ex. (1). I sell for 6*s*. that for which I gave 5*s*. What is my gain per cent.?

On an outlay of 5*s*. my gain is 1*s*.;

On an outlay of 1*s*. my gain is $\frac{1}{5}$*s*.;

On an outlay of 100*s*. my gain is $\frac{100}{5}$*s*., or, 20*s*.;

∴ I gain 20 per cent.

Ex. (2). I bought some goods for £4. 5*s*., how must I sell them in order to gain $17\frac{11}{17}$ per cent.?

That, for which I gave 100*s*., I must sell for $117\frac{11}{17}$*s*.;

That, for which I gave 1*s*., I must sell for $\frac{2000}{100 \times 17}$*s*.;

That, for which I gave 85*s*., I must sell for $\frac{2000 \times 85}{100 \times 17}$*s*., or £5.

Ex. (3). By selling goods for £1. 16*s*. I made a profit of 20 per cent. What did I give for them?

That, which I sold for 120*s*., I bought for 100*s*.;

That, which I sold for 1*s*., I bought for $\frac{100}{120}$*s*.;

That, which I sold for 36*s*., I bought for $\frac{100 \times 36}{120}$*s*., or £1. 10*s*.

Ex. (4). If by selling coffee at 1*s*. 7*d*. per lb. I lose 5 per cent., what must I sell it at to gain 5 per cent.?

That, which I sell at 95*d.*, I bought for

That, which I sell at 1*d.*, I bought for $\frac{100}{95}$*d.*;

That, which I sell at 19*d.*, I bought for $\frac{100 \times 19}{95}$*d.*, or 20*d.*

Having thus found *the cost price*, we proceed thus:

To gain 5 per cent.,

that, for which I gave 100*d.*, I must sell for 105*d.*;

that, for which I gave 1*d.*, I must sell for $\frac{105}{100}$

that for which I gave 20*d.*, I must :

186. When tea, spirits, wine, and such commodities are *mixed*, it must be observed that

quantity of ingredients = quantity of mixture,

cost of ingredients = cost of mixture.

Thus, if a mixture is made of 1 gallon of ale at 5*d.* a gallon, 3 at 9*d.*, 4 at 12*d.*, and 12 at 4*d.*,

quantity of ingredients = (1 + 3 + 4 + 12) gall., or 20 gall.,

cost of ingredients = (5 + 27 + 48 + 48)*d.*, or 128*d.*

If I want to know what gain per cent. I shall make by selling this mixture at 16*d.* a gallon, I reason thus:

20 gall. at 16*d.* will sell for 320*d.*;

∴ that, for which I gave 128*d.*, I sell for 320*d.*;

that, for which I gave 1*d.*, I sell for $\frac{320}{128}$*d.*;

that, for which I gave 100*d.*, I sell for $\frac{320 \times 100}{128}$*d.*, or 250*d.*

∴ I gain 150 per cent.

Examples. (cxv)

(1) If I buy an article for 16*s.* and sell it for £1, what is my gain per cent.?

(2) If I sell goods for £560 and gain 12 per cent., what was the cost price?

(3) If 375 yards of silk be sold for £490, and 20 per cent. profit be made, what did it cost per yard?

(4) If, by selling wine at 17*s*. 5*d*. a gallon, I lose 5 per cent., at what price must I sell it to gain 15 per cent.?

(5) If, by selling goods for £136, I lose 16 per cent., how much per cent. should I have lost or gained, if I had sold them for 160 guineas?

(6) The manufacturer will supply a certain article at $1\frac{1}{2}d.$; if a tradesman charge 2*d*., what profit per cent. will he make?

(7) A tradesman's prices are 20 per cent. above cost price. If he allow a customer 10 per cent. on his bill, what profit does he make?

(8) A tradesman's prices are 25 per cent. above cost price. If he allow a customer 12 per cent. on his bill, what profit does he make?

(9) A man buys goods at £23. 5*s*. 5*d*., and sells them at £22. 2*s*. $1\frac{3}{4}d.$ How much does he lose per cent.?

(10) A man buys goods at £15. 6*s*. 3*d*., and sells them again at £11. 15*s*. $9\frac{3}{4}d.$ How much does he lose per cent.?

(11) A man buys goods at the rate of £24 per cwt., and sells 2 tons 14 cwt. 3 qr. 12 lb. for £1500. How much has he gained or lost per cent. on his outlay?

(12) If 8 per cent. be gained by selling a piece of ground for £1031. 8*s*., what would be gained per cent. by selling it for £1050. 10*s*.?

(13) If 3 per cent. more be gained by selling a horse for £83. 5*s*. than by selling him for £81, what must his original price have been?

(14) A grocer mixes 12 lb. of tea at 2*s*. $6\frac{1}{2}d.$ per lb. with 4 lb. at 3*s*. $2\frac{1}{4}d.$ At what price must he sell the mixture so as to gain $33\frac{1}{3}$ per cent. upon his outlay?

(15) How many pounds of tobacco at 5*s*. 3*d*. a pound must a tobacconist mix with 4 lb. at 6*s*. 6*d*., that he may sell the mixture at 7*s*. 10*d*. per pound, and gain $33\frac{1}{3}$ per cent. upon his outlay?

(16) A spirit merchant buys 80 gallons of whisky at 18*s*. per gallon, and 180 gallons more at 15*s*. per gallon, and mixes them. At what price must he sell the mixture to gain $8\frac{1}{3}$ per cent. upon his outlay?

(17) I mix 80 gallons of gin at 15*s.* 6*d.* per gallon with 96 gallons at 17*s.* 1*d.*, and sell the mixture so as to gain 10 per cent. At what price per gallon do I sell it?

(18) A grocer buys two sorts of tea at 2*s.* 9*d.* and 3*s.* 1*d.* per lb. respectively. He mixes them so as to have 3 lb. of the dearer for every 1 lb. of the cheaper sort, and sells the mixture at 4*s.* per lb. What does he gain per cent.?

XXVI. Stocks and Shares.

187. In this Section we have to treat of transactions in the Public Debts of England and other countries, and in the Shares of Railway and other Companies.

The National Debt of Great Britain, which now amounts to about 773 millions, has been incurred by loans made to the State by individuals. Interest is paid upon the main part of this debt at the rate of 3 per cent. The names of the persons, who have a claim on the nation for such interest, are registered in books kept by the Bank of England on behalf of the Government. Such persons are called *Fundholders:* the debt itself is often called *The Funds:* and the interest, which is payable half-yearly, is called *Dividends.*

Suppose *A* to be a Fundholder in that particular part of the National Debt called *The Three per Cent. Consols*, and suppose the amount of the debt, which he is acknowledged by the Register to hold, to be £5000: he is then said to hold £5000 stock. *A* cannot demand the payment of 5000 sovereigns, or any smaller sum, from the Government, as a redemption of the debt, but the Government undertakes to pay him (or any one to whom he may assign his claim) 75 sovereigns, every half-year, that being the amount of interest on £5000 for half a year at 3 per cent.

Now suppose *A* to be desirous of selling his claim to *B*. The value of the claim does not vary much from

time to time in the case before us, for England is known to be willing and is acknowledged to be able to pay the interest on her debt, and the security of the claim makes the Fundholder satisfied with a low rate of interest, punctually paid and easily obtained. The value of £100 Stock in Consols is at the present time (Sept. 27, 1874) $92\frac{3}{8}$, that is, *A* can obtain £$92\frac{3}{8}$ for each £100 Stock that he holds, and *B* on the payment of $50 \times$ £$92\frac{3}{8}$, or £4618. 15*s*., can have the £5000 Stock, now held by *A*, transferred to him.

A's name is then removed from the Register, and *B*'s name is inserted in it, and the process is called a *Transfer*. *A* is said to *sell out* of the Funds and *B* is said to *invest* in them.

Three points must now be clearly marked:

(1) We shall know the amount of money received by *A* for any given amount of stock, if we know the price of the stock at the time of sale.

(2) We shall know how much stock can be bought by *B* for any given amount of money, if we know the price of the stock at the time of sale.

(3) We shall know the amount of income received by *A* (and subsequently by *B*) on any given amount of stock, if we know the rate of interest payable on the stock; the income depending in no way on the price of the stock.

These three cases we now proceed to illustrate:

Ex. (1). What is the value of £2500 stock in the 3 per cents. at $92\frac{1}{8}$?

The value of £100 stock is £$92\frac{1}{8}$

The value of £1 stock is £$\frac{92\frac{1}{8}}{100}$

The value of £2500 stock is £$\frac{2500 \times 92\frac{1}{8}}{100}$, or £2303. 2*s*. 6*d*.

Ex. (2). How much stock can be purchased at $92\frac{1}{2}$ for £740?

For £$92\frac{1}{2}$ I can purchase £100 stock

For £1 I can purchase £$\frac{100}{92\frac{1}{2}}$ stock

For £740 I can purchase £$\frac{740 \times 100}{92\frac{1}{2}}$ stock, or £800 stock.

Ex. (3). What annual income is derived from £3550 stock in the 3 per cents.?

Income on £100 stock is £3.

Income on £1 stock is £$\frac{3}{100}$.

Income on £3550 stock is £$\frac{3550 \times 3}{100}$, or £106. 10*s.*

This is merely a case of finding the Interest, where the stock is the Principal. (See Art. 173.)

Examples. (cxvi)

What must be given to buy

(1) £3850 stock in the 3 per cents. at 92?
(2) £475 stock in the 3 per cents. at 93?
(3) £572. 10*s.* stock in the 3 per cents. at $91\frac{1}{2}$?

When the 3 per cents. are

(4) At $91\frac{5}{8}$, how much stock can I buy for £2199?
(5) At $92\frac{1}{8}$, how much stock can I buy for £5527. 10*s.*?
(6) At $92\frac{3}{8}$, how much stock can I buy for £717. 8*s.* 11*d.*?

What is the half-yearly dividend

(7) On £3725 in the 3 per cents.?
(8) On £375 in the 3 per cents.?
(9) On £1406. 5*s.* in the 3 per cents.?

188. Stock is said to be at *par*, when the market price of £100 stock is £100.

It is said to be *at a premium*, when the market price of £100 stock is more than £100.

And *at a discount*, when the market price of £100 stock is less than £100.

The English funds are at a discount because other securities are always to be obtained in which the rate of interest is at least 4 per cent., and the risk but trifling. Still the price of our public funds varies so little in the time of peace, that we may say the current rate of interest, that is, the interest on capital invested in securities, which, according to general opinion, involve *no risk whatever*, is about $3\frac{1}{4}$ per cent.

189. Brokerage is a charge made by those agents who negotiate purchases and sales of stocks, and for such transactions in relation to the Funds the charge is $\frac{1}{8}$ per cent. on the stock sold or bought.

Hence, if *A sells* stock at $92\frac{3}{8}$, his broker deducts $\frac{1}{8}$ from the price, and the actual sum received by *A* is $92\frac{1}{4}$ per cent.

Again, if *B buys* stock at $92\frac{3}{8}$, his broker adds $\frac{1}{8}$ to the price, and the actual sum paid by *B* is $92\frac{1}{2}$ per cent.

190. In many Joint-Stock Companies the capital is held in *shares*. Thus the Capital of a Company being £200,000, it may be divided into 20,000 shares of £10 each, and the principles for calculating the effects of transactions in these shares are similar to those which regulate the calculations in reference to the Funds.

When all the Capital of a Company has been paid up, it is often changed from Shares to Stock, because in the case of Stock, transactions can be carried on with reference to *any portions of it*, whereas in the case of Shares, fractional parts of those Shares cannot be transferred.

Ex. (1). A person transfers £5000 stock from a 3 per cent. stock at 72, and invests the proceeds in a 4 per cent. stock at 90. Find the difference in his income.

First, he sells £5000 stock at 72, and gets £(72 × 50) or £3600.

Then he invests £3600 in the 4 per cent. stock at 90, and buys £$\frac{3600 \times 100}{90}$ stock, or £4000 stock.

Now his *first* income on the £5000 stock was £$\frac{5000 \times 3}{100}$,
or £150.

And his *second* income on the £4000 stock is £$\frac{4000 \times 4}{100}$,
or £160;

∴ he increases his income by £10.

Ex. (2). Which is the better investment, a 3 per cent. stock at $88\frac{1}{4}$, or a 4 per cent. stock at $117\frac{1}{2}$?

Income for £$88\frac{1}{4}$ invested in the 3 per cent. stock is £3,

∴ Income for £1 invested in the 3 per cent. stock

$$\text{is } £\frac{3}{88\frac{1}{4}}, \text{ or } £\frac{12}{353}.$$

Income for £1 invested in the 4 per cent. stock

$$\text{is } £\frac{4}{117\frac{1}{2}}, \text{ or } £\frac{8}{235}.$$

We have now to compare the fractions $\frac{12}{353}$ and $\frac{8}{235}$.

Reduced to a common *numerator*, these become

$$\frac{24}{706} \text{ and } \frac{24}{705};$$

and since the first of these is the smaller fraction (Art. 61),

∴ the 4 per cent. stock is the better investment.

Or we might have reasoned more simply thus:

To obtain an income of £3 I must pay £$88\frac{1}{4}$.
To obtain an income of £1 I must pay £$29\frac{5}{12}$.
To obtain an income of £4 I must pay £$117\frac{2}{3}$;

and since $117\frac{2}{3}$ is greater than $117\frac{1}{2}$,

∴ the 4 per cent. stock is the better investment.

Examples. (cxvii)

(1) Find the alteration in income occasioned by shifting £3200 stock from the 3 per cents. at $86\frac{3}{8}$, to 4 per cent. stock at $114\frac{7}{8}$: the brokerage being $\frac{1}{8}$ per cent.

(2) £5151 is invested in 5 per cent. stock at 101 : the stock rising to 105 it is sold out, and the proceeds invested in stock at 102, which gives $4\frac{3}{4}$ per cent. interest. Find the change in income.

(3) A person sells out £4800 in the 3 per cent. Consols, when they are at 93, and invests the proceeds in New Zealand 6 per cent. stock at 108. Find the alteration in his income.

(4) By selling out £4500 in the India 5 per cent. stock at $112\frac{1}{2}$, and investing the proceeds in Egyptian 7 per cent. stock, a person finds his income increased by £168. 15*s.* What is the price of the latter stock?

(5) A person sells out £2000 in the 3 per cents. at $94\frac{1}{2}$. Find the difference between the income derived from investing the proceeds in 6 per cent. stock at 126, and in 9 per cent stock at 210.

(6) What must be the price of 3 per cent. Consols so that, by investing £32,850, my income may be £1080 a year?

(7) A person has £5000 in the 3 per cent. Consols. He sells out when they are at $92\frac{1}{2}$, but on their falling to $88\frac{4}{5}$ he reinvests the money. What is the difference between his present and former incomes?

(8) The income derived from investing a certain sum in $3\frac{1}{2}$ per cent. stock at $106\frac{3}{4}$ is £120. 12*s.* Find the sum invested.

(9) What is the clear annual income derived from investing £6050 in the 3 per cents. at $90\frac{3}{4}$, after deducting an income-tax of 4*d.* in the pound?

(10) If a $3\frac{1}{2}$ per cent. stock be at 91, how much must I invest in it, so as to have a yearly income of £932, after paying 7*d.* in the pound income-tax?

(11) A person invests £9075 in the 3 per cents. at $90\frac{3}{4}$, and on their rising to 91 transfers it to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$. What increase does he make thereby in his annual income?

(12) The difference between the incomes derived from investing a certain sum in 6 per cent. stock at 126, and in 9 per cent. stock at 210, is £22. 10*s*. What is the amount invested?

(13) I sell out of the 3 per cents. at 96, and invest the proceeds in Railway 5 per cent. stock at par; find by how much per cent. on the stock bought my income is increased.

RATIO AND PROPORTION.

191. If A and B be quantities of the same kind, the relative greatness of A with respect to B is called the RATIO of A to B.

192. The ratio of one quantity to another quantity is represented in Arithmetic by the fraction, which expresses the measure of the first when the second is taken as the unit of measurement.

Thus if 5 shillings be the unit, the measure of 3 shillings is $\frac{3}{5}$, and the ratio of 3 shillings to 5 shillings is represented by the fraction $\frac{3}{5}$.

The words "the ratio of 3 shillings to 5 shillings" are abbreviated thus:

3 shillings : 5 shillings.

193. Ratios may be compared with each other by comparing the fractions by which they are represented.

Thus 2 pence : 5 pence is represented by $\frac{2}{5}$

and 3 pence : 7 pence is represented by $\frac{3}{7}$

Now $\frac{2}{5} = \frac{14}{35}$, and $\frac{3}{7} = \frac{15}{35}$,

$\therefore$ $\frac{3}{7}$ is greater than $\frac{2}{5}$

and $\therefore$ 3 pence : 7 pence is greater than 2 pence : 5 pence.

When we thus compare the ratios existing between two pairs of quantities, it is not necessary that all *four*

quantities should be of the same kind; it is only necessary that *each pair* should be of the same kind.

For example, we can compare the ratio of 4 shillings to 7 shillings with the ratio of 7 days to 12 days, and finding that $\frac{4}{7}$ is less than $\frac{7}{12}$, we may say that 4 shillings : 7 shillings is less than 7 days : 12 days.

194. When the ratio symbol (:) is placed between two *numbers* we may substitute for it the fraction symbol.

Thus if we have to compare the ratios 2 : 3 and 5 : 7, we effect it by comparing the fractions $\frac{2}{3}$ and $\frac{5}{7}$.

195. Ratios are *compounded* by multiplying together the fractions by which they are represented, and expressing the resulting fraction as a ratio.

Thus the ratio compounded of 2 : 3 and 5 : 7 is 10 : 21.

2 and 3 are called the TERMS of the ratio 2 : 3.

2 is called the ANTECEDENT and 3 the CONSEQUENT of the ratio.

196. PROPORTION consists in the equality of two ratios.

The Arithmetical test of Proportion is therefore *that the two fractions representing the ratios must be equal.*

Thus the ratio 6 : 12 is equal to the ratio 4 : 8, because the fraction $\frac{6}{12}$ = the fraction $\frac{4}{8}$.

The four numbers 6, 12, 4, 8, written in the order in which they stand in the ratios, are said to be *in proportion*, or *proportionals*, and this relation is thus expressed—

$$6 : 12 = 4 : 8.$$

The two terms 6 and 8 are called the EXTREMES.
12 and 4 the MEANS.

197. When four numbers are in proportion,
the product of the extremes = the product of the means.

For example, if $6 : 12 = 4 : 8$
$$6 \times 8 = 12 \times 4$$

For, since $\frac{6}{12} = \frac{4}{8}$, by hypothesis,
and $\frac{6 \times 8}{12 \times 8} = \frac{6}{12}$
and $\frac{4 \times 12}{8 \times 12} = \frac{4}{8}$

Now the *denominators* of these fractions are equal, and therefore the numerators must also be equal, that is
$$6 \times 8 = 4 \times 12.$$

From this it is evident that if three out of the four numbers that form a proportion are given, we can find the fourth.

Ex. (1). Find a fourth proportional to 3, 15, 7.

$3 : 15 = 7 :$ number required,
$\therefore 3 \times$ number required $= 15 \times 7$,
$\therefore$ number required $= \frac{15 \times 7}{3} = 35$.

Ex. (2). What number has the same ratio to 9 that 3 has to 5?

$3 : 5 =$ number required $: 9$,
$\therefore 5 \times$ number required $= 3 \times 9$,
$\therefore$ number required $= \frac{27}{5} = 5\frac{2}{5}$.

198. Three numbers are said to be in CONTINUED PROPORTION when the ratio of the first to the second is equal to the ratio of the second to the third.

Thus 3, 6, 12, are in continued proportion,
for $\frac{3}{6} = \frac{6}{12}$.

The second number is called a MEAN PROPORTIONAL between the first and the third.

Ex. (1). Find a mean proportional between 8 and 18.

8 : required number = required number : 18,

$\therefore$ required number $\times$ required number = 8×18,

$\therefore$ square of required number = 144,

$\therefore$ required number is 12.

Examples. (cxviii)

(1) Compare the ratios 2 : 5 and 4 : 9.

(2) Compare the ratios 17 : 39 and 19 : 41.

(3) Compare the ratios 4 : 7, 8 : 15 and 13 : 24.

(4) Compound the ratios 5 : 7, 13 : 15, 21 : 91, and 45 : 52.

(5) Find a fourth proportional to 5, 7, and 15.

(6) Find a fourth proportional to $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{5}{9}$.

(7) Find a fourth proportional to ·3, ·16, and ·09.

(8) Find a mean proportional to 14 and 56.

(9) Find a mean proportional to $\frac{5}{63}$ and $\frac{35}{81}$.

(10) Find a mean proportional to ·057 and ·513.

(11) If A = $3\frac{1}{3}$ of B and C = $5\frac{1}{5}$ of B, find the ratio of A to C.

(12) Divide £1587 between *A*, *B*, *C*, *D*, so that *A*'s share : *B*'s share = 6 : 5, *B*'s share : *C*'s share = 4 : 3, and *C*'s share : *D*'s share = 3 : 2.

XXVII. Division into Proportional Parts.

199. Suppose 3 persons, *A*, *B*, and *C*, to be in partnership, and that an arrangement is made that the profits of the business, in which they are engaged, are to be divided into 6 *equal* parts, of which *A* is to take 3 parts, *B* 2 parts, and *C* 1 part. The shares of *A*, *B*, and *C* are then said to be in the proportion of 3, 2, and 1.

Ex. (1). Divide £1275 among 3 persons, whose shares are to be in the proportion of 3, 5, and 7.

This may be regarded as a case in which one holds 3 shares, one 5, and one 7, and they hold 15 shares in all.

Hence, if we divide £1275 by 15, we get the amount of *one* share, that is, amount of one share = £$\frac{1275}{15}$ = £85.

Then one of the persons receives $3 \times$ £85, or £255;

the second receives $5 \times$ £85, or £425;

the third receives $7 \times$ £85, or £595.

Ex. (2). Divide £837 among three partners, whose shares are to be in the proportion of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$

The common denominator of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$ is 30;

$\therefore$ the shares are to be in the proportion of $\frac{15}{30}$, $\frac{10}{30}$ and $\frac{6}{30}$, that is, in the proportion of 15, 10, and 6;

Now $15 + 10 + 6 = 31$.

$\therefore$ amount of *one* share out of 31 shares = £$\frac{837}{31}$ = £27

Then one of the partners receives $15 \times$ £27, or £405;

the second receives $10 \times$ £27, or £270;

the third receives $6 \times$ £27, or £162.

Ex. (3). A rate of £4212 is to be paid by three townships, and the property on which it is levied is £24700 in the first, £37250 in the second, and £43350 in the third. What sum is paid by each?

Amount of property on which the rate is levied is £105300.

Then £105300 has to pay a rate of £4212.

$\therefore$ £1 has to pay a rate £$\frac{4212}{105300}$

$\therefore$ £24700 has to pay a rate £$\frac{24700 \times 4212}{105300}$ or £988;

£37250 has to pay a rate £$\frac{37250 \times 4212}{105300}$ or £1490

£43350 has to pay a rate £$\frac{43350 \times 4212}{105300}$ or £1734.

Examples. (cxix)

(1) Divide £60 into two parts proportional to 11 and 9.

(2) Divide £2500 into parts proportional to 2, 3, 7, 8.

(3) Divide £8470 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

(4) Gunpowder is made of saltpetre, sulphur and charcoal, in parts proportional to 75, 10 and 15: how many pounds of each are contained in 12 cwt. of gunpowder?

(5) The sides of a triangle are as 3, 4, 5, and the sum of the lengths of the sides is 480 yards: find the sides.

(6) Divide £640 between *A*, *B*, and *C*, so that *A* may have three times as much as *B*, and *C* as much as *A* and *B* together.

(7) Divide 100 apples between three boys, so that the first may receive 7 as often as the second receives 8, and the third may receive 5 as often as the second receives 4.

(8) A bankrupt owes £272. 10*s*. to *A*, £354. 5*s*. to *B*, and £490. 10*s*. to *C*: his assets are £418. 19*s*. $4\frac{1}{2}$*d*. What will each of the creditors receive?

(9) A force of police 1921 strong is to be distributed among 4 towns in proportion to the number of inhabitants in each: the population being 4150, 12450, 24900, and 29050, respectively. Determine the number of men sent to each.

(10) Divide £29 into an equal number of half-sovereigns, crowns, half-crowns, shillings, sixpences, and fourpences.

(11) A piece of land of 200 acres is to be divided among four persons, in proportion to their rentals from surrounding property: supposing these rents to be £500, £350, £800, and £90, how many acres must be allotted to each?

(12) Divide £2. 15*s*. between *A*, *B*, and *C*, so that for each threepenny piece received by *A*, *B* may receive a fourpenny piece, and that there are as many shillings in the sum received by *C* as there are sixpences in the sum received by *B*.

(13) Divide £2. 12*s*. between 5 men, 7 women, and 14 boys, so that each woman may have $\frac{5}{7}$ of each man's share, and each boy $\frac{2}{5}$ of each woman's share.

(14) A number of men, women, and children, are in the proportions 2, 3, 5; divide £129. 8*s*. 3*d*. among them, so that the shares of a man, a woman, and a child may be proportional to 3, 2, 1, there being 9 women.

NOTE.—In the course of trade it is often required to find the proportion in which goods of the same kind but different qualities must be mixed, so that the compound may be worth a particular price: the following are examples:

Ex. (1). In what proportion must beer worth 25 shillings a barrel and beer worth 50 shillings a barrel be mixed, so as to form a mixture worth 30 shillings a barrel?

On each barrel of the cheaper, that is sold at 30*s*., the gain will be 5*s*.; and on each barrel of the dearer, that is sold at 30*s*., the loss will be 20*s*.; hence, by selling 4 barrels of the cheaper, as much is gained as that which is lost by selling 1 barrel of the dearer;

∴ for every barrel at 50*s*., 4 barrels at 25*s*. must be put in the mixture.

Ex. (2). I have a quantity of beer worth 15 shillings a barrel, another quantity worth 25 shillings, and a third quantity worth 35 shillings a barrel. How must I mix them to get a mixture worth 30 shillings a barrel?

On each barrel of the first, sold at 30*s*., I shall gain 15*s*.
On each barrel of the second, sold at 30*s*., I shall gain 5*s*.
On each barrel of the third, sold at 30*s*., I shall lose 5*s*.

One way in which I can get a mixture worth 30*s*. a barrel will be by putting 3 barrels of the third for 1 of the first, and 1 of the third for 1 of the second, so that they are mixed in the proportion of 1 at 15*s*., 1 at 25*s*., and 4 at 35*s*.

Examples. (cxx)

(1) A grocer buys tea at 1*s*. 6*d*. per lb., and a better sort at 2*s*. 3*d*. per lb., in what proportion must he mix them to get a mixture worth 2*s*. per lb.?

(2) A brewer would mix beer at 23*s*. per barrel with beer worth 38*s*. per barrel, so as to make a mixture worth 30*s*. per barrel; how must he proceed?

(3) A merchant has a lot of currants which cost him 3*d*. a lb., and another lot which cost $4\frac{1}{4}d$. per lb.; how must he mix them so as to make the mixture worth 4*d*. per lb.?

(4) A man has whisky worth 16*s.* a gallon, and another lot worth 19*s.* a gallon; how must he mix these with water to make a mixture of 51 gallons worth 12*s.* a gallon?

PARTNERSHIP.

200. Suppose A and B to be partners in trading, that A has a capital of £2000 in the business for 9 months, and B a capital of £2500 for 12 months, at the end of which time the gain is £840. How should this be divided?

We reason thus:

£2000 for 9 months may be taken as £18000 for 1 month.

£2500 for 12 months may be taken as £30000 for 1 month.

A fair division of the gain will therefore be made by dividing it into two parts proportional to 18000 and 30000.

Hence A's share $= \frac{18000}{48000}$ of £840 $= \frac{3}{8}$ of £840 $=$ £315;

B's share $= \frac{30000}{48000}$ of £840 $= \frac{5}{8}$ of £840 $=$ £525.

And so, when *several* persons are engaged in a business, and they advance capital for different periods of time, the period of time must be made *the same for all* by multiplying the capital of each by the number which measures the years, months, or days, for which his capital is advanced, and then the profits may be divided by the method previously explained.

Examples. (cxxi)

(1) A starts a business with a capital of £2400 on the 19th of March, and on the 17th of July admits a partner B with a capital of £1800. The profits amount to £943 by the 31st of December. What is each person's share?

(2) D and E enter into partnership; D puts in £40 for 3 months, and E £75 for 4 months. They gain £70. What is each man's share in the gain?

(3) A, B, C are partners: A puts in £500 for 7 months, B £600 for 8 months, and C £900 for 9 months. The profit is £410; what is the share of each?

(4) Three graziers hire a pasture for their common use, for which they pay £26. 10*s.* One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent should each pay?

(5) Two men complete in a fortnight a piece of work for which they are paid £7. 7*s.* 9*d.* One of them works alternately 9 hours and 8 hours a day. The other works $9\frac{1}{2}$ hours for 5 days in the week, and does nothing on the remaining day. What part of the sum should each receive?

(6) *A* and *B* begin to trade in partnership. *A* puts in £400 at first, and £500 at the end of two months. *B* puts in £300 at first, and £600 at the end of three months. The profit at the end of the year is £470. How should this be divided?

XXVIII. Equation of Payments.

201. When several sums of money are due from *A* to *B*, payable at different times, it is often required to find the time, called the EQUATED TIME, at which all may be paid together, without injustice to *A* or *B*.

When great exactness is demanded, interest must be added to the sums paid after they are due, and discount subtracted from the sums paid before they are due. But in practice the following rule is sufficiently accurate:

Multiply each debt by the number of days [or months] after which it is due: add the results together: divide this sum by the sum of the debts: the quotient will be the number of days [or months] in the equated time.

Take the following Examples:

Ex. (1). If £300 be due from *A* to *B* at the end of 5 months, and £700 at the end of 9 months, when may both sums be paid in a single payment without unfairness to *A* or to *B*?

Number of months in equated time $= \frac{300 \times 5 + 700 \times 9}{300 + 700}$

$$= \frac{78000}{10000}$$

$$= \frac{78}{10}$$

$$= 7\tfrac{4}{5}$$

∴ the whole amount of the debt should be paid at the end of $7\frac{4}{5}$ months.

Ex. (2). *A* is indebted to *B* in the following amounts: £147. 10*s.* due 3 months hence; £243. 15*s.* due 6 months hence; £126. 14*s.* due 9 months hence; and £196. 17*s.* due 12 months hence. Find the time when all these payments should be made together.

Number of months in equated time

$$= \frac{147{\cdot}5 \times 3 + 243{\cdot}75 \times 6 + 126{\cdot}7 \times 9 + 196{\cdot}85 \times 12}{147{\cdot}5 + 243{\cdot}75 + 126{\cdot}7 + 196{\cdot}85}$$

$$= \frac{442{\cdot}5 + 1462{\cdot}5 + 1140{\cdot}3 + 2362{\cdot}2}{714{\cdot}8}$$

$$= \frac{5407{\cdot}5}{714{\cdot}8}$$

$$= 7\tfrac{4039}{7148}$$

∴ 7 months and 17 days may be taken as the time for the whole debt to be paid.

NOTE.—This method is but a rough approximation, and can only be taken as equitable when the various times of payment are not widely apart. It will, in short, be applicable only to cases which occur in the ordinary course of trade, and it is therefore all that we require in the present work.

It is also to be observed that the error involved in this method is *slightly in favour of the payer*, because interest is calculated on the payments made before they are due, instead of discount, in the algebraical process, from which the method is derived.

Examples. (cxxii)

Find, exactly or approximately, the equated times of payment of the following debts:

(1) £250 due 4 months hence, and £350 due 10 months hence.

(2) £300 due 3 months hence, £400 due 4 months hence, and £500 due 6 months hence.

(3) £275. 5*s.* due 2 months hence, £342. 16*s.* due 5 months hence, and £237. 10*s.* due 7 months hence.

(4) Find the equated time of payment of £1050, one-third of which is due in 3 months, two-fifths in 4 months and the rest in 6 months.

(5) *A* owes *B* on the 1st of March the following sums: £140 due on 20th of April, £120 due on 14th of May, £380 due on 15th of June. On what day may *B* pay these debts together?

(6) *M* buys goods of *N*, and has 6 months' credit from the date of invoice. The goods are delivered on 6 different days, to the following amounts: £101. 14*s.* 10*d.* on Aug. 8, £144. 2*s.* 10*d.* on Sept. 5, £303. 18*s.* 10*d.* on Sept. 18, £757. 0*s.* 8*d.* on Nov. 13, £123. 11*s.* 6*d.* on Nov. 28, £123. 11*s.* 6*d.* on Dec. 5. On the 13th of January, *N*, who desires to receive all the debts in one payment, reckons that this payment should be made in 100 days. Show that this is approximately correct.

XXIX. Exchange.

202. The term *Exchange* is here used for giving or receiving in the money of one country a sum equal in value to a sum of money of another country. For example, if an English merchant pays to a French merchant 100 sovereigns and receives in return 2500 francs, it is a case of Exchange.

The *Par of Exchange* between two countries denotes the nominal value of a unit of coinage in one country, as estimated in terms of a unit of coinage in the other country.

The *Course of Exchange* is the actual (and variable) value of a unit of coinage in one country, as estimated in terms of a unit of coinage in the other country.

Thus the Par of Exchange between Paris and London is given, if we know that £1 = 25 francs.

In the Course of Exchange £1 may be equal in value at one time to $25\frac{1}{2}$ francs, at another to $25\frac{1}{5}$ francs.

(1) What is the value in English money of 4528·7 francs, when the course of exchange between Paris and London is at 25·3 francs per pound sterling?

Since 25·3 francs = £1,

1 franc = £$\frac{1}{25\cdot3}$

∴ 4528·7 francs = £$\frac{4528\cdot7}{25\cdot3}$, or £179.

(2) A merchant pays a debt of 4379 milrees in Portugal with £971. 11*s.* $9\frac{3}{4}$*d.*; what is the course of exchange in pence per milree?

£971. 11*s.* $9\frac{3}{4}$*d.* = 932727 farthings.

Then since 4379 milrees = 932727 farthings,

1 milree = $\frac{932727}{4379}$ farthings, or 213 farthings;

∴ the course of exchange is $53\frac{1}{4}$ pence per milree.

(3) If 11·65 Dutch florins are given for 24·42 francs, 352 florins for 407 marks of Hamburg, and $58\frac{1}{4}$ marks for 32 silver rubles of Petersburg; how many francs should be given for 932 silver rubles?

Here 1 silver ruble = 3

1 mark = $\frac{352}{407}$ florins,

1 florin = $\frac{24\cdot42}{11\cdot65}$ francs;

∴ 1 silver ruble = $\frac{58.25}{32} \times \frac{352}{407} \times \frac{24\cdot42}{11\cdot65}$ francs, or 3·3 francs;

∴ 932 silver rubles = 932 × 3·3 francs, or 3075·6 francs.

Examples. (cxxiii)

(1) If £1 be worth 12 florins, and also worth 25 francs 56 centimes, how many francs and centimes is one florin worth? (N.B.—100 centimes = 1 franc.)

(2) If £1 be worth $25\frac{1}{2}$ francs, and be also worth 2244 copeks in Russian money, what is the value of the napoleon in Russian copeks? (N.B.—20 francs = 1 napoleon.)

(3) The French franc is divided into 100 centimes and the Frankfort florin into 60 kreutzers. When the pound sterling is worth 25·50 francs in Paris, and 11 fl. 54 kr. at Frankfort, what is the worth of the napoleon in florins and kreutzers?

(4) A tourist just before leaving London has £23. 12*s.* 6*d.*, and on his arrival at Paris he exchanges what remains for 23 nap. 12 fr. 50 cent.; find what he has already spent, the rate of exchange being 25 fr. 20 cent. per pound sterling.

(5) If £1 be worth 25·60 francs, and also worth 6 thalers 20 silbergroschen in Prussian money, how many francs and centimes is a thaler worth, the thaler being equal to 30 silbergroschen?

(6) If in London I get £1 for 25 francs 20 centimes, what shall I gain or lose per cent. by taking French money into Bavaria, where the exchange is 11 gulden 40 kreutzers for £1, and 8 gulden 20 kreutzers for a napoleon; a gulden being equal to 60 kreutzers?

(7) If £3 = 20 thalers; 25 thalers = 93 francs; 27 francs = 5 scudi; and 62 scudi = 135 gulden; how many gulden can I get in exchange for £5. 10*s.*?

(8) If 12 milrees be worth £2. 14*s.*, and a napoleon be worth 16*s.*, how many milrees ought I to receive for 270 napoleons?

(9) A person, on leaving England, exchanged his money for French money at the rate of 25 francs for a sovereign, and on arriving at Munich received 135 Bavarian gulden for 300 francs: what was his loss in English money, supposing a gulden to be worth 1*s.* $8\frac{1}{2}$*d.*?

(10) The exchange at Paris upon London is at the rate of 25 francs 70 centimes for £1 sterling, and the exchange at Vienna upon Paris is at the rate of $40\frac{1}{2}$ Austrian florins for 100 francs: find how many Austrian florins should be paid at Vienna for a £50 note.

MONETARY SYSTEMS OF FRANCE, ITALY, GERMANY, AND AUSTRIA.

The following is a brief sketch of the systems of Coinage of the European Countries chiefly visited by tourists:

France.

The unit of coinage is the FRANC, a silver coin, equal to $9\frac{2}{5}$*d.* nearly.

The gold coin most in use is the 20-franc piece, called a Napoleon. There are also gold pieces of 100, 50, 10 and 5 francs.

The copper coins are pieces of 10, 5, 2 and 1 centimes, 100 centimes being equal to a franc.

This coinage is used in Belgium and Switzerland.

Italy.

The unit is the LIRA, a silver coin of the same value as the franc.

The copper coins are Centesimi, of which 100 = 1 Lira.

German Empire.

The unit of the Monetary System of the German Empire is the MARK, a silver coin, equal to $11\frac{3}{4}$*d.* nearly.

The 20-mark gold piece is therefore rather less than our Sovereign.

In the copper coinage, 100 Pfennige = 1 Mark.

The old silver Thaler = $3\frac{1}{3}$ Marks.

Austria.

The unit of coinage is the FLORIN or GULDEN, a silver coin, equal to 2 shillings nearly.

The Ducat is a gold coin, equal in value to 9*s.* 5*d.*, nearly.

The copper coins are Kreuzers, of which 100 = 1 Florin.

THE METRIC SYSTEM.

The Metric System of Weights and Measures is now in use in many countries of Europe. The following is an account of the system as it is established in France, where it originated at the end of the last century.

The basis of all measurement is the MÈTRE, a measure of length equal to the ten-millionth part of the distance from the North Pole to the Equator.

The length of the Mètre in English Measure is 39·37 inches, nearly.

Units of Metric Measures.

1. LENGTH.—The MÈTRE.
2. SURFACE.—The ARE = 100 square mètres.
3. SOLIDITY.—The STÈRE = 1 cubic mètre.
4. CAPACITY.—The LITRE = the cube of the tenth part of a mètre.
5. WEIGHT.—The GRAMME, which is the weight of a quantity of distilled water which fills the cube of the hundredth part of a mètre.

The Tables of Weights and Measures under the Metric System are constructed upon one uniform principle. Prefixes derived from Greek and Latin are attached to each of the units.

Greek Prefixes.

Deca stands for	10 times	the unit.
Hecto stands for	100 times	
Kilo stands for	1000 times	
Myria stands for	10000 times	

Latin Prefixes.

	10th part	of the unit.
Centi stands for the	100th part	
Milli stands for the	1000th part	

Thus,

A decamètre	= 10 mètres.
A hectolitre	= 100 litres.
A kilogramme	= 1000 grammes.
A myriamètre	= 10000 mètres.

Also,

A decilitre	= ·1 litre.
A centimètre	= ·01 mètre.
A milligramme	= ·001 gramme.

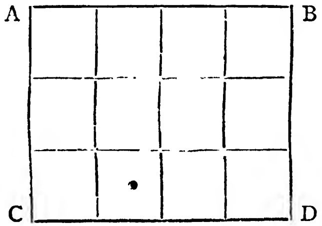
NOTE.—In English measures the following are rough approximations of some of the French measures:

The Kilogramme	is about $2\frac{1}{5}$ lb. Avoird.
The Litre	is about $1\frac{3}{4}$ pints.
The Kilomètre	is about 5 furlongs.
The Hectare	is about $2\frac{1}{2}$ acres.

XXX. Measurement of Area.

203. The Unit of Measurement, by which we measure Area or Surface, is derived from the unit of Length. Thus, if we take an inch as the unit of length, and construct a square whose side is an inch, this Square Inch may be taken as the Unit of Area, and the *measure* of any given area will be the number of times it contains this unit, in accordance with the remarks in Art. 52.

Let ABDC be a rectangle, and let the side AB be 4 inches in length, and the side AC 3 inches in length.



Then, if the Unit of Length be an inch, the *measure* of AB is 4, and the *measure* of AC is 3.

Divide AB, AC into four and three equal parts respectively, and draw lines through the points of division parallel to AC, AB respectively. Then the rectangle ABDC is divided into a number of equal *squares*, each of which is a square inch.

If one of these squares be taken as the Unit of Area, the *measure* of the area of ABDC will be the number of these squares.

Now this number is the same as that obtained by multiplying the measure of AB by the measure of AC:

that is, measure of ABDC $= 3 \times 4 = 12$;

$\therefore$ area of ABDC is 12 square inches.

Hence to find the area of a rectangle we multiply the measure of the length by the measure of the breadth, and the product will be the measure of the area.

Ex. (1). A rectangular garden is 48 feet long and 25 feet broad, what is its area?

Taking a foot as the unit of length, and therefore a square foot as the unit of area,

measure of the area $= 48 \times 25 = 1200$;

$\therefore$ the area is 1200 square feet.

Ex. (2). A rectangular board is 2 ft. 7 in. long and 1 ft. 4 in. broad, what is the area of its surface?

Taking 1 inch as the unit of length, and therefore 1 square inch as the unit of area,

measure of the area $= 31 \times 16 = 496$;

$\therefore$ the area is 496 square inches.

Or we might take 1 foot as the unit of length, and then

measure of area $= 2\frac{7}{12} \times 1\frac{1}{3} = \frac{31 \times 4}{12 \times 3} = \frac{31}{9} = 3\frac{4}{9}$;

$\therefore$ the area is $3\frac{4}{9}$ square feet.

Ex. (3). The length of the side of a square croquet-ground is 49 yards, what is its area?

Taking 1 yard as unit of length,

$$\text{area} = (49 \times 49) \text{ sq. yds.} = 2401 \text{ sq. yds.}$$

NOTE.—Observe the difference between the expressions 49 *yards square* and 49 *square yards*. The former refers to a square whose side is 49 yards, and whose area is 2401 square yards, the latter to a surface whose area is 49 square yards.

Ex. (4). A rectangular bowling-green is 56 yards long and 42 yards broad. Find the distance from corner to corner.

By Euclid I. 47, we know that in a right-angled triangle the square *on* the side opposite the right angle is equal to the sum of the squares *on* the sides containing the right angle.

Hence the square *of* the measure of the side opposite the right angle is equal to the sum of the squares *of* the measures of the sides containing the right angle.

Thus in our present example,

square of measure of distance from corner to corner

$$= (56 \times 56) + (42 \times 42) = 4900;$$

$$\therefore \text{distance is } 70 \text{ yards.}$$

Examples. (cxxiv)

Find the area of the rectangles having the following dimensions:

(1) 7 ft. by 5 ft. (2) $13\frac{1}{2}$ ft. by 10 ft.

(3) $22\frac{1}{4}$ ft. by $13\frac{1}{2}$ ft. (4) 5 ft. 4 in. by 2 ft. 3 in.

(5) 17 ft. 5 in. by 8 yd. 2 ft. (6) 5 yd. 1 ft. by 4 yd. 2 ft.

(7) 12 yd. 2 ft. by 5 yd. 1 ft.

(8) 6 yd. 2 ft. 3 in. by 2 yd. 1 ft. 5 in.

(9) 7 yd. 2 ft. by 5 yd. 2 ft. 6 in.

Find the area of the squares whose sides have the following lengths:

(10) $5\frac{1}{2}$ yd. (11) $37\frac{1}{4}$ yd. (12) $17\frac{3}{4}$ ft.
(13) $29\frac{1}{2}$ ft. (14) 9 ft. 7 in. (15) 3 ft. 4 in.
(16) 7 yd. 1 ft. 5 in. (17) 15 yd. 2 ft. 3 in.

Find the breadth of the following rectangles, having given the area and length:

(18) Area 176 sq. ft., length 11 ft.

(19) Area 71 sq. ft. 100 sq. in., length 9 ft. 8 in.

(20) Area 854 sq. ft. 84 sq. in., length 97 ft. 8 in.

(21) Area 1 acre, length 440 yd.

(22) Area 5 acres, length 275 yd.

(23) Area 5 ac. 1 ro. 36 po., length 267 yd. 2 ft.

What are the sides of the squares whose areas are

(24) 81 sq. ft. (25) 256 sq. ft.
(26) 1178 sq. yd. 7 sq. ft. (27) 33 ac. 4305 sq. yd.?

(28) A rectangular field is 225 yards in length and 120 yards in breadth; what will be the length of a straight path from corner to corner?

(29) A rectangular field is 300 yards long and 200 yards broad; find the distance from corner to corner.

(30) A rectangular plantation, whose width is 88 yards, contains $2\frac{1}{2}$ acres; find the distance from corner to corner.

(31) What is the length of the diagonal of a square, whose side is 5 inches?

(32) The area of a square is 390625 square feet; what is the length of the diagonal?

CARPETING ROOMS.

204. If we know the area of the floor of a room, we know how many square inches of carpet will be required to cover it. Carpets are sold in strips, and when the *width* of a strip is known, we shall know how much *length* of carpet will be required to cover a given surface.

For instance, if the surface be 159 square feet, and the carpet selected be 27 inches wide, we reason thus:

$$159 \text{ sq. ft.} = 159 \times 144 \text{ sq. inches;}$$

$$\therefore \text{ length of carpet reqd.} = \frac{159 \times 144}{27} \text{ inches} = 848 \text{ in.}$$

$$= 23 \text{ yd. } 1 \text{ ft. } 8 \text{ in.}$$

To find the cost of this carpet at 5*s.* a yard, that is, a yard in length, we may proceed by Practice, thus:

	s.	*d.*	
1 ft. = $\frac{1}{3}$ of a yd.	5 .	0	= cost of 1 yard.
		23	
	115 .	0	= cost of 23 yd.
6 in. = $\frac{1}{2}$ of 1 ft.	1 .	8	= cost of 1 ft.
2 in. = $\frac{1}{3}$ of 6 in.		10	= cost of 6 in.
		$3\frac{1}{3}$	= cost of 2 in.
2,0	117 .	$9\frac{1}{3}$	
	£5 . 17 .	$9\frac{1}{3}$	= cost of carpet.

Examples. (cxxv)

How many yards of carpet, 27 inches wide, will be required for rooms whose dimensions are:

(1) 15 ft. by 13 ft. (2) 25 ft. by 12 ft. 6 in.

(3) 22 ft. 4 in. by 20 ft. 3 in. (4) 27 ft. by $14\frac{1}{2}$ ft.

(5) 35 ft. 4 in. by 27 ft. 3 in.

Find the expense of carpeting rooms whose dimensions are:

(6) 18 ft. by 14 ft., with carpet 30 inches wide, at 5*s.* a yard.

(7) 22 ft. by $15\frac{1}{2}$ ft., with carpet 27 inches wide, at 4*s.* 6*d.* a yard.

(8) 29 ft. 9 in. by 23 ft. 6 in., with carpet a yard wide, at 5*s.* 9*d.* a yard.

(9) 34 ft. 8 in. by 13 ft. 3 in., with carpet $\frac{3}{4}$ yard wide, at 3*s.* $4\frac{1}{2}$*d.* a yard.

PAPERING THE WALLS OF A ROOM.

205. To find the quantity of paper required to cover *one* wall of a room, we find the area of the surface of the wall by taking the product of the measures of the length and breadth of that wall, the latter being the same as the height of the room. Hence we shall find the area of the *four* walls of the room *if we take the measure of the compass of the room and multiply it by the measure of the height.*

By the compass of a room we mean the length of a string stretched tight on the floor, going all round the room.

Deductions for doors and windows and fireplace must be made in practice.

Suppose then we have to find how much paper is required for the walls of a room, whose length is 22 ft. 3 in., breadth 17 ft. 4 in. and height 9 ft. 6 in.

We first find the compass of the room, thus:

$$\begin{array}{r@{\;.\;}l l} \text{ft.} & \text{in.} & \\ 22 & 3 & \\ 17 & 4 & \\ 22 & 3 & \\ 17 & 4 & \\ \hline 79 & 2 & \text{compass of the room.} \end{array} \quad \text{(the first four rows: dimensions of the four sides,)}$$

To get the area of paper required, we multiply the measure of the compass of the room by the measure of the height, thus:

$$\text{area} = (9\tfrac{1}{2} \times 79\tfrac{1}{6})\text{ sq. ft.} = \tfrac{19 \times 475}{12}\text{ sq. ft.} = 752\tfrac{1}{12}\text{ sq. ft.}$$

NOTE.—Papers, like carpets, are sold in strips, and if we know the width of a strip we shall know how many feet in length will be required to cover a given surface.

Thus, in the room under consideration, if the paper be 20 inches wide,

$$\text{length of paper required} = (752\tfrac{1}{12} \div \tfrac{20}{12})\text{ ft.} = \tfrac{9025}{20}\text{ ft.} = 451\tfrac{1}{4}\text{ ft.}$$

Examples. (cxxvi)

How many square feet of paper will be required for rooms whose dimensions are:

(1) Length, 19 ft.; breadth, 16 ft.; height 9 ft.?

(2) Length, $24\frac{1}{2}$ ft.; breadth, $18\frac{1}{4}$ ft.; height, 10 ft.?

(3) Length 25 ft. 7 in.; breadth, 19 ft. 4 in.; height, 9 ft. 9 in.?

(4) Length, 23 ft. 5 in.; breadth, 18 ft. 7 in.; height, 9 ft. 6 in.?

Find the expense of papering rooms whose dimensions are:

(5) Length, 18 ft.; breadth, 14 ft.; height, 8 ft.; with paper 16 inches wide, at a shilling a yard.

(6) Length, 20 ft. 6 in.; breadth, 17 ft. 4 in.; height 9 ft.; with paper 20 inches wide, at 5*d.* a yard.

(7) Length, 30 ft. 8 in.; breadth, 26 ft. 5 in.; height, 10 ft. 6 in.; with paper 2 ft. wide, at 8*d.* a yard.

(8) Length, 26 ft.; breadth, 21 ft.; height, 10 ft.; with paper 20 inches wide at 9*d.* a yard, allowing for a fireplace which is 5 ft. 3 in. by 4 ft., a door which is 7 ft. by $4\frac{1}{2}$ ft., and two windows each 6 ft. by $3\frac{1}{2}$ ft.

Miscellaneous Examples. (cxxvii)

(1) Find the cost of varnishing the floor of a room 14 ft. 4 in. broad, and 15 ft. 6 in. long, at 6*d.* per square yard.

(2) What will it cost to pave an area 146 ft. 9 in. long and 88 ft. 9 in. broad, at $11\frac{1}{4}$*d.* per square yard?

(3) The area of a square garden is 4 roods 1 pole 29 sq. yd. $6\frac{3}{4}$ sq. ft.; find the length of its side.

(4) Find the length of the side of a square whose area is 1 ro. 26 po. 28 sq. yd. $4\frac{1}{2}$ sq. ft.

(5) Find the expense of turfing a plot of ground, which is 40 yd. long and 100 ft. wide, with turfs each a yard in length and 1 ft. in breadth; the turfs, when laid, costing 6*s.* 9*d.* per hundred.

(6) A square room, whose floor measures 32 sq. yd. 1 sq. ft., is 11 ft. 6 in. in height: find the expense of whitewashing its ceiling and walls at $2\frac{1}{2}d.$ per square yard.

(7) It costs £22. 13*s.* 9*d.* to cover the floor of a room $8\frac{1}{4}$ yd. long by $6\frac{2}{3}$ yd. wide, with carpet 2 ft. wide. Find the price of the carpet per yard.

(8) If the cost of papering a room $8\frac{1}{4}$ yd. long and $6\frac{2}{3}$ yd. wide, with paper 2 ft. wide at 4*d.* per yard, be £2. 19*s.* 8*d.*, find the height of the room.

(9) A rectangular field, whose length is 997 yd. 1 ft., contains 12 acres, 4087 sq. yd. 1 sq. ft. Find the breadth of the field.

(10) How many acres are there in a square field each side of which is 330 yd.?

(11) The length of a room is 21 ft., and its height 10 ft. 6 in., and the area of the floor is $\frac{5}{11}$ of the area of the four walls. Find the breadth of the room.

(12) What length must be cut off a board, which is $6\frac{3}{4}$ inches broad, that the area may contain a square foot?

(13) What is the expense of papering a room 4 yd. $6\frac{3}{4}$ in. long, 3 yd. $11\frac{1}{4}$ in. wide, and 3 yd. 1 ft. high, with paper half a yard wide, at 3*d.* a yard?

(14) How many stones, each 2 feet long and $15\frac{1}{2}$ inches wide, would be required to pave a square courtyard, whose side is 124 feet?

(15) What is the cost of papering a room, 15 ft. long, 12 ft. wide, and 10 ft. high, with paper $\frac{5}{6}$ yd. wide, at $7\frac{1}{2}d.$ a yard?

(16) Find the cost of papering a room 21 ft. long, 15 ft. wide, and 12 ft. high, with paper $2\frac{1}{2}$ ft. wide, at 9*d.* a yard, allowing for a door 7 ft. high and 3 ft. wide, 2 windows each 5 ft. high and 3 ft. wide, and a panelling 2 feet high round the floor.

(17) The length of one side of a rectangular field is 572 yards, and the area of the field is 50 acres 2 ro. 32 po. Find the length of the other side, and of the diagonal.

(18) A rectangular field, 300 yards long and 150 broad, is separated into 4 equal parts by 2 bands of trees, 20 feet wide, parallel to the sides. How large will each part be, and what will be the area covered by the trees?

(19) A room, whose height is 11 feet, and length twice its breadth, takes 143 yards of paper 2 feet wide for its four walls; how many yards of gilt moulding will be required?

(20) What will be the cost of painting the walls and ceiling of a room, whose height, length, and breadth are 12 ft. 6 in., 27 ft. 4 in., and 20 ft. respectively, at $8\frac{1}{2}d.$ per square yard?

(21) Find the expense of carpeting a room 15 ft. 9 in. long and 13 ft. 4 in. broad, with carpet 27 inches wide, at 4*s.* 6*d.* per yard.

(22) Find the cost of carpeting a room 10 yd. 2 ft. long and 7 yd. 1 ft. broad, with carpet $\frac{3}{4}$ yd. wide, at 4*s.* 6*d.* a yard.

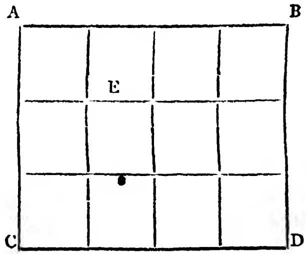
(23) If the cost of carpeting a room 11 yd. long and 8 yd. wide, with carpet at 3*s.* a yard, be £19. 16*s.*, find the width of the carpet.

(24) How many flag-stones, each 5·76 ft. long and 4·15 ft. wide, are requisite for paving a cloister, which incloses a rectangular court 45·77 yd. long and 41·93 yd. wide, the cloister being 12·45 ft. wide?

XXXI. Measurement of Solidity.

206. The Unit of Measurement, by which we measure the Volume of a Solid body or the Capacity of a vessel, is derived from the Unit of Length. Thus, if we take an inch as the unit of length, and construct a *cube*, each of whose edges is an inch in length, this Cubic Inch may be taken as the Unit of Volume, and the measure of any given volume will be the number of times it contains this unit.

207. Let ABDC be a rectangle, and let the side AB be 4 inches in length, and the side AC 3 inches in length.



Then ABDC will contain 12 square inches. (Art. 203.)

Now suppose we construct a number of blocks of wood, perfect cubes, whose volume is a cubic inch, and place one of these on each of the squares in ABDC: and then place another of the blocks on the top of each of the first set; and so on, till we have piled up 5 layers. Then we shall have constructed a rectangular solid, whose length is 4 inches, breadth 3 inches, and depth or thickness 5 inches.

Now the number of cubic inches in this solid we estimate in the following way: for *each* of the squares in ABDC we shall have a pile of 5 cubic inches: therefore the number of cubic inches in the solid will be 5×12 or 60.

Hence we obtain the following Rule:

To find the cubic content of a rectangular solid, find the continued product of the measures of the length, breadth, and thickness, and the result is the measure of the cubic content.

Ex. (1). Find the cubic content of a rectangular piece of timber whose length is 47 ft., breadth 4 ft., and thickness 3 ft.

Taking a foot as the unit of length, and therefore a cubic foot as the unit of cubic content,

$$\text{measure of cubic content} = 47 \times 4 \times 3 = 564;$$
$$\therefore \text{ the cubic content is 564 cubic feet.}$$

Ex. (2). What is the cubic content of a room whose length is 22 ft. 6 in., breadth 18 ft. 3 in. and height 10 ft.?

$$\text{Cubic content} = (22\tfrac{1}{2} \times 18\tfrac{1}{4} \times 10) \text{ cubic ft.,}$$
$$= \frac{45 \times 73 \times 80}{2 \times 4} \text{ cub. ft.} = 4106\tfrac{1}{4} \text{ cub. ft.}$$

Ex. (3). A rectangular sheet of water, of uniform depth, is 430 yards long, 270 yards broad, and contains 7314300 cubic feet of water; what is its depth?

Reducing the length and breadth to feet,

$$\text{Area of surface} = (430 \times 3 \times 270 \times 3) \text{ sq. ft.};$$
$$\therefore \text{ depth} = \frac{7314300}{430 \times 3 \times 270 \times 3} \text{ ft.} = 7 \text{ ft.}$$

Examples. (cxxviii)

Find the cubic content of the rectangular solids whose dimensions are

(1) 8 ft., 7 ft., 6 ft. (2) $10\frac{1}{2}$ ft., $8\frac{1}{4}$ ft., $6\frac{1}{3}$ ft.

(3) 5 ft. 6 in., 4 ft. 3 in., 3 ft. 7 in.

(4) 11 ft. 8 in., 9 ft. 10 in., 7 ft. 5 in.

(5) 6 yd. 2 ft. 4 in., 3 yd. 1 ft. 7 in., 4 ft. 11 in.

(6) How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 18 inches thick; each brick being 9 inches long, $4\frac{1}{2}$ inches wide, and 3 inches deep?

(7) A lake, whose area is 45 acres, is covered with ice 3 inches thick: find the weight of the ice in tons, if a cubic foot of ice weigh 920 oz. avoird.

(8) If 500 men excavate a basin 800 yd. long, 500 yd. wide, and 40 yd. deep in 4 months, how many men will be required to excavate a basin 1000 yd. long, 400 yd. wide, and 50 yd. deep in 5 months?

(9) A square block of stone, 2 feet in thickness, is in cubic content 5 cub. ft. 24 in.: what is the length of its edge?

(10) What weight of water will a rectangular cistern contain, the length being 4 ft., the breadth 2 ft. 6 in., and the depth 3 ft. 3 in., when a cubic foot of water weighs 1000 oz.?

(11) A block of stone is 4 ft. long, $2\frac{1}{2}$ ft. broad, and $1\frac{1}{4}$ ft. thick; it weighs 27 cwt. Find the weight of 100 cubic inches of the stone.

(12) A cubic foot of water weighs 1000 oz. Find the length of the side of a cubic vessel whose contents (water) weigh 4 tons 12 cwt. 3 qr. 10 lb. 7 oz.

(13) If 120 men can make an embankment $\frac{3}{4}$ of a mile long, 30 yards wide, and 7 yards high, in 42 days, how many men would it take to make an embankment 1000 yards long, 36 yards wide, and 22 feet high, in 30 days?

(14) A rectangular cistern, 9 feet long, 5 ft. 4 in. wide, and 2 ft. 3 in. deep, is filled with liquid which weighs 2520 pounds. How deep must a rectangular cistern be, which will hold 3850 pounds of the same liquid, its length being 8 feet, and its width 5 feet 6 inches?

(15) Find the cost of making a road 110 yards in length, and 18 ft. wide: the soil being first excavated to the depth of 1 ft., at a cost of 1*s.* per cubic yard: rubble being then laid 8 inches deep, at 1*s.* per cubic yard, and gravel placed on the top, 9 inches thick, at 2*s.* 6*d.* per cubic yard.

EXAMINATION PAPERS.

(1) EXPRESS in words, 20103007; and in figures, nineteen millions one thousand and six.

(2) Find the sum of 42734, 50269, 7005, 80007 and 40000.

(3) By how many does 42704 exceed 37859?

(4) Multiply 7256 by 892, and 49382 by 375.

(5) Divide 173432 by 532, and 2667640 by 340.

(6) Write in figures, two hundred and twenty-five millions three thousand and ten; and in words, 999990090.

(7) From eight millions and eight subtract seventy thousand and seventy-seven.

(8) Multiply 8649 by 4237, and 7650 by 389.

(9) Divide 50222122 by 598, and 38871923744 by 40072.

(10) The sum of two numbers is 58, and one of them is 23, what is the other?

(11) What number must be subtracted from one million two hundred and fifty thousand to leave seven hundred thousand nine hundred and twenty-one?

(12) Divide 436694 by 54, using short division, and explaining how the remainder is obtained.

(13) Find the H. C. F. of 5947 and 5985.

(14) Multiply 869875 by 78946, and test the result by "casting out the nines."

(15) Divide thirty-nine millions three hundred and forty-two thousand one hundred and fifty-four by CCCCXXIII.

(16) Add together three millions five hundred thousand and fifteen, three hundred and fifteen thousand and five, and fifty-nine thousand and ninety-five; and subtract from the

sum the difference of nine millions eight thousand and ten, and eight millions five hundred and ninety-seven thousand six hundred and twenty-eight.

(17) The difference between two numbers is 47, and the smaller is 59, what is the other?

(18) Find the H. C. F. of 4941 and 1485; and also of 504, 5292 and 1520.

(19) Reduce 48602728 farthings to pounds, &c.

(20) Divide £825. 1*s*. 10$\frac{3}{4}$*d*. by 19.

(21) Divide two thousand six hundred millions five hundred and nine thousand and fifty by two thousand two hundred and fourteen.

(22) Reduce £123. 17*s*. 11$\frac{1}{4}$*d*. to farthings.

(23) Find the H. C. F. of 19527 and 23667.

(24) Find the L. C. M. of 42, 56, 126 and 154.

(25) The sum of two numbers is 148, and one of them exceeds the other by 16, what are the numbers?

(26) Reduce to their lowest terms $\frac{185}{333}$ and $\frac{2772}{8064}$.

(27) Find the continued product of 87, 56 and 409.

(28) Multiply £27. 14*s*. 7$\frac{1}{2}$*d*. by 77.

(29) Find the L. C. M. of 9, 12, 18, 30, 48 and 60.

(30) There were 900 millions of penny postage stamps issued in 1870. What was their value in pounds, shillings and pence?

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(31) Add twelve hundred thousand six hundred and seventy-eight to seventy millions seven hundred thousand and seven; subtract ninety thousand from the sum, and write down the remainder in words.

(32) The thirty-fifth part of a number is 43, what is the number?

(33) Find the H. C. F. of 11431 and 12006.

(34) Find the L. C. M. of 1, 2, 3, 4, 5, 6, 7, 8, 9.

(35) Reduce £14789. 19*s*. 11$\frac{3}{4}$*d*. to farthings.

(36) Simplify $\frac{54792}{89154}$, and also $\frac{4}{51} \times \frac{3}{64} \times \frac{340}{25}$.

(37) Multiply five hundred and sixty-three millions four hundred and one by thirty-six thousand eight hundred, and divide the result by seventy-nine millions.

(38) Multiply £2. 13*s*. 4*d*. by 15.

(39) Divide £17. 13*s*. 4*d*. by 20.

(40) Divide 436694 by 35, employing short division.

(41) A soldier receives $8\frac{1}{2}$*d*. per day: find the amount of his pay for a year.

(42) Add together $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{7}{8}$.

(43) How many farthings are there in five half-sovereigns, five half-crowns, five shillings and five pence?

(44) If £275. 15*s*. $1\frac{1}{2}$*d*. be equally divided amongst 18 people, how much will each receive?

(45) Multiply MDCCLXXVII by twelve thousand twelve hundred and twelve; and express the result in words.

(46) Find a number such that if it be added twenty-three times to 37601 the sum will be 40200.

(47) Find, as a vulgar fraction in its lowest terms, the value of $\frac{12}{13}$ of $\frac{84175}{99900}$.

(48) Add together $\frac{1}{5}$, $13\frac{3}{10}$ and $\frac{2}{7}$ of $9\frac{2}{5}$.

(49) Multiply and divide £961. 4*s*. $5\frac{3}{4}$*d*. by 35.

(50) Reduce 8795342 oz. to tons.

(51) Multiply together 1, 2, 3, 4, 5, 6, 7, 8, 9; and divide the product by 105, by short division.

(52) A person bought 500 yards of cloth at 15*s*. 9*d*. per yard, and retailed it at 16*s*. 3*d*. per yard: what was his profit?

(53) Find the H. C. F. of 372, 837, 248; and arrange the three fractions $\frac{7}{9}$, $\frac{11}{13}$, $\frac{24}{29}$ in order of magnitude.

(54) Multiply 21·945 by 5·08 and divide the result by ·381,

(55) A soldier's pay for a year is £9. 17*s*. $8\frac{1}{2}$*d*.: how much does he receive per day?

(56) Reduce 7 cwt. 3 qr. 24 lb. to ounces, and 7805491 ounces to tons.

(57) Multiply £35. 17*s*. $4\frac{1}{2}$*d*. by 315.

(58) How many times is £325. 13*s*. 4*d*. contained in £7816?

(59) Add $2\frac{11}{12} + \frac{17}{16} + \frac{59}{60} + \frac{47}{48} + 4\frac{59}{60}$.

(60) What is the difference between ·979 and 1?

(61) Express in figures one hundred and seven millions twenty thousand and sixty; and in words, 21637037.

(62) Divide £23596. 11*s*. $8\frac{1}{4}$*d*. by 63.

(63) Multiply 3·073 by 3·15, and divide the result by 131·7.

(64) A man spends £561. 3*s*. 9*d*. in a year of 365 days: how much does he spend in a week?

(65) A soldier takes 7920 paces in a march of $3\frac{3}{4}$ miles: find his length of pace.

(66) From 261 times £35. 4*s*. 2*d*. take £9089. 5*s*., and divide the remainder by 89.

(67) If 1 ton 17 lb. be made up into 16 equal parcels, how much does each weigh?

(68) Divide 9366 farthings into an equal number of sovereigns, half-sovereigns, half-crowns and farthings.

(69) Add together $\frac{1}{8}$, $\frac{1}{2}$ of $3\frac{5}{8}$ and $\dfrac{1}{3\frac{1}{5}} - \dfrac{2\frac{1}{4}}{9}$.

(70) Divide 1·26 by ·00021 and ·126 by 210.

(71) How many pounds are there in ten thousand tons, and how many tons in ten thousand pounds?

(72) What is the whole extent of a farm, consisting of 275 ac. 3 ro. 21 po. downs, 56 ac. 2 ro. 12 po. meadow, 187 ac. 1 ro. arable, 99 ac. 33 po. woodland, and $2\frac{1}{2}$ ro. orchard?

(73) Multiply £546. 10*s*. $9\frac{1}{2}$*d*. by 143 and by $71\frac{1}{2}$.

(74) A person gave £20 for 48 cwt. of goods; what does he gain by selling them at 10*s*. a hundredweight?

(75) Simplify $\frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48} + \frac{1}{63}$.

(76) Multiply 1 ac. 1 ro. 1 po. 1 yd. 1 ft. 1 in. by 30.

(77) Divide ·14 by 7, 140 by ·07 and ·014 by 7000; add the results together, and turn the decimal into a vulgar fraction.

(78) A man spends £10. 11*s*. 9*d*. in a week: how much does he spend in a year of 365 days?

(79) The polar diameter of the earth is 41707796 feet; reduce this to miles.

(80) Divide 54 tons 19 cwt. 3 qr. 27 lb. by 72.

(81) Find the H. C. F. and also the L. C. M. of 228, 304, and 342.

(82) Write in figures three hundred and twenty millions, nine thousand and nine; and in words ·00099002.

(83) If telegraph posts are placed 66 yards apart, and a train passes one in every 3 seconds: how many miles an hour is the train running?

(84) Divide ·24 by 012, 240 by ·12, and ·024 by 120.

(85) Find the sum, difference, and product of $\frac{5}{8}$ and $\frac{42}{75}$.

(86) Find the H. C. F. of 78489 and 648720.

(87) Simplify the expression $7 \cdot \dot{5}\dot{7} \times \cdot 3\dot{6} - 2 \cdot 3\dot{4}\dot{5}$.

(88) How many times is 7 gall. 5 pt. contained in 20 bus. 5 gall. 3 pt.?

(89) If a person spends in four months as much as he earns in three, how much can he lay by annually, supposing that he earns £250. 10*s*. every six months?

(90) Add together $1000\frac{2}{5}$, $74\frac{5}{9}$, and $6\frac{2}{45}$, and subtract $81\frac{7}{12}$ from the sum.

(91) Divide 76·57 by ·0019 and multiply the quotient by $\frac{3}{7}$ of ·0008568.

(92) Reduce to Vulgar Fractions in their lowest terms ·0075, $\cdot 1\dot{3}\dot{6}$, $\cdot \dot{0}0\dot{9}$.

(93) Find the whole annual cost of a house, of which the rent is £36; the poor-rate being 3*s*. 4*d*. in the pound, the gas-rate two-thirds of the poor-rate, and the paving-rate three-fifths of the gas-rate.

(94) How many steps does a man whose length of pace is 32 inches take in $4\frac{3}{4}$ miles?

(95) Reduce 17 cwt. 3 qr. 15 lb. 8 oz. to drams.

(96) Divide £7. 6*s*. $6\frac{3}{4}$*d*. between two men, so that one may receive a third as much again as the other.

(97) Divide $\frac{41}{162} - \frac{0}{49} - \frac{3}{54}$ by $\frac{4}{9} + \frac{1}{2} - \frac{3}{14}$; and express the result as a decimal of four places.

(98) Multiply 238·7 by 14·1, and divide the result by 1·023.

(99) A man spends £26. 8*s*. every 30 days, and his annual income is £400; how much does he save in a year?

(100) Simplify the expression $1\cdot\dot{3} \times (2\cdot\dot{4} + 7\cdot\dot{5}) + 2\cdot3\dot{6}\dot{4} - 1\cdot6\dot{9}\dot{7}$.

(101) Express 3 tons 11 cwt. 3 qr. 27 lb. 8 oz. 4 dr. in drams.

(102) If *A* had £400 more, he could pay a debt of £1500, and have £29 over; how much has he?

(103) Find the H. C. F. and also the L. C. M. of 128, 384, 768, 2304.

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(104) Find the square root of 152399025.

(105) A boy receiving 6*d*. a week has 3*d*. stopped every third week: if there are 39 weeks in the school-year how much does he realise in 4 years?

(106) Multiply ·5678 by ·08765 and divide ·75445 by ·00625.

(107) Find by Practice the cost of 164 things at £1. 8*s* $5\frac{1}{2}$*d*. each.

(108) Reduce 11 ro. 11 po. 11 yd. to inches, and find what fraction the result is of 3 acres.

(109) How many inches are there in ·1215625 of a mile?

(110) Divide ·68921 by ·0041, and prove that the product of $\cdot\dot{1}4285\dot{7}$ and $\cdot\dot{0}7692\dot{3}$ is $\cdot\dot{0}1098\dot{9}$.

(111) A moidore being worth 27 shillings, what is the least debt in dollars, worth 4*s*. 2*d*., which can be paid in moidores?

(112) A, B, and C start on a tour, each with £40 in his pocket, and agree to divide their expenses equally. When they return, A has £7. 3*s.* 6*d.*, B £4. 10*s.* and C £1. 13*s.* 9*d.* What ought A and B to pay to C to settle their accounts?

(113) Add together $\frac{7}{42}$, $\frac{12}{40}$, $\frac{12}{45}$, $\frac{1}{35}$; and simplify

$$\tfrac{3}{8} \text{ of } \tfrac{4}{7} - \tfrac{2}{11} \text{ of } 3\tfrac{1}{7} + \tfrac{5}{9} \text{ of } 3\tfrac{3}{8}.$$

(114) Express the difference between $\frac{5}{6}$ of 2 guineas and $\frac{3}{4}$ of 6*s.* 8*d.* as the fraction of £3.

(115) Find the square root of 531118116.

(116) Find by Practice the cost of 175 cwt. 14 lb. 12 oz. at £1. 13*s.* 4*d.* per cwt.

(117) If 45 yards of silk cost £17. 9*s.* $8\frac{1}{4}$*d.*, how many yards can be bought for £1. 18*s.* $10\frac{1}{4}$*d.*?

(118) Divide ·00918 by ·017 and by 170.

(119) Multiply £37. 18*s.* $2\frac{3}{4}$*d.* by $81\frac{1}{2}$.

(120) From the sum of ten millions two hundred and three thousand five hundred and nine, and nine millions eight hundred and fifty-six thousand and seven, take their differences. Express the result in words.

(121) If the tonnage of the Great Eastern ship be 20000 tons, find how many pins she will hold, 200 to the oz.

(122) Add together 14·94, ·0093, 100, and 26·005.

Divide ·025075 by 1·003; and multiply the quotient by 1000.

(123) Find the value of $\frac{3}{5}$ of a ton + $\frac{5}{8}$ of a cwt. + $\frac{2}{3}$ of a lb.

Express £2. 11*s.* $6\frac{3}{4}$*d.* as the decimal of £1. 10*s.*

(124) Find by Practice the value of 5 cwt. 2 qr. 21 lb. at £4. 10*s.* per cwt.

(125) Extract the square root of 7468·4164.

(126) If 8 oz. of sugar be worth ·5625*s.*, what is the value of ·75 of a ton?

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(127) Express in figures, two hundred and forty-seven millions one hundred and thirty-one thousand and twelve. And in words, 1247681354211.

(128) If 5 cwt. 3 qr. 18 lb. 8 oz. of lead cost £24. 7*s*. 6*d*., what will be the price of 12 cwt. 2 qr. 14 lb. 6 oz.?

(129) Find the value of

$$\frac{2\frac{1}{3} + \frac{4}{5} \text{ of } 3\frac{1}{3} \text{ of } \frac{3}{4} - 2\frac{1}{2}}{\frac{9\frac{1}{6}}{5}}$$

(130) Multiply ·024 by 1·04; divide 2·499 by ·00525; and find the value of 1·045 of £5. 7*s*. 6*d*.

(131) Find the value of $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{2}{3}$ of $\frac{9}{16}$ of £2. 5*s*. 6*d*.

(132) If $\frac{2}{5}$ of $\frac{3}{4}$ of a ship cost £4000, what will $\frac{7}{8}$ of $\frac{3}{35}$ of the ship cost?

(133) If 24 men can do a piece of work in 12 days of 10 hours each, how many men can do three times as much in 10 days of 8 hours each?

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(134) Find the value of

$$\frac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$$

and express the result as a decimal.

(135) Simplify

$$\frac{(3 \cdot 2 - 2 \cdot 9) \times 147}{\cdot 003 \times \cdot 0005}$$

(136) Find the value of

$\frac{5}{9}$ of a guinea + $\frac{3}{16}$ of £1 + $\frac{5}{8}$ of a shilling.

(137) Find by Practice the value of 12 cwt. 3 qr. 16 lb. at £2. 17*s*. 6*d*. per cwt.

(138) If ·3 of an estate is worth £450. 15*s*., what is the value of ·48 of the estate?

(139) What decimal of 1 cwt. is 2 qr. $16\frac{4}{5}$ lb.? And what is its value if ·13 of a ton costs £2. 12*s*.?

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(140) Find the value of

$$\frac{1\frac{1}{4} - \frac{5}{12}}{1\frac{1}{4} + \frac{5}{12}} + \frac{7}{6} \text{ of } \frac{9 \times 5}{14 \times 3} - \frac{11\frac{1}{4}}{15}$$

and reduce to its lowest terms $\frac{2622}{3381}$

(141) Express as vulgar fractions in the lowest terms 24·0025 and ·0008125; and divide 1·1214 by 5·34, and 1121·4 by ·534.

(142) Find the value of ·875 of a pound + ·75 of a crown − 1·625 of a shilling; and reduce the result to the decimal of £1. 2*s.* 6*d.*

(143) A gunboat's crew, consisting of a lieutenant, a gunner, and fifteen seamen, captured a prize worth £399. 7*s.*; the lieutenant's share is 10 times and the gunner's share 3 times as much as that of each seaman. What is the value of each person's share?

(144) Find by Practice the value of 6 oz. 18 dwt. 20 gr. at 7*s.* 9*d.* per oz.

(145) Extract the square root of 167·9616, and of $\frac{529}{2401}$.

(146) Reduce $\frac{1}{16}$ of 10*s.* + $\frac{4}{9}$ of 5*s.* 3*d.* + $\frac{8}{15}$ of half-a-crown to the decimal of £1. 5*s.* 9*d.*

(147) What fraction is 7 cwt. 4 lb. of 3 tons 1 qr.? How often must one go round a square field of 10 acres to run 1 mile?

(148) Find the interest on £880 for $1\frac{1}{4}$ year at $4\frac{1}{2}$ per cent., and the discount on £929. 10*s.* 0*d.*, for $2\frac{1}{4}$ years, at $2\frac{1}{2}$ per cent.

(149) Find the whole annual cost of a house of which the rent is £41; the poor-rate being 3*s.* in the pound, the gas-rate $\frac{7}{9}$ of the poor-rate, and the paving-rate $\frac{4}{7}$ of the gas-rate.

(150) A clock which loses 4 minutes in 12 hours is 10 minutes fast at midnight on Sunday. What o'clock will it indicate at 6 o'clock on Wednesday evening?

(151) If the 4 per cents. are at 96, how much cash will be received for £1700 stock? What annual income will it produce in a 5 per cent. stock at par?

(152) Reduce $\frac{747}{960}$, and $\frac{2}{5}$ of $\frac{7}{12}$ of $\frac{11}{21}$ of $2\frac{1}{2}$ to decimal fractions.

(153) Add together

$$\frac{13}{24} \text{ of } \frac{125}{156} \text{ of } \frac{7}{9\frac{3}{8}} \text{ and } \frac{14}{15} \text{ of } \frac{25}{42} \text{ of } \frac{4\frac{11}{16}}{6\frac{12}{16}}$$

(154) Find the value of ·375 of a guinea + $\cdot\dot{5}\dot{4}$ of 8*s*. 3*d*. + $\cdot 0\dot{2}\dot{7}$ of £2. 15*s*. Express the result as a fraction of a guinea and a half.

(155) If the income-tax were $2\frac{1}{4}$ per cent. on a man's income instead of 5*d*. in the £, what difference would it make to a man with a gross annual income of £368. 15*s*.?

(156) If £1 be worth 11·57 Dutch guilders, 101 thalers worth 175 guilders, and 2 thalers worth 7·35 francs: how many francs should be received for £40?

(157) If a premium on Fire Insurance is 2*s*. 3*d*. per cent., what sum has to be paid on £4325. 17*s*. 6*d*.?

(158) Find the simple interest on £845. 10*s*. for 6 years at $4\frac{1}{2}$ per cent.

(159) What is the difference between simple interest, compound interest, and discount? Find the difference between the simple interest and the true discount on £2000 for $1\frac{3}{4}$ years at 3 per cent.

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(160) Simplify

(1) $\frac{5}{8}$ of $1\frac{1}{3}$ of $2\frac{1}{4}$ of $3\frac{1}{5}$;

(2) $$\frac{17}{\frac{1}{2}+\frac{1}{3}+\frac{7}{8}+\frac{5}{12}} - \frac{7\frac{1}{5}}{3\frac{3}{5}}$$

(161) How many ninths of a shilling are there in $9\frac{1}{9}$ shillings? how many ninetieths? and what is that number whose ninth part is nine dozen and nine?

(162) The distance between two wickets was marked out for twenty-two yards, but the yard measure was $\frac{5}{12}$ of an inch too short: what was the actual distance?

(163) Express 13 po. 2 yd. 1 ft. 4 in. as the vulgar and decimal fractions of 1 furlong 36 po. 2 yd. 5 in. Also find what part of a pound is $2\frac{1}{2}d$.

(164) Extract the square root of 137641 and ·4 to four figures.

(165) Compare the simple and the compound interest on £21. 10*s*. at the end of 3 years, at 4 per cent. per annum.

(166) What is the present value of a bill of £21. 10*s*., due in 3 months, reckoning money at 4 per cent. per annum?

(167) If by selling my horse for £17. 4*s*. I lose 20 per cent., what did my horse cost?

(168) Divide £5. 9*s*. $10\frac{3}{4}$*d*. between two men, so that one may receive half as much again as the other.

(169) Add together $3\frac{1}{5}$ of a crown, $\frac{119}{120}$ of half-a-sovereign, $\frac{17}{84}$ of a guinea, 1·375 of a pound, and $\cdot 2\dot{3}$ of ten shillings.

(170) Find the value of 3·75 of 8*s*. 4*d*. + $\cdot 5\dot{9}$ of £2. 15*s*. + $\cdot\dot{0}2\dot{7}$ of 18*s*. 6*d*. Express the result as a fraction of £7. 4*s*. 11*d*.

(171) If the income-tax were 2 per cent. on a man's income, instead of $4\frac{1}{2}$*d*. in the £, what difference would it make to a man whose annual income is £455. 16*s*. 8*d*.?

(172) If £1 be worth 25·2 francs, $9\frac{1}{2}$ thalers worth 35 francs, and 60 thalers worth 107 Austrian paper florins, find how many Austrian paper florins should be received for £10.

(173) Find the present worth of £810. 16*s*. 8*d*. due $2\frac{1}{4}$ years hence at $5\frac{1}{3}$ per cent. simple interest.

(174) A man has £3430 stock in the $3\frac{1}{2}$ per cents. at $83\frac{1}{2}$: when the stock rises 2 per cent. he transfers his capital to the 4 per cents. at 98: find the alteration in his income.

(175) The weight of the water contained in a rectangular cistern 8 ft. long, 7 ft. wide, is $93\frac{3}{4}$ cwt. If a cubic foot of water weigh 1000 oz., find the depth of water in the cistern.

(176) Add together $4\frac{1}{7}$, $8\frac{1}{3}$, $3\frac{3}{14}$, $9\frac{1}{6}$; and divide $5\frac{7}{15}$ of $1\frac{1}{41}$ by $1\frac{3}{5}-1\frac{1}{4}$

(177) Reduce $\frac{2}{17}$ of $6\frac{3}{8}$ of £3. 5*s*. to the fraction of £4. 17*s*. 6*d*.

(178) A man buys goods at the rate of £24 per cwt. and sells 2 tons 14 cwt. 3 qr. 12 lb. for £1500. How much has he gained or lost per cent. on the outlay?

(179) Find the discount on £68. 3*s*. 6*d*. due $6\frac{2}{3}$ months hence at $3\frac{3}{5}$ per cent.

(180) Which is the better investment, the $3\frac{1}{2}$ per cents. at 91, or the 4 per cents. at 103?

How much must a man invest in the former that he may have a yearly income of £470, after paying 5*d*. in the pound for income-tax?

(181) Extract the square root of 1798281; and of 2·5.

(182) The height of a tower on a river's bank is 55 feet, the length of a line from the top to the opposite bank is 78 feet: what is the breadth of the river?

(183) Express $3\frac{1}{2}$ crowns as the decimal of $7\frac{1}{2}$ guineas.

(184) £530 is to be raised from three towns, whose inhabitants number 2500, 3000, 4200 respectively: how much has each town to pay?

(185) £1600 stock gives £166 interest (simple) in $4\frac{1}{2}$ years: find the rate per cent. per annum.

(186) Simplify $\dfrac{7\left(1\frac{1}{2} \text{ of } \dfrac{3}{14}\right)}{\dfrac{1}{6}\left(\dfrac{3}{3\frac{1}{2}} \text{ of } 7\right)} \div \dfrac{9}{14}$

(187) Find the cost of a chest, the dimensions of which are 3 feet $4\frac{1}{2}$ inches high, 4 feet $2\frac{1}{4}$ inches long, 2 feet $1\frac{3}{4}$ inches wide, at $5\frac{3}{4}d.$ per square foot.

(188) From $2{\cdot}75$ of $2\frac{1}{2}$ square yards take $9{\cdot}45$ square feet.

(189) Find the vulgar fraction equivalent to $\dfrac{1{\cdot}010\dot{1}\dot{5}}{{\cdot}55}$

(190) A train leaves London at 10 A.M., reaching Portsmouth at 1.15 P.M., and another train leaves Portsmouth at 6.15 A.M., reaching London at 11.30 A.M.: at what time do they meet?

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(191) Simplify $\dfrac{5 - 2\frac{4}{5} \text{ of } 1\frac{2}{3}}{\frac{7}{9} \text{ of } \frac{8}{5} - \frac{41}{45}} - \dfrac{2\frac{1}{5}}{\frac{4}{5} + 1\frac{2}{5}}$

(192) Reduce 17s. $8\frac{1}{4}d.$ to the decimal of £5, and express the result as a fraction in its lowest terms.

(193) If $\frac{4}{7}$ of a vessel be worth £67. 2s., what is the value of $\frac{2}{11}$ of the same?

(194) Find the value of $3{\cdot}14\dot{5}$ of $69\frac{1}{2}$ miles, and of $3{\cdot}1\dot{4}\dot{5}$ of £1. 2s. 11d.

(195) Two ships get under weigh at the same time for the same port, distant 1200 miles; the faster vessel averages 10 miles an hour, and arrives at the port a day and a half before the other: what will the latter vessel average an hour?

(196) Find the 4th root of 1679616.

(197) How many yards of matting $2{\cdot}\dot{4}$ feet broad will cover a floor that is $27{\cdot}\dot{3}$ feet long and $20{\cdot}16$ feet broad?

(198) Simplify the fraction

$$\frac{1\frac{1}{4} \text{ of } 1\frac{4}{5} - \frac{5}{3} \text{ of } 1\frac{1}{5}}{\frac{3\frac{1}{2}}{2} + \frac{4\frac{1}{4}}{11\frac{1}{3}}} \div \frac{2}{17}$$

(199) If $\frac{2}{5}$ of $1\frac{1}{4}$ of an estate be worth £300, what will be the value of $\frac{2\frac{1}{2}}{\frac{5}{14}}$ of the estate?

(200) At what price must an article, which cost 15*s.*, be sold so as to gain 10 per cent.?

(201) What is the price of a piece of timber 27 ft. long, 1 ft. 9 in. thick, and 1 ft. 2 in. broad, at 4*s.* per cubic yard?

(202) Of an electric cable $\frac{11}{13}$ rests on the bottom of the sea, $\frac{1}{15}$ hangs in the water, and $234\frac{2}{3}$ yards are employed on land: what is the length of the cable?

(203) The length of a rectangular field which contains 4 ac. 3 ro. 14 po. $26\frac{1}{2}$ sq. yd. is 260 yd. 1 ft. 4 in., what is its breadth?

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(204) Reduce one thousand and eighty, divided by thirty-seven millions two hundred and fifty thousand three hundred and sixty, to a decimal to seven places.

(205) Reduce 1 qr. 13 lb. $7\frac{1}{5}$ oz. to the fraction of a ton.

(206) Multiply 1·23 by ·0059, and divide the product by ·005.

(207) Extract the cube root of 16777216.

(208) A cargo, valued at £3561. 11*s.* $7\frac{1}{2}$*d.*, being insured at $8\frac{1}{3}$ per cent., what is the insurance?

(209) What is the value of 65 napoleons, if one napoleon = 20·8 fr., and £1 = 25·45 fr.?

(210) The number of disposable seamen at Portsmouth is 800, at Plymouth 756, and at Sheerness 404. A ship is commissioned, whose complement is 490 seamen. How many must be drafted from each place so as to take an equal proportion?

(211) What principal at 5 per cent. will bring a yearly income of £341. 5*s.*?

(212) Add 24, ·24, ·024, 1·0024, and reduce the result to a vulgar fraction.

(213) (α) Divide 72·36 by 3·6 and by ·0036.

(β) Find the value of $\dfrac{4\cdot4 + \frac{3}{5}}{7\cdot375 - \frac{1}{8} + \frac{3}{4}}$

(γ) Reduce $\frac{2}{9}$ of a guinea + $\frac{1}{2}$ of $\frac{3}{4}$ of £1 + $\frac{4}{6}$ of $\frac{3}{4}$ of a crown + $\frac{1}{3}$ of $\frac{5}{8}$ of 1*s.* to decimals of £1 and of £100.

(214) Two ships start from the same port due east and due north respectively, at the rates of 12 and 9 miles an hour, how far are they apart at the end of six hours?

(215) (α) Find the difference between the simple and compound interest of £416. 13*s.* 4*d.* for 2 years at 4 per cent.

(β) Find the rate of interest, when the discount on £226. 2*s.* 8*d.* due at the end of $1\frac{1}{2}$ years is £12. 16*s.* 0*d.*

(216) A room is 14 ft. 3 in. high, 20 ft. wide, 24 ft. long, what will it cost to paper it with a paper $2\frac{1}{2}$ ft. wide, whose price is $11\frac{1}{4}$*d.* per yard; allowing 8 ft. by 5 ft. 3 in. for each of four doors, 10 ft. by 6 ft. 8 in. for each of two windows, and 6 ft. 6 in. by 5 ft. for a fireplace?

(217) Multiply £217. 11*s.* 4*d.* by $21\frac{3}{4}$; and find the value of 67 cwt. 3 qr. 24 lb. at £1. 2*s.* 4*d.* per qr.

(218) Reduce 3 roods 7 poles 28 yards 2 feet 36 inches to the fraction of and to the decimal of 1 acre 31 poles 27 yards 2 feet 36 inches.

(219) Reduce $\cdot0\dot{1}\dot{4}$ and $\cdot\dot{1}05\dot{6}$ to vulgar fractions; and find the square root of 184·117761.

(220) Find the interest on £127750 for 60 days at $3\frac{1}{2}$ per cent. per annum; and the discount on £1302. 1*s.* 2*d.*, due $2\frac{1}{4}$ years hence, at $3\frac{1}{4}$ per cent.

(221) What is the cost of papering a room 21 ft. long, $16\frac{1}{2}$ ft. wide, $10\frac{1}{4}$ feet high, at $6\frac{1}{2}$*d.* per square yard?

(222) What sum will amount to £425. 19*s.* $4\frac{4}{5}$*d.* in 10 years at $3\frac{1}{2}$ per cent. simple interest?

(223) Find the income arising from the produce of 529 acres of wheat at 5 qr. 1 bus. 3 pk. per acre, when wheat is at 60*s.* a quarter.

(224) If an income of £600 pays £10 for income-tax, how much must be paid on an income of £345. 10*s.* when the tax is half as much again?

		tons	cwt.	qr.	lb.	oz.
(225)	From	567	10	2	14	11
	Take	378	16	3	22	13

(226) A crew of 567 men receive each £8. 1*s.* $1\frac{1}{2}$*d.* prize money : find the whole value of the prize.

(227) If $\frac{9}{64}$ of a cwt. of powder costs £16. 10*s.* 9*d.*, what is the cost of 1 oz. ?

(228) How many yards of carpet, 2 ft. 4 in. wide, will it take to cover a floor 26 ft. by 35 ft. ?

(229) Simplify $\left(1 - \frac{426}{697} + \frac{2\frac{1}{2}}{8\frac{1}{2}}\right) \div \frac{3\frac{1}{2}}{5\frac{1}{8}}$

(230) Express $22\frac{1}{2}$ cubic inches as the decimal of $1\frac{1}{2}$ cubic yards to six places.

(231) The rent of a farm is £850, and the taxes are $12\frac{2}{5}$ per cent. on the rent : find the amount of rent and taxes together.

(232) *A* can mow 5 acres of grass in 3 days, *B* 7 acres in 9 days, *C* 11 acres in 12 days : in how many days can they jointly mow 121 acres ?

(233) Add together three hundred and sixty millions ninety-eight thousand and six, and forty thousand five hundred and sixty-three, expressing the result in words.

(234) Simplify the fraction

$$\frac{\frac{3-\frac{1}{3}}{3+\frac{1}{3}} \text{ of } \frac{2-\frac{1}{2}}{2+\frac{1}{2}}}{\frac{3+\frac{1}{3}}{3-\frac{1}{3}} \text{ of } \frac{2+\frac{1}{2}}{2-\frac{1}{2}}}$$

(235) Find the value of $\frac{5}{14}$ of a guinea + $\frac{5}{8}$ of a shilling + $\frac{1}{12}$ of half-a-crown, and reduce the result to the decimal of £1.

(236) Multiply ·0214 by 2·14 and divide 17·82 by ·0011. Find the value of ·05 of £1 + ·3 of a guinea + ·75 of a crown + ·8 of a shilling.

(237) Find by Practice the value of 5 lb. 9 oz. 7 dwt. 12 gr. of gold, at £3. 17*s.* 10*d.* per oz.

(238) The rates of the express and mail trains on a railway are 40 and 28 miles an hour respectively. What time is saved by taking the former for a journey of 192 miles ?

(239) What is the interest on £750 for 16 months, at $3\frac{1}{4}$ per cent.?

(240) Three persons divide the cost of an entertainment amongst them in such a manner that the first pays $\frac{1}{3}$ of the whole, and the second $\frac{6}{5}$ of what the first pays, and the third pays the remainder, which is £2. 10*s.*: what is the amount of the bill?

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(241) Simplify the fraction

$$\frac{\left(\frac{4}{5} + \frac{5}{\frac{11}{13}} \text{ of } 2\frac{1}{5} - \frac{6}{7}\right)}{1\frac{208}{245}}$$

(242) Multiply 2·45 by ·05, and divide ·0225 by 2·5, and find the value of ·025 of 2 qr. 6 lb. 4 oz.

(243) If $\frac{5}{7}$ of the cargo of a ship be worth £16000, what will be the value of $\frac{2}{3}$ of $\frac{7}{8}$ of the remainder?

(244) The solid content of a piece of timber is 40 cubic yards, its length is 54 feet, and breadth 5 feet: find its thickness.

(245) Find the value of

$$\cdot 00\dot{3} \text{ of } £1.\ 5s. + \cdot 06\dot{9} \text{ of } £5 - \cdot\dot{8} \text{ of } 2s.\ 3d.$$

(246) What is the cost of the carpet for a room, the dimensions of which are 21 feet long, $15\frac{5}{8}$ feet wide, at 1*s.* $2\frac{2}{5}$*d.* per square yard?

(247) A farmer bought a load of wheat at £12. 15*s.* per load; at what price must he sell it in order to gain 10 per cent.?

(248) A watch, which is 5 m. 40 s. fast on Monday at noon, is 2 m. 51 s. fast at midnight on the following Sunday: what did it lose in a day?

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(249) Simplify $\dfrac{\frac{3}{17} + \frac{1}{3}}{5 - \frac{2}{3}} \div \dfrac{\frac{3}{4}}{4\frac{1}{4}}$

(250) Reduce $\frac{4}{7}$ of 1*s.* $5\frac{1}{2}$*d.* to the fraction of 3*s.* 4*d.*

(251) What is the value of $\cdot 27\dot{7}$ of 4·5 of a pound sterling?

(252) A piece of wood measures superficially 3·5 feet by 4·5 yards: what is it worth at 1*s.* 6*d.* a square yard?

(253) Extract the square root of 1·014049, and of $175\frac{9}{16}$

(254) *A* invests £552 in the $3\frac{1}{4}$ per cents. when they are at 92; *B* invests £679 in the 3 per cents. when they are at 97. Find the difference of their incomes.

(255) If 6 men earn £8 for 5 days' work, how long will it take 15 men to earn £4?

(256) Two pipes together fill a cistern in 1 hour: one of them alone fills it in $1\frac{1}{2}$ hour. How long will it take the other to fill it?

(257) If $\frac{4}{49}$ of an estate be worth £46. 7*s.* 8*d.*, what is the value of $\frac{2}{11}$ of the estate?

(258) A regiment marching $3\frac{1}{2}$ miles an hour makes 110 steps in a minute: what is the length of the step?

(259) What principal will give £208 simple interest in $5\frac{1}{2}$ years at $3\frac{1}{4}$ per cent.?

(260) A street being 850 feet long, and the width of the pavement on each side being 5 ft. 3 in., find the cost of paving it at 1*s.* $3\frac{1}{4}$*d.* a square foot?

(261) Add together $\frac{5}{12}$ of a cwt., $\frac{2}{7}$ of a quarter, $\frac{7}{8}$ of a lb., and express the result as the decimal of a ton to eight places.

(262) The ten thousandth part of a cubic foot of gold gilding being 4026 square inches, what is the thickness of the gold leaf?

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(263) Find the value of $\dfrac{(2+\frac{1}{5}) \div (3+\frac{1}{7})}{(\frac{1}{2}-\frac{1}{3}) \times (4-3\frac{3}{7})}$

(264) What fraction of £1 is $\frac{3}{14}$ of a penny?

(265) Reduce 1 foot 3 inches to the decimal of a yard; and find the number of pence in ·0208 of £1.

(266) *A*, *B*, and *C* are in partnership; *A*'s capital was £2000, *B*'s £2500, and *C*'s £3000. A profit of £1000 is to be divided among them. What is the share of each?

(267) Find the hypotenuse of a right-angled triangle, whose base is 30 feet, and whose perpendicular is 40 feet.

(268) How long would a column of men, extending 3420 feet in length, take to march through a street a mile long at the rate of 58 paces in a minute, each pace being $2\frac{1}{2}$ feet?

(269) A log of timber is 18 feet long, 1 foot 4 inches wide, and 15 inches thick. If a piece, containing $2\frac{1}{2}$ solid feet, be cut off the end of it, what length will be left?

(270) Simplify:

$$\left(\frac{2\frac{3}{4}+3\frac{2}{5}}{4\frac{1}{5}+5\frac{1}{4}}+\frac{3\frac{2}{3}}{10\frac{1}{2}}\right)\times\left(\frac{2\frac{4}{11}}{2\frac{3}{5}}\div\frac{2\frac{7}{11}}{8\frac{7}{10}}\right)-\frac{\cdot 281}{1\cdot 405}$$

(271) Find by Practice the rent of 134 acres, 3 roods, 16 poles, at £3. 13*s.* 4*d.* per acre.

(272) Add together $\frac{7}{8}$ of £2. 10*s.*, $\frac{1}{7}$ of £19. 6*s.* $5\frac{1}{2}$*d.*, and $\frac{2\frac{1}{2}}{12}$ of 5*s.*

(273) Reduce $\frac{1}{9}$ of 1 oz. 13 dwt. to the decimal of $1\frac{1}{3}$ of 5 dwt. 15 gr.

(274) At what rate will the simple interest on £125 amount to £13. 2*s.* 6*d.* in 3 years?

(275) A person sells £530 3 per cent. stock at 86, and invests the proceeds at $3\frac{3}{4}$ per cent., what is the alteration in his income?

(276) How many hours a day must 42 boys work, to do in 45 days what 27 men can do in 28 days of 10 hours long; the work of a boy being half that of a man?

(277) If 8 guineas be expended in purchasing Brussels carpet, $\frac{3}{4}$ yd. wide, at 3*s.* 6*d.* a yd., for a room 20 ft. long and 16 ft. 9 in. broad; how much of the floor will remain uncovered?

(278) Simplify:

$$\frac{\left(\frac{7}{11}\times\frac{22}{23}\times\frac{46}{40}\right)+7\frac{17}{40}\div 1\frac{43}{77}}{\left(4-\frac{5}{7}\right)+\left(\frac{10}{20}\text{ of }\frac{86}{129}\right)}$$

(279) Reduce 10*s.* 9*d.* to the decimal of £1; and express 1 qr. 22 lb. as the fraction of 120 lb.

(280) Find the interest on £39. 12*s.* 6*d.*, at $5\frac{1}{2}$ per cent., for 4 years.

(281) If $\frac{7}{8}$ lb. cost $\frac{13}{14}$*s.*, what is the cost of $2\frac{11}{12}$ cwt.?

(282) Express $\cdot 4\dot{7}$ and $\cdot 68\dot{4}9\dot{4}$ as vulgar fractions; and find the value of $\cdot\dot{4}1\dot{3}$ of a cwt.

(283) How much tin and copper does a bell weighing 150 lb. contain, bell metal being composed of three-parts copper and one of tin?

(284) How many yards of carpet half-a-yard wide will cover the floor of a room $22\frac{1}{2}$ ft. by 19 ft.?

(285) Simplify $\dfrac{1\frac{1}{2} + 2\frac{1}{3}}{2\frac{1}{3} + 3\frac{1}{4}} \div \dfrac{\frac{5}{7}}{1 + \dfrac{1}{2 + \frac{1}{3}}} - \dfrac{\cdot 06}{\cdot 6}$

(286) Multiply the sum of three tenths, three hundredths, three thousandths, and three ten thousandths by thirty thousand, and divide the product by nine thousandths.

(287) What is the interest of £750 for $2\frac{3}{4}$ years at $3\frac{1}{4}$ per cent.?

(288) Reduce £1. 6*s*. 8*d*. to the decimal of a crown, and express $\frac{3}{4}$ of a crown + $\frac{7}{9}$ of half-a-sovereign as a fraction of a pound.

Find the value of

$\cdot 02$ of £1 + $\cdot 03$ of 7*s*. 6*d*. + $\cdot 01\dot{4}$ of 2*s*. 9*d*.

(289) Find, by Practice, the cost of 3 cwt. 3 qr. 17 lb., at £1. 7*s*. 9*d*. per cwt.

(290) What will be the cost of papering a room 21 ft. long by 15 ft. broad and 11 ft. high, which has two windows each 9 ft. high and 3 ft. wide, a door 7 ft. high and 3 ft. 6 in. wide, and a fireplace 4 ft. high by 4 ft. 0 in. wide, with paper, 2 ft. 3 in. wide, at 9*s*. a piece; the price of putting it on being 6*d*. per piece, and each piece containing 12 yards?

(291) If the gas for five burners, five hours every day, for ten days, cost 4*s*. 3*d*., how many burners may be lighted four hours every evening for fifteen days at a cost of £3. 16*s*. 6*d*.?

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(292) Reduce $\dfrac{3\frac{1}{2} + 2\frac{1}{3}}{\frac{3}{4} \text{ of } 9\frac{1}{6}}$ to a simple fraction.

(293) Reduce $\frac{1}{64}$ to a decimal, and multiply the result by $\cdot 0064$.

(294) What fraction of 2*s*. 6*d*. is $\frac{5}{16}$ of 3*s*. 4*d*.?

(295) Reduce 3*s*. $11\frac{1}{4}$*d*. to the decimal of a guinea, and take from it $\frac{3}{4}$ of $\frac{1}{8}$ of 1*s*. 3*d*.

(296) A reservoir is 26 ft. 8 in. long by 12 ft. 9 in. wide; how many cubic feet of water must be drawn off to make the surface sink 1 foot?

(297) Required the value of 97 cwt. of cheese at £1. 5*s*. 3*d*. per cwt.

(298) If 14 men can mow 35 acres of grass in 6 days of 10 hours each, in how many days of 12 hours each can 3 men mow 24 acres?

(299) A father left his eldest son £5000 more than he left his second son; and the second son 1500 guineas more than he left his third; his property was £17,900. What did each son receive?

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(300) Simplify

(i) $\frac{7}{8} \times 2\frac{1}{7} \times \frac{3}{4} \times 1\frac{7}{9} \times \frac{2}{5}$

(ii) $$\frac{1\frac{1}{7} \times \dfrac{4\frac{2}{3}}{3\frac{3}{7}} \times (2 - \frac{2}{7}) \div 1\frac{3}{5}}{1 + \dfrac{1}{1+\frac{1}{2}}}$$

(301) Find the value of $\frac{7}{8}$ of $(1 - \frac{7}{9})$ of a guinea, and reduce 1 ton 8 cwt. 3 qr. 17 lb. to the fraction of 3 tons.

(302) Divide

$\cdot 0001$ by $\cdot 00002$,
1000 by $\cdot 0001$,
$\cdot 00\dot{1}$ by $\cdot 0\dot{0}0\dot{1}$.

(303) Find the value of ·175 of a ton, and of ·006785 of £2. 6*s*. 8*d*.

(304) A bankrupt owes £4608, and pays 13*s*. 10¼*d*. in the pound. How much do his creditors jointly lose?

(305) Find (α) the total surface, (β) the cubical content of a log which is 8 yards long, and 2 feet thick and deep.

(306) Extract the square root of 30712·5625 of $\frac{625}{2401}$; and of ·000000133225.

(307) Find the interest of 125 guineas for $2\frac{1}{2}$ years at 4 per cent.: and the discount of £63. 15*s*. for 15 months at 5 per cent.

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(308) Divide the sum of $3\frac{3}{5}$ and $7\frac{7}{15}$ by their difference.

(309) Find the cost of a chest 3 ft. 4 in. high, 4 ft. 2 in. long, 2 ft. 1 in. wide, at 6½*d*. a square foot.

(310) If 9 men or 16 women could do a piece of work in 144 days, in what time would 7 men and 9 women do it, working together

(311) Find the gain or loss per cent. in buying oranges at 5*s.* 6*d.* a hundred and selling them eight for $2\frac{1}{2}d.$

(312) Divide £1220 amongst *A*, *B*, and *C*, in the proportion of ·7, ·28, and ·056.

(313) A man walks 1 m. 1 fur. 7 p. in 20 minutes: how long will it take him to walk 41 m. 2 fur. 12 p.?

(314) A clock which gains $7\frac{1}{2}$ minutes in 24 hours is 12 minutes fast at midnight on Sunday. What o'clock will it indicate at 4 o'clock on Wednesday afternoon?

(315) How much 4 per cent. stock at 96 can be bought for £1000? What annual income will it produce?

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(316) Add together ·00125 of a ton, ·0125 cwt. and ·025 lb.; and reduce the result to a decimal of 2 qr. 9 lb.

(317) A man owns $\frac{3}{16}$ of a mine, and sells $\cdot 135\dot{1}$ of his share; what fraction of the mine has he left?

(318) If 1 ton 16 cwt. 3 qr. 9 lb. of coal cost £2. 10*s.*, what will 3 tons 6 cwt. cost?

(319) Gunpowder being composed of 33 parts of nitre, 7 of charcoal and 5 of sulphur; find how many pounds of each will be required to make 30 lb. of powder.

(320) Find by Practice the cost of $2645\frac{1}{4}$ articles at £2. 12*s.* 11*d.* each.

(321) What is the difference between Interest and Discount? Which of the two is greater?

Find the difference between the Interest and Discount on £160. 6*s.* 3*d.* for $4\frac{3}{4}$ months at $3\frac{3}{10}$ per cent.

(322) Extract the square root of 173889; and of $134\frac{1}{1}\frac{5}{6}\frac{5}{9}$

(323) Find the cost of carpeting a room 10 yd. 2 ft. long, and 7 yd. 1 ft. broad, with carpet $\frac{3}{4}$ yd. wide at 4*s.* 6*d.* a yard.

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(324) Add together $1\frac{2}{5}$, $1\frac{1}{3}$, $\frac{1}{6}$ and $\frac{1}{30}$, and divide $\frac{4}{5}$ of $3\frac{1}{2}$ by $\frac{4}{13}$ of $2\frac{2}{5}$

(325) Reduce $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards to the fraction of a furlong.

How many pence are there in 1·05625 of 6*s.* 8*d.*?

(326) Find by Practice the cost of 960 things at £2. 4*s.* $4\frac{1}{2}d.$ each; and the rent of 21 ac. 3 ro. 5 po. at £2. 13*s.* 4*d.* per acre.

(327) Give the rule for Division of Decimals.

(328) Find the present worth of £68. 13*s*. 4*d*. due $9\frac{3}{5}$ months hence at $3\frac{3}{4}$ per cent.

(329) Which is the better investment, the $4\frac{3}{4}$ per cents. at 95 or the 5 per cents. at 104?

How much must a man invest in the former that he may have a yearly income of £613. 12*s*. after paying 4*d*. in the pound income-tax?

(330) Extract the square root of 5345344 and of $1\cdot\dot{6}$.

(331) Find by Practice the value of

(α) 9836 articles at 7*s*. $2\frac{1}{2}$*d*. each.

(β) 6 tons 7 cwt. 2 qr. 17 lb. at £3. 10*s*. 7*d*. per cwt.

(332) Simplify

$$(1)\quad \frac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}}$$

$$(2)\quad \frac{(3\cdot71 - 1\cdot908) \times 7\cdot03}{2\cdot\dot{2} - \frac{74}{333}}$$

(333) If a pole 10 feet high cast a shadow 12 ft. 8 in. long, how high is a tower whose shadow at the same time is 57 feet long?

(334) Twenty dormice bought at 2*d*. each are sold 2 for 7*d*., find the total gain and the gain per cent.

(335) Find the square root of 90306·2601, and also the cube root of 20·570824.

(336) If 2 guineas make 3 napoleons, and 15 rix-dollars make 4 napoleons, and 6 ducats make 7 rix-dollars, how many ducats are there in £490?

(337) At what time are the hands of a watch together between 7 and 8?

(338) Which is the better investment, bank stock paying 10 per cent. at $234\frac{1}{2}$, or Consols paying 3 per cent. at $92\frac{1}{2}$?

(339) Express in figures thirteen hundred millions five thousand and one; and in words 5030090501.

(340) A farm lets for £240 a year; the land-tax for 9 months is £5. 1*s*. $7\frac{1}{2}$*d*.; find the net annual income and the rate per pound of the tax.

(341) Find the Simple Interest upon £328500, at 5 per cent., for 200 days.

(342) Having given that the weight of a cubic foot of water is 1000 oz., and that the imperial gallon contains 277·274 cubic inches, find the weight of a pint of water.

(343) A room is 22 ft. 6 in. long, 20 ft. 3 in. wide, and 10 ft. 9 in. high. Find the cost of carpeting the room at 5*s.* 6*d.* a square yard, and of papering the walls at 10*d.* a square yard.

(344) *A* and *B* can do a piece of work in 8 days, *B* and *C* can do it in 12 days, and *A*, *B*, and *C* can do it in 6 days. In how many days can *A* and *C* do it?

(345) If 3 boats take 6000 herrings in 8 days, how long will 600 boats be in taking 20,000 barrels, each containing 700 herrings?

(346) A person invests £375 at simple interest, and at the end of 3 years and 8 months he finds that he possesses £423. 2*s.* 6*d.*; at what rate per cent. per annum was his profit?

(347) In a compound metal the proportion of tin to copper is 7·75 to 92·25. Find to the nearest penny the value of 8 cwt. 3 qr. of it, if tin is at £140 per ton, and copper at £80.

(348) Show that $3 \times 5 = 5 \times 3$; and multiply three millions nine thousand and seven by five millions six hundred thousand one hundred and fifty.

(349) The quotient in a division sum equals six times the divisor, and the divisor equals six times the remainder; the three amount together to 516; find the dividend.

(350) Find the H. C. F. of 1235192 and 411355; and the L. C. M. of 10, 11, 12, 13, 14, 15.

(351) Add together ·60625 of £1 + $\cdot\dot{1}4285\dot{7}$ of 14*s.* $10\frac{1}{2}d.$, and $\frac{23}{71}$ of $\frac{3}{71}$ of £3. 5*s.* 1*d.*, and express the result as the decimal of 27 shillings.

(352) *Seven times the square root of a number is 161*; find the number.

(353) A clock gains $3\frac{1}{2}$ minutes a day; how must the hands be placed at noon so as to point to true time at 7 h. 30 m. P.M.?

(354) A person's income is derived from the proceeds of £4550 at a certain rate per cent., and £5420 at 1 per cent. more than the former. His whole income is £453. Determine the rates.

(355) The discount on £516. 7*s.* 3*d.* for 3 months is £5. 2*s.* 3*d.*; at what rate per cent. is it calculated?

(356) How many heaps, $6\frac{1}{4}$ yards apart, can be placed symmetrically on a rectangular field of 10 ac. 3 ro. 36 p. 6 yd.?

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(357) Three horses are worth 5 cows, and 4 cows cost £17; how much are 20 horses worth?

(358) Simplify:

$$\frac{\cdot 004 \div \cdot 0005}{2\cdot 4\dot{2}\dot{3} + 3\cdot\dot{5}7\dot{6} + 2\cdot 0\dot{0}0191\dot{1}}$$

(359) Find by Practice the value of 9 cub. yd. 21 c. ft. 432 c. in. of timber, at £4. 14*s.* 6*d.* per cubic yard.

(360) Extract the square roots of 102·01 and $6\cdot 24\dot{9}$.

(361) What will be the cost of enclosing a rectangular garden, 90 yd. long and 30 yd. 2 ft. 3 in. broad, with a wall 8 ft. 4 in. high, at the rate of 6*s.* per superficial square yard?

(362) A person invests £10000 in 3 per cents. at 75, and when they rise to 78 he sells out and invests the produce in bank shares at £208 each, which pay a dividend of £8 per share. Show that his income is not altered.

(363) What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 5 or 6 deep, and also of its being formed into a solid square?

(364) Find the simple interest on £2733. 6*s.* 8*d.* at 4 per cent. for 3 years and 9 months; and determine what sum will amount to £926 2*s.* in 3 years at 5 per cent. compound interest.

(365) If the rent of 12 ac. 3 ro. is 16 guineas, find the rent of a farm of 288·46875 acres.

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(366) Simplify $8\cdot 3 - 1\frac{1}{9}$ of $2\frac{5}{20}$ of $1\frac{1}{5} + 2\frac{1}{2} \div \frac{3}{14} - 7$.

(367) Multiply 57875 by 729819, with three lines of multiplication, and divide 123456 by 63, using short division.

(368) Find the price of $7254\frac{2}{3}$ articles at £2. 8*s*. $3\frac{1}{2}$*d*. each, and also at £3. 6 f. 5 c. 4 m. each.

(369) Find the cost of papering a room 14 ft. 5 in. long, 13 ft. 7 in. broad, and 12 ft. 3 in. high, with paper at $3\frac{1}{2}$*d*. per square yard. In the room are 4 windows 4 ft. by 3 ft., 2 doors 6 ft. 6 in. by 2 ft. 5 in., and a fireplace 5 ft. by 4 ft.

(370) *A* and *B* can do a piece of work in 4 days, *B* and *C* in $5\frac{3}{4}$ days, and *A* and *C* in $4\frac{3}{5}$ days. In what time can each do the work separately?

(371) *M* starts from C and travels towards D at a rate of 6 miles per hour; two hours afterwards *N* starts from C, and going 10 miles per hour reaches D 4 hours before *M*. Find the distance from C to D.

(372) Find the difference between the discount on £1161. 11*s*. 3*d* for 14 months at 5 per cent. per ann. and the interest on £1475 for 15 months at $3\frac{1}{2}$ per cent. per ann.

(373) A woman buys a certain number of apples at 3 a penny, and the same number at 2 a penny; she then mixes them and sells them at 5 for twopence. How much does she gain or lose per cent.?

(374) Find the cube root of 198155287.

(375) Multiply 45 tons 14 cwt. 2 qr. 5 lb. by 205.

(376) Find the L. C. M. of 34, 51, 52, 425, 1200.

(377) Find the price of 7865 articles at £2. 19*s*. 6*d*. each, using only one aliquot part.

(378) *A*, *B*, and *C* are employed on a piece of work. After 15 days *A* is discharged, one-third of the work being done. *B* and *C* continue at the work, and after 20 days more *B* is discharged, one-third more of he work being done. *C* finishes the work in 30 days. In what time would the work have been done, if *A* and *B* had continued to work?

(379) Find the true discount on a £100 bill, due 2 months hence, the interest of money being taken at 6 per cent. per ann. If the discount were taken at £1, what would be the rate of interest?

(380) Simplify $(25\cdot4)^2 + (24\cdot6)^2 - 12\cdot7 \times 98\cdot4 + (\cdot6)^2$

(381) Divide £31. 17*s*. 6*d*. into two sums of money, one of which shall contain as many sixpences as the other contains fourpenny pieces.

(382) A man sold a horse for £41. 17*s.*, and by so doing lost 7 per cent. on what it cost him: find the cost price.

(383) A French mètre = 1·0936 of a yard, and a centimètre is the hundredth part of a mètre. Find a centimètre in decimals of an inch to 4 places.

(384) Find the sum of $6 \cdot 2\dot{7}$, $18 \cdot \dot{6}5\dot{1}$ and $12 \cdot 3\dot{4}\dot{5}$; and the difference between $\cdot 3402\dot{7}$ and $\cdot 2\dot{7}$.

(385) Find the value of 50 qr. 7 bus. $3\frac{1}{2}$ pk. of wheat at 6*s.* 8*d.* per bushel.

(386) A person, by disposing of goods for £182, loses 9 per cent. What ought they to have been sold at to realise a profit of 7 per cent.?

(387) The external dimensions of a box without a lid are, length 4 feet, breadth 3 feet, depth 2 feet, and the thickness of the sides and bottom is the same, namely 1 inch; if the cost of a cubic yard of the material is 9*s.*, and the cost of making the box = $\frac{1}{11}$ of the cost of the material, what will the box cost?

(388) The receipts of a company average £522. 12*s.* on a week-day, and half that sum on a Sunday, and their weekly expenses are £1396. 19*s.*: if at the end of a year a dividend of 5 per cent. be declared on their capital, £2,000,000, how much can they carry to their reserve fund?

(389) Find the cube root of 674526133.

(390) Show that the simple interest on £625 for 8 months at $4\frac{1}{2}$ per cent. is equal to that on £1875 at 3 per cent. for 4 months.

(391) Determine the greatest and least of the ratios 17 : 24, 35 : 49, 52 : 73.

(392) The cost price of a book is 4*s.* 9*d.*, expense of the sale 6 per cent., profit 24 per cent.: what is the retail price?

(393) Divide 27·72 by 8·1 and also by ·081.

(394) Reduce 3 cwt. 4 lb. to the decimal of 5 tons 7 cwt. 1 st. 2 lb.

(395) Find the present value of £1031. 17*s.* due 6 months hence at 3 per cent.

(396) Eight bells begin tolling together at the same instant, and they toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds

respectively: after what time will they be again tolling at the same instant?

(397) A cistern, whose length, depth, and breadth are 6 ft. 3 in., 5 ft., and 4 ft. 2 in., is filled with water, and leaks till the water sinks 7 in.; find the volume of water left.

(398) If 22 men can dig a trench 420 yards long, 5 wide, and 3 deep, in 350 days of 9 hours each; in how many days of 11 hours each will 252 men dig a trench 210 yards long, 3 wide, and 2 deep?

(399) A sum of money amounts in 3 years at 5 per cent. compound interest to £926. 2*s.*; what would be its amount in 5 years?

(400) If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days, how long will it take 2 men and 2 boys to reap 10 acres?

(401) Divide £7817. 12*s.* 10½*d.* by 127.

(402) How many ducats of 4*s.* 11¾*d.* each are worth 55926 rix-dollars of 4*s.* 10½*d.* each?

(403) Add two-thirds of three-fourths of a florin to four-fifths of seven-eighths of a crown; and convert the result to the decimal of a pound sterling.

(404) Extract the square root of 9·8626044 to four places of decimals.

(405) A gallon of water weighs 146·17 ounces troy, 7000 grains troy = 1 lb. avoirdupois. Find the weight of a cubic foot of water in ounces avoirdupois, if a pint contains 34·66 cubic inches.

(406) A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides, of 6 ft. in breadth, and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1*s.* 8*d.* per square foot, and of the grass 3*s.* per square yard, find the whole cost of laying out the court.

(407) One clock gains 4 minutes in 12 hours, and another loses 4 minutes in 24 hours. They are set right at noon on Monday. Determine the time indicated by each clock, when the one appears to have gained 16¼ minutes on the other.

(408) The gold coinage of one nation contains 1 part of silver to 11 parts of gold without any alloy; that of another

nation, 1 part of alloy to 23 parts of gold. It is found that 46 of the first weigh as much as $88\frac{1}{2}$ of the second. The intrinsic value of silver is one-sixteenth that of gold. Determine the par of exchange.

(409) Find the discount on £690. 4*s*. due 6 months hence, at 3 per cent. simple interest.

(410) Find a mean proportional between ·6 and ·024.

(411) Divide 6*s*. 4*d*. $3\frac{2}{9}$*q*. by 7, and add the result to $\frac{2}{3}$ of $\frac{37}{24}$ of a penny.

(412) Divide ·00108 by 270; 62500 by ·25; and 2·64 by ·32.

(413) A person invests £5000 in Turkish 6 per cent. stock at 80; find the rate of interest he gets for his money. When his stock has risen to 104 he sells out, and buys £20 railway shares at £18, which pay dividend at the rate of $4\frac{1}{2}$ per cent. Find the alteration in his income.

(414) The cost of 155 yards of cloth is £178·25; how many yards can be bought for £74·875?

(415) Simplify

$$\frac{1\frac{3}{4} - \frac{7}{6} \text{ of } \frac{18}{28}}{\frac{5}{6} \text{ of } \frac{12}{20} + 5\frac{1}{2}} \div \frac{1}{6} - \left\{\frac{3\frac{1}{3}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{18} \text{ of } \frac{4}{7}\right\} \div \frac{4}{7}$$

(416) If 20 men do a piece of work in 14 weeks, working 6 days a week and 8 hours a day, in how many weeks will 24 men do it, working 5 days a week and 7 hours a day?

(417) A railway train having left a terminus at noon is overtaken at 6 P.M. by another train, which left the same terminus at 1 P.M. If the former train had been 10 miles farther on the road when the latter started, it would not have been overtaken till 8 P.M. Find the rates of the trains.

(418) How many yards of paper $\frac{5}{8}$ yd. wide will cover the walls of a room $20\frac{5}{8}$ ft. long, $11\frac{1}{8}$ ft. wide, $12\frac{1}{2}$ ft. high; and what will be the cost of it at $2\frac{1}{4}$*d*. per yard?

(419) *A*, *B*, and *C* are partners. *A* receives two-fifths of the profits, *B* and *C* dividing the remainder equally. *A*'s income is increased by £220 when the rate of profit rises from 8 to 10 per cent. Find the capital of *B* and *C*.

(420) Find the cube root of 159837789483.

(421) A butcher bought 160 lb of mutton for £4, and sold two-thirds of it at 8*d*. per lb.; at what price per lb. must he sell the remainder so as to gain £2. 5*s*. on the whole?

(422) Reduce $3\frac{1}{2}$ qr. to the decimal of 15 tons; and find the value of $\cdot 126\dot{1}\dot{2}$ of 9*s*. 3*d*.

(423) A creditor, agreeing to receive £25 for a debt, finds that he has been paid at the rate of 12*s*. 6*d*. in the pound; how much was the debt?

(424) *A*, *B*, and *C* rent a meadow for £43. *A* puts in 10 horses for 1 month, *B* 12 oxen for 2 months. and *C* 20 sheep for 3 months. How should the expense be divided, if the quantities eaten by a horse, an ox, and a sheep, during the same time, be in the ratio of 4, 3, and 1?

(425) If the price of 9760 bricks, of which the length, breadth, and thickness are 20 inches, 10 inches, and $12\frac{1}{2}$ inches respectively, be £50. 16*s*. 8*d*., what will be the price of 100 bricks, which are one-fifth smaller in every dimension?

(426) How many years' purchase should I give for an estate, so as to get $3\frac{1}{3}$ per cent. interest for my money?

(427) How much stock must be sold out of the 3 per cents. at 90, in order to pay a legacy of £9000, free of legacy-duty, the duty being 10 per cent.?

(428) How often between 11 and 12 are the hands of a clock an integral number of minute spaces apart?

(429) *A* and *B* walk a race of 25 miles; *A* gives *B* 45 minutes' start; *A* walks uniformly a mile in 11 minutes and catches *B* at the 20th milestone: find *B*'s rate and by how much he lost in time and space.

(430) A debt is due at the end of $4\frac{1}{2}$ months; $\frac{1}{4}$ is paid immediately, and $\frac{1}{4}$ at the end of 3 months; when ought the remainder to be paid?

(431) How many times does the 29th day of the month occur in 400 consecutive years?

(432) Divide £16. 17*s*. 6*d*. into 4 parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$

(433) What fraction of 2 mi. 5 fur. 9 po. 1 ft. 6 in. is $\frac{5}{11}$ of 7 furlongs?

(434) Multiply 10·375 by ·0074, and $1\cdot\dot{2}\dot{7}$ by $\cdot 04\dot{5}8$. Divide 1080 by ·008, and $1\cdot 2\dot{3}12\dot{3}$ by $3\cdot 6\dot{3}$.

(435) Two cogwheels work together, one having 360 teeth, and the other 100; if the first revolves 20 times in one minute, how many times will the second revolve in an hour? and if

the latter turn a drum, whose circumference is five feet, how many miles of thread will it wind upon the drum in a working day of 11 hours?

(436) A man, by selling out of a 3 per cent. stock at 99, gains 10 per cent. on his investment. At what price did he buy, and what was his income, supposing that he realised £15345?

(437) A tank is 8 ft. long, 5 ft. 4 in. wide, 4 ft. 6 in. deep. Find the number of gallons it contains, having given that 1 cub. ft. of water weighs 1000 oz., and that a pint of water weighs a pound and a quarter.

If, when the cistern is three-quarters full, and water running in at the rate of 18 quarts per minute, a leak should break out, which discharges 7 gallons per minute, how long will it be before the cistern is half full?

(438) An agent has to receive a rent paid in corn from a tenant, and to deliver it to the landlord. At both payments he uses, so as to benefit himself, a false balance, such that 9 lb. in one scale balances 10 lb. in the other. Corn being worth 49*s*. a quarter, the value of his plunder is £46. 11*s*. What is the corn-rent?

(439) What are the times between 3 and 4 when the hands of a watch are equally distant from fig. III.

(440) If the length of a square reservoir be 12 ft. 8 in., what weight of water must be drawn off that it may sink 3 ft. 9 in.?

(N.B.—1 cubic ft. of water weighs 1000 oz.)

(441) In how many years will £320 double itself, at $2\frac{1}{2}$ per cent. simple interest?

(442) If three men working 11 hours a day can reap a field of 20 acres in 11 days, in how many days can 9 men working 12 hours a day reap a field 360 yards long and 320 broad?

(443) A level reach in a canal, 14 miles 6 furlongs long, and 48 feet broad, is kept up by a lock 80 feet long, 12 feet broad, and having a fall of 8 ft. 6 in.; how many barges might pass through the lock before the water in the upper canal was lowered one inch?

(444) Simplify

$$\frac{1}{7\frac{1}{4} \text{ of } 3\frac{3}{11} + 3\frac{3}{11}} \div \left(\frac{3}{13} - \frac{2}{9}\right) - \left(\frac{13}{3} + \frac{1}{6}\right) \div \frac{2}{3} \text{ of } \frac{3}{8} \text{ of } 63.$$

(445) Find accurately the value of $\cdot\dot{8}\dot{1}$ of £14. 13*s*. 4*d*.; find the approximate value when only four places of decimals are taken, and express the difference between the approximate and true values as a fraction of the true value.

(446) A person invests the present value of £2358 due two years hence at 4 per cent. in gas-shares, which pay at the rate of nine per cent.; he gives £144 for each share of £100; what is his annual income, and what rate per cent. does he make of his money invested in the gas-shares?

(447) At billiards *A* can give *B* 5 points in a game of 50, and *C* 10 points in 50; how many points can *B* give *C* in a game of 90

(448) In a dormitory $\frac{11}{23}$ of the boys are in the upper school, $\frac{2}{3}$ of the remainder in the middle, and the rest, 8 in number, in the lower; find the number in the dormitory.

(449) Find the square root of 5290455·409801.

(450) At what rate per cent. will £1250 amount to £1531. 5*s*. in 5 years?

(451) How often is £11. 8*s*. $7\frac{1}{2}$*d*. contained in £502. 19*s*. 6*d*.?

(452) Divide $\frac{1}{3}[3 + \frac{1}{3}\{3 + \frac{1}{3}(3 + 1\frac{1}{2})\}]$ by ·125.

(453) Multiply 79·347 by 23·15; and divide ·00178986144 by ·00464.

(454) Reduce 15*s*. $3\frac{1}{4}$*d*. to the decimal of a guinea; and find the value of $\cdot0\dot{9}$ of £5.

(455) An army loses 12000 men in battle, one-sixth of the remainder in a forced march, and then it has 60000 men left. Of how many men did it consist at first?

(456) How much money must one invest in 3 per cent. Consols, when they are at 10 per cent. below par, in order to have an income of £2000 a year?

(457) The circumference of the fore-wheel of a carriage is 8 feet, and that of the hind-wheel is 10 feet; in what distance will the fore-wheel make 100 revolutions more than the hind-wheel?

(458) A mixture of a certain quantity of brandy with 20 gallons of water is worth 25 shillings per gallon. If the brandy pure be worth 30 shillings per gallon, how much brandy is there in the mixture?

(459) Two trains start at the same time from London and Edinburgh, and proceed towards each other at the rates of 30 and 50 miles per hour respectively. When they meet, it is found that one train has run 100 miles farther than the other. Find the distance between London and Edinburgh.

(460) Which is the better stock for investment, $3\frac{1}{4}$ per cents. at $92\frac{5}{8}$, or $3\frac{1}{2}$ per cents. at par?

(461) What number is that which exceeds the sum of its fourth, fifth, and sixth parts by 161?

(462) Find the value of $\dfrac{3\frac{1}{7}}{7\frac{4}{5} \text{ of } \frac{3}{4}} \times \dfrac{\frac{5}{7} - \frac{1}{4}}{\frac{3}{8} + \frac{3}{7}}$ of £9.

(463) Divide 171·99 by 27·3, and find the square root of the quotient to two places of decimals.

(464) A reservoir is 56 ft. 8 in. long, and 19 ft. 6 in. broad; how many cubic feet of water must be drawn off to make the surface sink 2 ft. 6 in.?

(465) Two persons buy respectively with the same sums into the 3 and $3\frac{1}{2}$ per cents., and get the same amount of interest. The 3 per cents. are at 75: at what price are the $3\frac{1}{2}$ per cents.?

(466) Three lines of paling run side by side for a distance of 90 feet. The rails are respectively 2, 3, and 5 feet apart. How often will a person walking outside the palings, on looking across them, see three rails in a line?

(467) Find the cost of painting the walls of a square room 14 ft. high and 18 ft. long, with two doors 8 feet by 4, and three windows 10 feet by 5, the amount saved by each window being £2. 16*s*. 3*d*. What additional height would increase the cost by 9 shillings?

(468) *A* can do a piece of work in 6 days, which *B* can destroy in 4. *A* has worked for 10 days, during the last 5 of which *B* has been destroying: how many days must *A* now work alone, in order to complete his task?

(469) If 7 men working $10\frac{3}{5}$ hours a day can earn £4. 15*s*. 3*d*. in $5\frac{1}{4}$ days, what sum will 28 men earn in $15\frac{1}{4}$ days, if they work $5\frac{3}{10}$ hours a day?

(470) Find the cube root of 273359449.

(471) What must be added to $\frac{3}{7}$ of $\frac{5}{8}$ to make it equal to $\frac{9}{10}$ of $3\frac{3}{4}$?

(472) Divide $\cdot 14$ by 7, 140 by $\cdot 07$, and $\cdot 014$ by 7000; add the results together, and turn the decimal into a vulgar fraction.

(473) A grocer buys half a ton of sugar for £29. 10*s*., and retails it at $7\frac{1}{4}d.$ a pound: how much does he gain by it?

(474) Which is more valuable, so far as principal is concerned, £850 stock in the 3 per cents. at 93, or £817. 13*s*. to be paid in a year's time, interest being reckoned at $3\frac{1}{2}$ per cent.? What income would be derived from each?

(475) *A*, *B*, and *C*, are three workmen: *A* can do half a piece of work in 3 hours, doing twice as much as *B* can do; and *A*, *B*, and *C* can together do the whole in $2\frac{1}{2}$ hours. Show that *C* can do in 5 hours as much as *B* can do in 9 hours.

(476) If for a sovereign one can buy 11 gulden 12 kreutzers or $25 \cdot 5$ francs, and for one 20-franc piece 9 gulden 20 kreutzers, how much per cent. is gained by buying French gold with English gold before buying German money? (N.B. 1 gulden = 60 kreutzers.)

(477) A bankrupt, whose effects are worth £201, owes his creditors £133, £212, and £325 respectively; what will each receive?

(478) If 3 men, 4 women, 5 boys, or 6 girls, can perform a piece of work in 60 days, how long will it take 1 man, 2 women, 3 boys, and 4 girls, all working together?

(479) The circular piers of Hungerford Bridge are 15 feet in diameter, and will bear a pressure of 750 tons to the square inch; taking the area of a circle as $3 \cdot 14159$ times the square on the radius, what pressure may be put on a pier?

(480) Express $69\frac{1}{2}$ miles in mètres, 32 mètres being taken to be equivalent to 35 yards.

(481) If $1\frac{2}{3}$ of a sum of money $= \frac{3}{7}$ of 5*s*. 10*d*., find the sum.

(482) Multiply $\cdot 5\dot{7}1428\dot{5}$ by $\dot{6}\dot{3}$, and show that the result is $\cdot \dot{3}\dot{6}$.

(483) Two lines are $41 \cdot 06328$ inches and $\cdot 0438$ of an inch long respectively. How many lines as long as the latter can be cut off from the former, and what will be the length of the remaining line?

(484) A hollow cubical box, made of material which is $1 \cdot 3$ inches in thickness, has an interior capacity of $50 \cdot 653$ cubic feet: determine the length of the outside edge of the box.

(485) A man invested £8063 in the 3 per cents. at 91, the brokerage being $\frac{1}{8}$ per cent.; what will be his clear income after an income-tax of 5*d.* in the pound is deducted?

(486) The expense of painting the wainscot of a room is £16. 15*s.* 3*d.*, at 2*s.* 3*d.* per square foot, the wainscot being 2 feet deep. What will be the expense of carpeting the room with carpet 27 inches broad at 4*s.* 6*d.* per yard, the breadth of the room being to its length as 62 is to 87?

(487) *A* and *B* start to run a race; their speeds are as 17 to 18. *A* runs $2\frac{1}{3}$ miles in 16 min. 48 sec.; *B* finishes the course in 34 minutes: determine the length of the course.

(488) A boat's crew row over a course of a mile and a quarter against a stream which flows at the rate of 2 miles an hour, in 10 minutes. The usual rate of the stream is half a mile an hour. Find the time which the boat would take in the usual state of the river.

(489) A cistern 12 ft. long, 2 ft. 4 in. wide, and 9 in. deep, contains pulp for making paper. If half the volume of the pulp is lost in the process of drying, how many sheets of paper, 8 inches by 6, will be obtained, if 300 sheets in thickness go to the inch?

(490) At what times between 5 and 6 are the hands of a watch at right angles?

(491) Express $\frac{3}{8}$ of $1\frac{5}{9}$ of a mile in terms of a mètre, supposing 32 mètres = 35 yards.

(492) How many times does a wheel of 4 feet diameter revolve in a mile, supposing the circumference of the wheel to be $6 \cdot 28$ times as great as the radius?

(493) A person pays one tax of 10*d.* in the £, and another of 5 per cent. on his income. His remaining income is £545. What was his original income?

(494) Find the difference between the interest and discount of £455 for 3 months at 4 per cent. per annum.

(495) If an American dollar be worth 4*s.* $3\frac{3}{5}$*d.*, and be also worth 5·42 francs, find the number of francs in £1. sterling.

(496) A soldier has 5 hours' leave of absence: how far may he ride on a coach which travels 10 miles an hour, so as to return to the camp in time, walking at the rate of 5 miles an hour?

(497) A man sells out of the 3 per cent. Consols at 90, and invests the proceeds in the Russian 4 per cents. at par, thereby adding £200 to his income. How much of the Russian stock does he hold?

(498) Two trains start at the same time, the one from London to Norwich, the other from Norwich to London. If they arrive in Norwich and London respectively 1 hour and 4 hours after they passed each other, show that one travels twice as fast as the other.

(499) *A* and *B* can do a piece of work in $6\frac{2}{3}$ days, *A* and *C* in $4\frac{4}{5}$ days, *A*, *B*, and *C* in $3\frac{3}{4}$ days. In how many days could *A* do it alone?

(500) When £170 will purchase 4233 francs, what is the *course* of exchange between London and Paris? And if 503 gold pieces of 20 francs contain as much pure gold as 400 sovereigns, what is the *par* of exchange between London and Paris?

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(501) Simplify $\dfrac{2\frac{1}{4}}{2\frac{2}{3}} + \dfrac{2\frac{1}{2} + 5\frac{1}{5}}{3\frac{1}{3} + 9\frac{1}{2}} + \frac{1}{2} + \frac{3}{8}$ of $\frac{3}{20}$

(502) Multiply together ·01 and ·000001, ·01 and 1000000, ·000001 and 1000.

(503) Two blocks of equal weight are divided, the one into 73 parts of equal weight, and the other into 193 parts of equal weight: which will be the heavier, 14 of the former parts or 37 of the latter?

(504) If 13 denarii are worth 11 drachmae, 80 drachmae worth three darics, and 11 darics worth 25 half-sovereigns, how many denarii are there in £1630?

(505) Two equal wine-glasses are filled with mixtures of spirit and water in the ratios of 1 of spirit to 3 of water and 1 of spirit to 4 of water: when the contents are mixed in a tumbler, find the strength of the mixture.

(506) Two men set out to meet each other. The one walks 30 miles each day. The other walks 15 miles the first day, 18 the second, 21 the third, and so on. They meet half-way. Find how long and how far each has travelled.

(507) A piece of work must be finished in 36 days, and 15 men are set to do it, working 9 hours a day; but after 24 days it is found that only three-fifths of the work is done. If 3 additional men be then put on, how many hours a day will they all have to labour, in order to finish the work in time?

(508) When the three per cents. are at $91\frac{5}{8}$, and the $3\frac{1}{4}$ per cents. at $99\frac{1}{4}$, which is the better investment? How much is one investing, when the difference in income is a shilling?

(509) Find the square root of 60481729 and the cube root of 387420489.

(510) Of two stalactites hanging from the flat roof of a cavern, one is 1·02 inches longer than the other, and the shorter one increases in length at the rate of 3·014 inches in a century. Find the rate of increase of the other, in order that they may be of the same length at the end of 125 years.

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(511) Simplify $\left(6\frac{3}{7} \text{ of } \dfrac{5\frac{1}{6} - 4\frac{7}{12}}{12\frac{2}{3} - 7\frac{5}{12}}\right) \div \dfrac{1 + \dfrac{1}{2\frac{1}{3}}}{2}$

(512) Find all the numbers which will divide 258380 and 36036.

(513) A room is 15 ft. 6 in. long and 12 ft. 9 in. broad; find the expense of covering the floor with carpet 24 in. wide, at 7*s*. 8*d*. per yard.

(514) If I pay £375 now for a debt of £381. 5*s*. not yet payable, and money be considered worth 4 per cent. per ann., when will the debt be due?

(515) The cost of a cube of metal, at £3. 10*s*. 4*d*. per cubic inch, is £1206. 4*s*. 4*d*. Find the cost of gilding it over at a halfpenny per square inch.

(516) Gold of the value of £423267 arrives from Australia; what is its weight in lb. avoirdupois, the price being £3. 18*s*. per oz. troy, and 1 lb. avoirdupois being equal to $29\frac{1}{6}$ oz. troy?

(517) *A* can do one-half of a piece of work in 1 hour, *B* can do three-fourths of the remainder in an hour, and *C* can finish it in 20 minutes: how long would *A*, *B*, and *C* together take to do it?

(518) A person can read a book containing 220 pages, each of which contains 28 lines, and each line on an average 12 words, in $5\frac{1}{2}$ hours; how long will it take him to read a book containing 400 pages, each of which contains 36 lines, and each line on an average 14 words.

(519) The whole time occupied by a train 120 yards long, travelling at the rate of 20 miles an hour, in crossing a bridge is 18 seconds: find the length of the bridge.

(520) If 20 men, 40 women, and 50 children receive £350 among them for 7 weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a woman receive per week?

(521) Reduce $\dfrac{2\frac{3}{4} - 1\frac{3}{9}}{\frac{10}{36} + \frac{3}{2} \text{ of } \frac{1}{4}} \div 1\frac{9}{13}$ to its simplest form.

(522) Find the square root of 10747·4689, and the cube root of 189119224.

(523) Four bells toll at intervals of 3, 7, 12, 14 seconds respectively, and begin to toll at the same instant; when will they next toll together?

(524) How much 5 per cent stock can be purchased by the transfer of £1000 stock from the 3 per cents. at 96 to the 5 per cents. at 108? What will be the difference of income arising from the exchange?

(525) There are two rectangular fields equal in area; the sides of one are 945 yards and 1344 yards in length, and the longer side of the second is 1134 yards; what is the length of its shorter side, and how many acres are there in each field?

(526) A room is 20 ft. long, 16 broad, and 12 high. If pure gold be worth £4. 5*s.* per oz. troy, and a cubic foot of gold weigh 19260 oz. avoirdupois, what is the value of the gold which will exactly fill the room?

(527) The number of coins in a bag is 150, consisting of sovereigns, half-crowns and shillings. If the sum represented by the half-crowns and shillings be equal to that represented by the sovereigns, and the number of sovereigns be two-thirds that of the half-crowns, find the numbers of the

(529) A coal-truck is exactly filled by barrows which hold each 9 cwt.; it is exactly emptied again by sacks which hold each 5 cwt.; given that the contents of the truck were between 4 and 5 tons, find the exact amount.

(530) Two clocks begin to strike twelve together: one strikes in 35 seconds, the other in 25; what fraction of a minute is there between their seventh strokes?

(531) Multiply 3004072905 by 512648 in three lines of multiplication.

(532) Take $\frac{2}{31}$ of £4. 0*s.* 1*d.* from $\frac{5}{43}$ of £7. 14*s.* 1*d.*

(533) Find by Practice the cost of 15 cwt. 3 qr. 21 lb. at £2. 3 cents per cwt.

(534) The area of a rectangular field, whose length is three times its breadth, is 6 ac. 960 yd.; find the distance from corner to corner.

(535) A person saw the flash of a gun fired from a frigate at sea, distant 1 mile 480 yards, and 2 seconds afterwards saw the flash of another gun fired from a vessel in a line between the frigate and himself, and 4 seconds later still heard the two reports simultaneously: what was the distance between the vessels?

(536) *A* and *B* fire at targets, and have 55 cartridges each. *A* fires twice in 3 minutes, and *B* three times in 5 minutes: how many times will *B* have to fire after *A* has finished?

(537) A speculator bought 43 shares in a mine at $35\frac{1}{4}$, and kept them till they dropped to $11\frac{1}{2}$, when he sold out and bought with the proceeds 6 per cent. railway stock at 28 premium: find his annual income from the latter investment.

(538) Two clocks strike 9 together on Tuesday morning. On Wednesday morning one wants 10 minutes to 11 when the other strikes 11. How much must the faster be put back, that they may strike 9 together on Wednesday evening?

(539) How much ore must one raise, that on losing $\frac{17}{40}$ in roasting and $\frac{8}{19}$ of the residue in smelting, there may result 506 tons of pure metal?

(540) If a population is now ten millions, and the births are 1 in 20 and the deaths 1 in 30 annually, what will the population become in 5 years?

(541) Multiply 13 tons 5 cwt. 3 qr. 11 lb. by 24.

(542) Divide £65931. 12*s*. 9*d*. by $6\frac{3}{4}$.

(543) If 135 cwt. carried 65 miles cost £1. 9*s*. 3*d*., what will 60 cwt. carried 180 miles cost?

(544) If 16 darics make 17 guineas, 19 guineas make 24 pistoles, 31 pistoles make 38 sequins, then how many sequins are there in 1581 darics?

(545) What must be the gross produce of an estate, that after paying a 10 per cent. income-tax, and a rate of 2*s*. $1\frac{1}{2}$*d*. on £1 on the residue, there may remain £2574 per annum?

(546) What sum must a man invest in the 3 per cents. at $91\frac{1}{2}$ in order to have a clear income of £230, after paying an income-tax of 10*d*. in the pound?

(547) A merchant sells tea to a tradesman at a profit of 60 per cent., but the tradesman becoming a bankrupt, pays only 7*s*. 6*d*. in the pound. How much per cent. does the merchant gain or lose by the sale?

(548) The cubic content of a room 20 ft. long, and $12\frac{1}{2}$ ft. high, is 4875 cubic feet; find the cost of painting its walls at 9*d*. the square yard.

(549) The masters of a school are $\cdot 041\dot{6}$ of its whole number, but after 40 new boys have been added the masters became ·0375 of the whole. How many boys and masters were there before the new boys came?

(550) An express train starts from a station A at one o'clock for a station B at 30 miles an hour, and in 15 minutes is followed by an ordinary train at 20 miles an hour. A train from B to A at 25 miles an hour, after travelling one hour, meets the express, and in twenty minutes more meets the ordinary train. At what time did the train leave B?

(551) What decimal of £2. 10*s*. is 3*s*. 6*d*.?

(552) Divide $2\frac{3}{4}$ of $7\frac{7}{11}$ by $\frac{1}{2}$ of $\frac{3}{4}$ of $18\frac{2}{3}$.

(553) The length of $\frac{1}{360}$ of the earth's circumference is $69\frac{1}{12}$ miles nearly. What is the earth's diameter, assuming that the diameter of a circle is $\frac{7}{22}$ of the circumference?

(554) A person bought a box of 100 oranges at 16 a shilling, just before a rise in price of $1\frac{1}{4}$*d*. a dozen. How much did he save by buying before the rise?

(555) The telegraph posts on railways are generally erected at intervals of 60 yards. Shew that if a traveller count the number of the posts, which pass his eye in two minutes, that number will nearly express in miles per hour the speed of the train?

(556) A person sells out of the $3\frac{1}{2}$ per cents. at $92\frac{3}{4}$, and realises £18550. If he invests two-fifths of the produce in the 4 per cents. at 96, and the remainder in the 3 per cents. at 90, find the alteration in his income.

(557) If I can write 360 pages of a book in 2 months, working $7\frac{1}{2}$ hours a day, in how many months could I write 450 pages, working 9 hours a day?

(558) Divide £8. 6*s.* 8*d.* among 4 persons, so that *B* may have three times as much as *A*, *C* half as much again as *A* and *B* together, and *D* as much as *A*, *B*, and *C* together.

(559) By selling a horse for 37 guineas I lost $7\frac{1}{2}$ per cent.; what must I have sold him for to have gained $12\frac{1}{2}$ per cent.?

(560) A rectangular piece of ground 72 yards by 45 yards is to be laid out in 4 plots of grass, each 27 ft. by $13\frac{1}{2}$ ft., and a pond in the centre 6 yards square, to contain 252 cubic yards of water; find the expense of gravelling the remainder at 1*d.* per square yard, and the depth of the pond.

(561) Divide 36·6327 by ·07254, and 15 by ·000032.

(562) Find the number of square feet in $\frac{5}{66}$ of $3\frac{3}{11}$ of an acre.

(563) What is the average annual profit of a business, when a partner, entitled to $\frac{2}{7}$ of the profits, receives as his share for 2 years and 4 months the sum of £3598. 13*s.* 5*d.*?

(564) If a tradesman adds to the cost price of his goods a profit of $12\frac{1}{2}$ per cent., what is the cost price of an article which he sells for £1. 18*s.* 3*d.*?

(565) A room, 10 ft. high and 20 ft. long, requires 760 square feet of paper to cover its walls, without deduction for windows and doors. What is the cost of the carpet which will cover the floor, at the price of 6*s.* 3*d.* per square yard?

(566) An estate is bought at 25 years' purchase for £15000, two-thirds of the purchase-money remaining at mortgage at 3 per cent. The cost of repairs averages £100 per annum. What interest does the purchaser make on his investment?

(567) A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses 20 per cent. The price of flour falls 50 per cent., and other trade expenses are thereby reduced 25 per cent. What reduction should he make in the price of a fivepenny loaf, allowing him still to realise the same amount of profit?

(568) Two ships are built. Twice as many ship-carpenters are employed about the first as about the second; the first is built in 9 months, the second in 8 months; the wages of each man of the first set are 7*d.* per hour, and they work 12 hours a day; the wages of each of the second set are 6*d.* per hour, and they work $10\frac{1}{2}$ hours a day. The cost of the first in carpenters' wages was £6000; what was that of the second?

(569) Find the difference between the interest and discount on £403. 4*s.* for 73 days at 4 per cent.

(570) The compound interest on a certain sum at 4 per cent. for 2 years exceeds the simple interest for the same time at the same rate by £6; what is the sum?

(571) Four-fifths of the cube of a certain number is 337500, what is the number?

(572) Find a fourth proportional to 3·125, ·000215 and ·0732.

(573) Find the principal sum on which the simple interest in $2\frac{1}{4}$ years at 4 per cent. per ann. is £11. 17*s.* 9*d.*

(574) If a loaf that weighs $4\frac{1}{2}$ lb. is sold for 6*d.* when wheat is 6*s.* per bushel, what should be the price of 150 lb. of bread when wheat is 72*s.* a quarter?

(575) A person leaves £12670 to be divided among his five children and three brothers, so that, after the legacy duty has been paid, each child's share shall be twice as great as each brother's. The legacy duty on a child's share being one per cent., and on a brother's three per cent., find what each will receive?

(576) In a certain estate one-fifth is pasture, three-eighths arable land, and the remainder, consisting of woods, covers 50 acres, what is the size of the estate?

(577) Two persons, *A* and *B*, meet to settle their accounts *A* has $3\frac{1}{4}$ years previously lent *B* £500; and *B* has a bill of £320 against *A*, for which he is to allow nine months' discount; if the *interest* in each case is 4 per cent. per ann., what has *B* to pay *A*?

(578) The weight of a cubic foot of water being 1000 oz., find the weight of a rectangular block of gold, 8 inches in length, 2 inches in thickness, and 3 in breadth; the weight of a mass of gold being 19·26 times the weight of an equal bulk of water.

(579) If 12 men or 18 boys can do $\frac{3}{4}$ of a piece of work in $6\frac{1}{2}$ hours, in what time will 11 men and 9 boys do the rest?

(580) The content of a cistern is the sum of two cubes whose edges are 10 inches and 2 inches, and the area of its base is the difference between two squares whose sides are $1\frac{1}{9}$ and $1\frac{2}{9}$ feet. Find its depth.

(581) Reduce $\frac{3}{5}$ of a guinea to the fraction of a pound; and 20 feet $7\frac{1}{2}$ inches to the fraction of a mile.

(582) Find the L. C. M. of 2, 3, 8, 9, 11, 20, 21, 22.

(583) A grocer buys 4 cwt. 3 qr. 14 lb. of sugar at £1. 16*s.* 8*d.* per cwt., and sells it at $4\frac{1}{2}$*d.* per lb.; how much does he gain or lose per cent.?

(584) If 7 men mow 22 acres in 8 days, working 11 hours a day, in how many days, working 10 hours a day, will 12 men mow 360 acres?

(585) A brick is 9 inches long, $4\frac{1}{2}$ inches wide, and 3 inches thick; how many bricks will be required to build a wall, 520 yards 9 inches long, 15 feet high, and 18 inches thick?

(586) 20,000 copies of a penny newspaper weigh $\frac{1}{4}$ of a ton, and when the paper duty was removed the profit on the receipts was increased $4\frac{1}{6}$ per cent. What was the duty per pound on paper?

(587) I buy wheat at 39*s.* a quarter, and some of a superior quality at 6*s.* per bushel: in what proportions must I mix them, so as to gain 25 per cent. by selling the mixture at 57*s.* 6*d.* per quarter?

(588) If when 25 per cent. is lost in grinding wheat, a country has to import 10,000,000 quarters, but can maintain itself on its own produce if only 5 per cent. be lost, find the quantity of wheat grown in the country.

(589) A man rows down a river 18 miles in 4 hours with the stream, and returns in 12 hours; find the rate at which he rows, and the rate at which the stream flows.

(590) A person having £20000 in the 3 per cents. sells out at $94\frac{1}{2}$, and invests the produce in bank stock at 225, from which he can get $8\frac{1}{2}$ per cent. interest. Find the change in his income.

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(591) Find the value of

$$\frac{5\frac{1}{2} \text{ of } \frac{2}{9} \text{ of } 2\frac{4}{7} - 1 \div \left(\frac{1}{5} + \frac{1}{2}\right)}{1 - \frac{3}{14} \text{ of } \left\{\frac{1}{2} + \frac{1}{2} \text{ of } \dfrac{\frac{1}{20}}{\frac{1}{7} \text{ of } 1\frac{1}{20}}\right\}}$$

(592) Express 9*s.* 2*d.* as the decimal of £4. 11*s.* 8*d.*

(593) The floor of a room, 14 ft. by $12\frac{1}{2}$ ft., being partly covered by a carpet measuring $11\frac{1}{2}$ ft. by $9\frac{2}{3}$ ft., how much floor-cloth will complete the covering, and what will it cost at 6*s.* per square yard?

(594) If with a capital of £500 a tradesman gain £50 in 7 months, in what time will he gain £60. 10*s.* with a capital of £385?

(595) If the rent of land in France be 130 francs per hectare, calculate the rent per acre in English money, 25 francs being equal to 20 shillings, and 100 hectares equal to 247 acres.

(596) What sum of money put out at interest for 9 months at 4 per cent. per ann. will amount to £193. 2*s.* 6*d.*?

(597) *A* and *B* can do a piece of work in 6 days, *B* and *C* in 8 days, *A*, *B*, and *C* in 4 days. How long would *A* and *C* take to do it?

(598) Three farmers with 460, 805, and 391 acres respectively, agree to employ 72 labourers in proportion to their occupation; what is the number employed by each?

(599) If, by selling an article for £38. 5*s.*, 8 per cent. is lost, what per cent. is gained or lost by selling it for £57?

(600) If gold can be beaten out so thin that a grain will form a leaf of 56 square inches, how many of these leaves will be required to make up the thickness of a sheet of paper, the weight of a cubic foot of gold being 1215 lb. Troy and 400 sheets of paper making a book 1 inch thick?

(601) Divide ·22101 by 139, 5329 by ·0073, and 1 by ·0625.

(602) Extract the square root of 1657059849, and the cube root of 48228544.

(603) If 13 solid inches of copper balance 17 of iron, and 15 of iron balance 16 of tin, and 19 of tin balance 12 of zinc, how many solid inches of zinc balance 2470 solid inches of copper?

(604) A tradesman marks his goods with two prices: the one for ready money, the other for 6 months' credit, the rate of interest being 5 per cent. per ann.; if the credit price of an article be 6*s.* 10*d.*, what ought its ready-money price to be?

(605) A man invests £3000 in the 3 per cents. at 80; after the funds have risen 4 per cent. he transfers his stock to the $3\frac{1}{2}$ per cents. at $96\frac{1}{4}$: what will be the alteration in his income?

(606) If 16 men can reap 152 acres of wheat in 8 days, how many men will reap 228 acres in 12 days?

(607) Show that compound interest reckoned quarterly at £1. 4*s.* $6\frac{1}{2}$*d.* per cent. is very nearly equivalent to interest reckoned yearly at 5 per cent.

(608) A French mètre contains 39·371 English inches: express to three decimal places an English mile in mètres.

(609) A franc being worth $9\frac{3}{4}$*d.*, find the sum of money which can be paid by an exact number of either shillings or francs, the number of francs exceeding the number of shillings by 27.

(610) A viaduct consists of 3 series of arches built upon each other, the breadths of the arches in each being respectively 8 yd. 2 ft., 6 yd., and 5 yd.: whenever the piers in all the series are vertically above each other, there occurs a mass of masonry 4 yd. wide; of such there are 3; find the length of the viaduct.

(611) Show that

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1}}}}}+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1}}}}}=\frac{8}{9}$$

(612) What sum must be paid down in order to receive £374. 18*s*. $6\frac{9}{10}$*d*. two years hence, allowing $3\frac{1}{2}$ per cent. compound interest?

(613) Find the length of a square field containing 4466 sq. ft. 100 sq. in.

(614) Divide £4089. 9*s*. 2*d*. among 1 man, 8 women, and 4 boys, so that the shares of the man, each woman and each boy may be in the proportion of 10, 8, and 6.

(615) How much stock must be sold from an investment at $92\frac{1}{4}$ to produce £276. 15*s*. a year, when the proceeds of the sale are laid out in another investment at 96, which pays interest at the rate of $4\frac{1}{2}$ per cent.?

(616) Find the value of $\cdot 857\dot{1}4285\dot{7}$ of £10. 14*s*. 1*d*. accurately; and show that the error committed by neglecting all decimals of an order higher than the fifth is less than $\frac{1}{125}$ of a penny.

(617) If 11 lb. of coffee be equal in value to 2 lb. of tea, 3 lb. of tea to 25 lb. of sugar, and if the price of a cwt. of sugar be £3. 17*s*., find the price of 1 lb. of coffee.

(618) The sum of £327 is borrowed at the beginning of a year at interest, and after 9 months have passed £400 more is borrowed at a rate of interest double that which the former sum bears. At the end of the year the interest on both loans is £13. 3*s*. 6*d*. What is the rate of interest in each case?

(619) A dealer purchases a liquid at 4*s*. the gallon, and dilutes it with so much water that, when he sells the compound at 3*s*. a gallon, he gains 20 per cent. on his outlay. How much water is there in every gallon of the compound sold?

(620) Find to three places of decimals the cube root of 7.

(621) Reduce to a vulgar fraction in its lowest terms $\cdot 58\dot{3}$, and find the number of grains it represents when the unit is 3 oz. 5 dwt.

(622) A contractor bought 250 sheep, and sold them for £532. 5*s*. 10*d*. at a gain of $16\frac{2}{3}$ per cent.; what was the cost price of each sheep?

(623) If 25 gas-burners, which are lighted 5 hours every evening for 20 days, consume a quantity of gas that costs £2. 2*s*. 6*d*., how many burners may be lighted 4 hours every evening for 30 days, at a cost of £7. 13*s*.?

(624) The discount on £566. 10*s.* for 9 months is £16. 10*s.*: find the rate of interest.

(625) A merchant lost a cargo at sea which he had insured; the broker offered him a sum of money for his loss, which the merchant refused as being 10 per cent. below the estimated value of his loss; the broker then offered £379. 15*s.* more than he offered at first, and the whole amount of the second offer was $5\frac{1}{2}$ per cent. in excess of the estimated value. What was that value?

(626) A labourer has to spend 3*s.* $9\frac{1}{2}$*d.* a week on bread when it is at $6\frac{1}{2}$*d.* the quartern. If it rises to 7*d.* how much less bread must his family eat in a week, so that it may cost him the same as before?

(627) If a cubic foot contains 343 units of volume, find the number of units of area in a rectangle, whose sides are 60 ft. and 100 ft.

(628) By selling tea at 4*s.* 8*d.* a pound, a grocer clears one-seventh of his outlay; what profit per cent. will he clear by selling the same tea at 5*s.* a pound?

(629) I have £12. 4*s.* in half-crowns, florins, and shillings, and the numbers of coins of each kind are respectively as the numbers 7, 5, and 3. Find how many of each kind I have.

(630) A picture-gallery consists of three large rooms; the first is 20 yd. long, 20 yd. broad, and 6 yd. high; the other two are 20 yd. long, 20 yd. broad and 5 yd. high. Supposing the walls to be covered with pictures, except the doors, which are 8 ft. high and 3 ft. wide, and of which each room has two, what will be the number of pictures, the average size being 8 feet by 3 feet?

(631) Find by Practice the value of $5354\frac{3}{4}$ cwt. of soap at £4. 4*s.* 8*d.* per cwt.

(632) The sidereal year is 365 days 6 hr. 9 min. 9·6 sec., the solar year is 365 days, 5 hr. 48 min. 49·7 sec.; reduce their difference to the decimal of a solar year.

(633) If the three per cent. Consols be at $93\frac{1}{2}$, what should be the price of a £50 ailway share paying 4 per cent., so that the two investments may give equal interest on the money invested?

(634) If the carriage of 41 cwt. 1 lb. for 49 miles cost £20. 9*s.* 6*d.*, what must be paid for the carriage of 13 cwt. 2 qr. 19 lb. for 35 miles?

(635) The net income of an estate, after deducting 10*d.* in the pound for income-tax, and 4 per cent. on the remainder for expenses of collection, is £437. What is the gross rental?

(636) How many Swedish dollars and skillings, at $13\frac{1}{2}d.$ the dollar, must be given in exchange for 2 Russian rubles and 25 kopeks, at 3*s.* 2*d.* to the ruble, a dollar being worth 48 skillings, and a ruble 100 kopeks?

(637) If 15 masons, working 10 hours a day, can build a wall, 6 ft. high and 100 yd. long, in 6 days, how long will it take 7 masons, working 9 hours a day, to build a wall 9 ft. high and 140 yd. long?

(638) A bankrupt's assets are £675, out of which he pays 15*s.* in the pound on half his debts, and 12*s.* on the other half. What is the amount of his debts?

(639) A grocer buys 567 cwt. of sugar at £1. 19*s.* $10\frac{1}{2}d.$ per cwt., and mixes it with 1161 cwt. bought at £2. 2*s.* $6\frac{1}{2}d.$ per cwt.; at what price per lb. must he sell the mixture to realise a profit of 12 per cent.?

(640) Compare the cash and credit prices of the same article, credit being given for 9 months, and simple interest at 6 per cent. per ann. being allowed. Find the cash price of articles, the credit price of which amounts to £114. 1*s.* 7*d.*

(641) Find the value of $\sqrt{3 + \sqrt{3}}$ to six places of decimals.

(642) A railway carriage travels 77 miles in 87 minutes, find how many feet it passes over in a second; and if the circumference of a wheel be 11 ft. 6 in., how many times does it revolve?

(643) If wages vary as the price of wheat, and 18 men working for 4 weeks receive £43. 4*s.* when wheat is 64 shillings a quarter, find the price of wheat when 16 men working for 5 weeks obtain £67. 10*s.*

(644) A contractor sends in a tender of £5000 for a certain work; a second sends in a tender of £4850, but stipulates to be paid £500 every three months; find the difference between the tenders, supposing the work in both cases to be finished in two years, and money to be worth 4 per cent. simple interest.

(645) What sum of money must be left in order that, after a legacy duty of 10 per cent. has been paid, the remainder being invested in the 3 per cents. at $91\frac{1}{8}$ may give a yearly income of 100 guineas.

(646) If in a box 3 ft. 6 in. long, 1 ft. 7 in. deep, and 1 ft. 10 in. wide I pack 160 books, each $9\frac{1}{2}$ in. long, $5\frac{1}{4}$ in. wide, and $1\frac{3}{4}$ in. thick, find how many more, of a size 7 in. long, $4\frac{1}{2}$ in. wide, and 1 inch thick, are required to fill it.

(647) If two boys and one man can do a piece of work in 4 hours, and two men and one boy can do the same in 3 hours, find in what times a man, a boy, and a man and a boy together, respectively, could do the same.

(648) What is the interest on 30029 rupees, 4 annas, 6 pice, at $4\frac{1}{2}$ per cent.? Give the result in English money, when the rupee is worth 2*s.* $4\frac{1}{2}$*d.*, the rupee containing 16 annas, and the anna 12 pice.

(649) A person sells out £1250 stock of 3 per cent. Consols when they are at 96, and invests the proceeds in railway stock at 75, paying an annual dividend of $2\frac{1}{2}$ per cent. What is his increase of income?

(650) Find the square roots of 15376·248001 and $\frac{31.36}{39\cdot 69}$

(651) If 8 dwt. of silver be worth 2*s.* 2*d.*, what is the value of 5 lb. 3 oz.?

(652) Multiply 350·4 by ·0105 and divide the product by ·0000219.

(653) If 7800 men on full rations consume 910 quarters of wheat in 49 days, in how many days will 6440 men on half-rations consume 690 quarters?

(654) A general, after losing a battle, found that he had only two-thirds of his army left fit for action; one-ninth of the army had been wounded, and the remainder, 2000 men, killed or missing; of how many did the army consist before the battle?

(655) A luggage-train leaves a station and travels at the rate of 10 miles an hour; after 4 hours another train follows from the same station, travelling $16\frac{2}{3}$ miles an hour; how far must the second train travel before it comes up with the first?

(656) If 15 pumps working 8 hours a day can raise 1260 tons of water in 7 days, how many pumps working 12 hours a day will be required to raise 7560 tons of water in 14 days?

(657) Show that the interest on £266. 13*s*. 4*d*. for 3 months at $4\frac{1}{2}$ per cent. is equal to the discount on £83 for 15 months at 3 per cent.

(658) A piece of work has to be finished in 36 days, and 15 men are set to do it, working 9 hours a day; but after 24 days it is found that only three-fifths of the work is done; if 3 additional men be then put on, how many hours a day will all have to work so as to finish the task in time?

(659) I buy corn in France at 29 francs per hectolitre; at what price must I sell it in England per imperial bushel, so that at least I may not lose? The cost of freight is 1 franc per 100 hectolitres; the commission is $2\frac{1}{2}$ per cent.; insurance 1 per cent.; the course of exchange 25 fr. 30 cent. for £1 sterling; and 64 imperial bushels = 22 hectolitres.

(660) In a game of rackets *A* can give *B* 3 points out of 15, and *A* can give *C* 7 points; how many points can *B* give *C*, so as to make an even match?

(661) Divide 480 into two parts, so that one is three-fifths of the other.

(662) Simplify $\dfrac{3\cdot\dot{5} - 1\cdot8\dot{3}}{9\cdot\dot{7} - 6\cdot\dot{4}} \times \dfrac{1}{\frac{4}{71}} \div \dfrac{3\cdot1 \times \cdot\dot{1}0\dot{1}}{2\cdot\dot{1}\dot{5}}$

(663) The breadth of a room is twice its height and half its length, and the contents are 4096 cubic feet. Find the dimensions of the room.

(664) If 10 scudi = 52·5 francs, 16 shillings = 20 francs, and 12 carlini = 4*s.* 2*d.*; how many carlini are equivalent to 500 scudi?

(665) The interest on a certain sum at simple interest is £28, and the discount £21. 17*s.* 6*d.* for the same time. What is the sum?

(666) A spirit merchant buys two sorts of spirits in equal quantities, one at a shilling a gallon more than the other: he mixes them and sells the mixture for 16*s.* 6*d.* per gallon, gaining 10 per cent. on his outlay. What was the price paid per gallon by the merchant?

(667) A man buys a farm of 150 acres for £4624, and after repairing the buildings lets it at 30*s.* per acre, thereby getting a return of $4\frac{1}{2}$ per cent. on his money: how much did he spend on repairs?

(668) If a bushel of barley weighs $60\frac{1}{2}$ lb., and the produce of an acre is 5 qr., what is the average weight of produce on a square foot?

(669) The price of 13 sheep and 11 pigs is £49, and that of 7 sheep and 33 pigs is £67; find the values of a sheep and a pig.

(670) How many bricks, 9 inches long, $4\frac{1}{2}$ broad, and 4 thick, will be required for a wall 60 feet long, 20 ft. high, and 4 ft. thick, allowing $6\frac{1}{4}$ per cent. of the space for mortar?

(671) Find the value of

(*a*) $\frac{20}{39}$ of £6. 6*s.* 9*d.* – $\frac{10}{33}$ of £4. 10*s.* 9*d.*

(*β*) ·65 of £4. 10*s.* + ·0125 of £5. 13*s.* 4*d.*

(672) Define an *aliquot part.* Find by Practice the price of 7 lb. 5 oz. 12 dwt. 12 gr. at £4. 2*s.* 4*d.* per lb.

(673) Divide £69 between *A*, *B*, and *C*, so that where *A* receives £1, *B* may receive £3, and where *B* receives £2, *C* may receive £5.

(674) If a cubic foot of iron weigh 7·8 times as much as a cubic foot of water, find the weight of a block of iron 20·28 ft. long, 2·58 ft. broad, and 2·5 ft. thick, supposing a cubic foot of water to weigh 1000 oz.

(675) *M* invests one-third of his property in bank stock, one-sixth in Consols, and the remainder in railway shares. When he sells out he makes a profit of 5 per cent., 3 per cent., and 2 per cent. respectively on the investments, and realises £6190. Required the amount of his property originally.

(676) A tradesman makes a deduction of 10 per cent. for ready money on a bill of £28 due in 12 months, receiving £25. 4*s*. Find the difference between this sum and the present worth of the debt, reckoning interest at 10 per cent.

(677) At a certain battle two-thirds of the defeated army ran away with their arms, five-sevenths of the remainder left their arms on the field, and of the rest seven-eighths were missing, the remaining 500 being either killed or wounded. Find the whole number of the army.

(678) Nine horses, having 4 feeds a day each, can be kept for 3 weeks for £12. 8*s*. $0\frac{3}{4}$*d*. What will be the cost of 15 horses for 36 days with 3 feeds a day?

(679) Show that the difference between the interest and the discount on the same sum for the same time is the interest of the discount.

(680) The surface of Great Britain contains 83827 square miles; find its value at £11. 7*s*. 6*d*. per acre.

(681) If 1 lb. of tea be worth 50 oranges, and 70 oranges be worth 84 lemons, what is the value of a pound of tea when a lemon is worth a penny?

(682) Decompose 831600 into its prime factors, and find the least multiplier of it, which will make the product a perfect cube.

(683) By investing a certain sum of money in the 3 per cents. at $91\frac{1}{2}$ a man obtains an income of £464: what would he obtain by investing an equal sum in the 4 per cents. at 96?

(684) If gold be at a premium of 20 per cent., and a person buy goods marked 135 dollars, and offer gold to the amount of 135 dollars, what change ought he to receive in notes, 5 per cent. being abated for ready payment?

(685) A merchant buys two butts of wine, one for £120, and one for £110; he also buys a third, and after mixing the three, retails the wine at 45*s.* a dozen, making $12\frac{1}{2}$ per cent. on his outlay: supposing the number of dozens in a butt to be 52, find the price of the third butt.

(686) The price of oats being 28*s.* per quarter, it costs 17*s.* 3*d.* per week to keep a horse; if oats only cost 24*s.* per quarter the expense would be 15*s.* $11\frac{1}{4}$*d.*; what quantity of oats does a horse eat in a year?

(687) The breadth of a room is two-thirds of its length and three-halves of its height, and the contents are 5832 cubic feet. Find the dimensions of the room.

(688) If 72 carlini be worth 25 shillings, 4 shillings worth 5 francs, and 8 scudi worth 45 francs, how many carlini are equal to 100 scudi?

(689) The interest on a sum at simple interest is £7, and the discount for the same time is £5. 9*s.* $4\frac{1}{2}$*d.* What is the

(690) A grocer buys twice as much black tea as green, giving two shillings a pound more for the green than the black; he retails it when mixed at 5*s.* a pound, and makes 25 per cent. on his outlay. What did he give for each sort of tea?

(691) Find the cube root of 365525875.

(692) A legacy of £146000 is left to three sons in the proportion of $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$ respectively: how much will each receive?

(693) The price of gold in this country is £3. 17*s.* $10\frac{1}{2}$*d.* an ounce; find the least number of ounces which can be coined into an exact number of sovereigns, and the number of sovereigns so coined.

(694) A man hired a labourer to do a certain amount of work, on the agreement that for every day he worked he should have 2 shillings, but that for every day he absented himself he should lose ninepence. He worked twice as many days as he absented himself, and received on the whole £1. 19*s.* Find how long he was doing the work.

(695) A work can be accomplished by A and B in 4 days; by A and C in 6 days; by B and C in 8 days. Find in what time it would be accomplished by all working together.

(696) If 35 metres = 39 yards, and 17 metres = 9 toises, and 5 plethra = 124 toises, how many yards are there in 1575 plethra?

(697) If I invest in the 3 per cents. at 93, what interest per cent. per ann. shall I get for my money, supposing that I invest immediately after the payment of the dividend, and that the dividends are payable half-yearly?

(698) Three persons, A, B, and C, who can walk respectively at the rate of 3, 4, and 5 miles per hour, start from the same place P at intervals of one hour. A starts first, and as soon as B has caught him up, B returns to the station P; find where he will meet C.

(699) A man has three vessels, P, Q, R, holding 1, 2, 4 gallons respectively; P is empty, Q is full of water, R is full of wine. He fills P from Q, replenishes Q from R, and then empties P into R. When he has performed this operation twice, what will be the proportion of the wine in Q to the water in R?

(700) A cubic foot of water weighs 1000 oz. avoirdupois. Find in tons the weight of a rainfall of one inch over an acre of ground.

(701) What is the value of

$$\cdot 25 \text{ of } \tfrac{11}{14} \text{ of } \frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{2}-\frac{1}{3}} \text{ of 8 guineas?}$$

(702) A person spends $\frac{1}{8}$ of his income in rent, $\frac{1}{3}$ in household expenses, and $\frac{1}{20}$ in dress. How much has he left, supposing his income is £500 a year?

(703) A person, who has £1900 Russian 4 per cent. stock, sells out at 104, and devotes £962. 13*s*. 4*d*. to the purchase of 3 per cent. Consols at 95, and lends the rest of the sum realised on mortgage. What interest must he ask for his money, that his income may be the same as before?

(704) Find the prime factors of 210, 294, and 336, and thence write down all the numbers which will divide them all without remainder, and also the smallest number of which they are all exact divisors.

(705) State the connection between Troy and Avoirdupois weights. A ring weighs 1 dwt. 4 gr., and is worth £1. 2*s.*; if 1050 of such rings be packed in a box weighing $3\frac{1}{2}$ lb., what would it cost to convey them 144 miles at the rate of 5*s.* per ton per mile, insurance being demanded at the rate of $\frac{1}{8}$ per cent.?

(706) How long will it be before £2500 put out to compound interest at 10 per cent. per ann. will obtain £1727. 11*s.* 9·3*d.* as interest?

(707) *A* and *B* rent a field for £60. *A* puts in 10 horses for $1\frac{1}{2}$ months, 30 oxen for 2 months, and 100 sheep for $3\frac{1}{4}$ months. *B* puts in 20 horses for 1 month, 40 oxen for $1\frac{1}{2}$ months, and 200 sheep for 4 months. If the food consumed by a horse, an ox and a sheep, in the same time, be in the proportion of 3, 2, and 1, find how much of the rent each should pay.

(708) One clock strikes four times while another strikes three. It is observed that they both begin striking a certain hour together, and that the last stroke of one is simultaneous with the last stroke but two of the other. What o'clock is it?

(709) A man and a boy are to work on alternate days at a piece of work, which would have occupied the boy alone 13 days. If the boy take the first day, the work will be finished half a day later than if the man commences. Find how long they would take to do it working together.

(710) One company pays 5 per cent. on shares of £100 each; another pays at the rate of $4\frac{5}{8}$ per cent. on shares of £7. 10*s.* each: the price of the former is £124. 10*s.*, and of the latter £8. 10*s.* Compare the rates of interest which the shares return to purchasers.

(711) Find by Practice the value of 11 tons 17 cwt. 1 qr. at £7. 7*s.* $4\frac{1}{2}$*d.* per ton.

(712) How long will it take to walk round a square field containing 13 acres 81 yards, at the rate of $3\frac{1}{2}$ miles an hour?

(713) Which is the more advantageous stock to invest in, the 3 per cents. at 94, or the $2\frac{1}{2}$ per cents. at 79?

(714) Find the sum of money which in 4 years at 5 per cent. compound interest will amount to £881. 4*s.* $10\frac{7}{80}$*d.*

(715) A piece of work can be done by 35 men in 50 days; but, after working together for 12 days, 16 of the men leave off work. In how many days will the remaining men finish it?

(716) Two men invest £300 and £100 in a machine; it works 6 months for each of them: determine what one must pay the other, if they would have made 30 per cent. on the money by letting the machine.

(717) Three men the lengths of whose strides are 2 ft. 8 in., 3 ft., and 3 ft. 4 in., walk a mile. Find how often they will step together.

(718) *A* owes *B* £2725, and offers to pay him at a certain rate of discount instantly, instead of at the end of two years, when the debt will be due. *B* can place out the money, which he will receive, at 5 per cent. interest, and by that means gain £25 on the transaction. At what rate is the discount calculated?

(719) If 5 pumps, each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine; how many pumps with a length of stroke of $2\frac{1}{2}$ feet, working 10 hours a day for 12 days, will be required to empty the same mine; the strokes of the former set of pumps being performed four times as fast as the other?

(720) A man sells out of the $3\frac{1}{2}$ per cents. at $92\frac{3}{4}$, and realises £18550; if he invest one-fifth of the produce in the 4 per cents. at 96, and the remainder in the 3 per cents. at 90, find the alteration in his income.

(721) Add together $\frac{3}{4\frac{1}{2}}$ of 4*s.* 6*d.*, 2·35 of 2*s.* 6*d.*, and ·2375 of £2; and reduce the result to the decimal of half-a-guinea.

(722) If 36 men, working 8 hours a day for 16 days, can dig a trench 72 yards long, 18 wide, and 12 deep, in how many days will 32 men, working 12 hours a day, dig a trench 64 yards long, 27 wide, and 18 deep?

(723) A person buys a farm of 240 acres for £6500, and, after laying out money on repairs, lets it at 24 shillings per acre, getting a return of $3\frac{3}{4}$ per cent. for his money: how much did he expend on repairs?

(724) A man having his property in a railway £100 stock, which is at $52\frac{1}{2}$ and pays a dividend of $1\frac{5}{8}$ per cent., sells out and invests in the 3 per cents. at $95\frac{5}{8}$. Will he increase or diminish his income?

(725) If 5 men and 3 boys can reap 23 acres in 4 days, and if 3 men and 2 boys can reap 7 acres in 2 days, how many boys must assist 7 men, in order that they may reap 45 acres in 6 days?

(726) A cubic foot of water weighs 1000 ounces; a pipe, whose bore is 5 square inches, discharges $312\frac{1}{2}$ lb. of water per minute; find the rate per hour at which the water issues.

(727) The price of 2 turkeys and 9 fowls is £3, and the price of 5 turkeys and 3 fowls is £4. 5*s.*; find the price of a turkey and a fowl.

(728) A man allows his agent 5 per cent. on his gross income for collecting his rents, he spends $\frac{1}{7}$ of his net income in assuring his own life, and these portions of his income are in consequence exempt from income-tax. The income-tax being 5*d.* in the pound, and his income-tax amounting to £19. 19*s.*, find his gross income.

(729) A man discounts a bill of £180 drawn at 4 months at 60 per cent. per ann., and insists on giving in part payment 5-dozen of wine, which he charges at 4 guineas a dozen, and a picture, which he charges at £19. How much ready money does he pay? If the cost to the man of the wine and the picture be only one-fourth of the sum he has charged for them, what is the real interest the man has been charging?

(730) Find the cube root of 28 to four places of decimals.

(731) Multiply 32856 by 121711, using 3 lines of multiplication only.

(732) Simplify $\dfrac{2\cdot 8 \text{ of } 2\cdot\dot{2}\dot{7}}{1\cdot 1\dot{3}\dot{6}} + \dfrac{4\cdot\dot{4} - 2\cdot 8\dot{3}}{1\cdot\dot{6} + 2\cdot\dot{6}2\dot{9}} \text{ of } \dfrac{6\cdot 8 \text{ of } 3}{2\cdot 25}$

(733) The supply of a number of persons with bread at $7\frac{1}{2}d.$ the loaf for 31 days cost £27. 18*s.*; what will it cost to supply $\frac{2}{3}$ of that number for 20 days at $6\frac{1}{4}d.$ the loaf?

(734) In 1248 the usurers were not allowed to charge the Oxford students a higher rate of interest than 2*d.* for the use of £1 for a week. How much is this per cent. per annum?

(735) A man buys an article and sells it again so as to gain 5 per cent. If he had bought it at 5 per cent. less, and sold it for 1*s.* less, he would have gained 10 per cent. Find the cost price.

(736) A man has £5000 stock in the 3 per cents., which he sells, and invests the proceeds in the $3\frac{1}{2}$ per cents. at $87\frac{1}{2}$. If the increase in his income be £5, what is the price of the 3 per cents.?

(737) If the difference between the simple and compound interest on a sum of money for two years at 5 per cent. be £5. 18*s.* $9\frac{3}{4}$*d.*, find the sum.

(738) A monolith of granite in the Isle of Mull is said to be about 108 feet in length, and to have an average transverse section of 113 square feet. If shaped for an obelisk, it would probably lose one-third of its bulk, and then weigh 600 tons. Determine the number of cubic yards in such an obelisk and the weight in pounds of a cubic foot of granite.

(739) In a hundred yards race *A* can give *B* four and *C* five yards start: if *B* were to race *C*, giving him 1 yard in a hundred, which would win?

(740) What is the capacity of a vessel, out of which, when it is half full, 9 gallons being drawn, there remains $\frac{1}{5}$ of the whole content.

(741) A rectangular field measures 12 chains 25 links by 5 chains 60 links; find its area in acres, roods, and poles.

(742) What fraction of a guinea together with £4·125 is equivalent to £5. 14*s.*?

(743) What must be paid for the carriage of 17 cwt. 3 qr. a distance of $7\frac{1}{2}$ miles, if the carriage of 5 cwt. 1 qr. for 15 miles cost 8*s.* 2*d.*?

(744) *A* and *B* can together do a piece of work in 2 hours, *B* and *C* in $1\frac{3}{5}$ hours, *A*, *B*, and *C* in $1\frac{1}{3}$ hours: in what time can *A* and *C* do it?

(745) If 7 per cent. be gained by selling goods for £69. 11*s.*, what will be gained or lost per cent. by selling them for £61. 15*s.*?

(746) Two persons, walking at the rate of 3 and 4 miles per hour respectively, set off from the same place in opposite directions to walk round a park, and meet in 10 minutes. Find the length of the walk round the park.

(747) A person invests £5187. 10*s.* in the 3 per cents. at 83, and when they have risen to 85, he transfers three-fifths of his capital to the 4 per cents. at 96; find the alteration in his income.

(748) A banker in discounting a bill due in 3 months at 4 per cent., charges 5*s.* $1\frac{1}{4}$*d.* more than the true discount. Find the amount of the bill.

(749) A grocer buys two cwt. of tea; the first he sells at 5 per cent. profit, and the second, which cost £1 more, at 12 per cent. profit. The difference in the retail price being 4*d.* per lb., find the cost price of each cwt.

(750) One-tenth of a rod is coloured red, one-twentieth orange, one-thirtieth yellow, one-fortieth green, one-fiftieth blue, one-sixtieth indigo, and the remainder, which is 302 inches long, violet, what is the length of the rod?

(751) If £10. 1*s.* 3*d.* be a person's income-tax at 6*d.* in the pound, how much in the pound is it when his income-tax is £12. 11*s.* $6\frac{3}{4}$*d.*?

(752) If 20 men can reap a field of 105 acres in 6 days of 12 hours each, how long will it take 16 men to reap 126 acres, working 16 hours a day?

(753) If 9 tons $7\frac{1}{2}$ cwt. of iron be sold for £245, and the gain on it be 20 per cent., what was the cost per cwt.?

(754) In what time will £12500 amount to £15185. 9*s.* 0·96*d.* at 6 per cent. compound interest?

(755) A man invests £14970 in the purchase of 3 per cents. at 90, and of $3\frac{1}{4}$ per cents. at 97. His total income being £500, how much did he invest in each stock?

(756) A grocer mixes 18 pounds of coffee at 1*s.* $1\frac{1}{2}$*d.* a pound with 12 pounds of chicory at $3\frac{3}{4}$*d.* a pound: at what price must he sell the mixture to gain 25 per cent.?

(757) I buy a field for £1000, for which I receive £30 a year rent, which I invest as soon as I receive it at 4 per cent. compound interest. At the end of 3 years I sell the field for £1030. What have I lost or gained by buying the field, instead of investing the purchase money on the same terms that I invested the rent?

(758) The discount due on a certain sum due 9 months hence is £20, and the interest on the same sum for the same time is £20. 15*s*. Find the sum and the rate of interest.

(759) After paying income-tax at 7*d*. in the pound, and $3\frac{1}{2}$ per cent. on a mortgage of £5000, the net income from a farm is £747. 5*s*. 10*d*.; find the amount of the income-tax and the gross income.

(760) The following rule has been given to divide by $3\cdot14159$: "Multiply by 7, divide by 11, then by 2, and add $\frac{1}{8}$th of $\frac{1}{1000}$th of the result." Find the error made in obtaining $1 \div 3\cdot14159$ by this process.

(761) Find the product of $\frac{5}{9}$ of $\frac{3}{8}$ of $\frac{11}{15}$ and $\frac{2}{7}$ of $\frac{13}{18}$ of $\frac{27}{39}$; add the result to the difference between $\cdot014$ and $\frac{3}{250}$; and express the result as a decimal.

(762) A plate of gold, 3 inches square and one-eighth of an inch thick, is extended by hammering so as to cover a surface of 7 yards square; find its present thickness.

(763) The estate of a bankrupt (value £21000) is to be divided among four creditors, whose claims are, *A*'s to *B*'s as 2 to 3, *B*'s to *C*'s as 4 to 5, *C*'s to *D*'s as 6 to 7. What must each receive?

(764) The discount on a certain sum of money due 3 years hence is £30, and the interest on the same sum for the same time is £33. 12*s*. Find the sum and the rate of interest.

(765) The railway fare from Paris to Rennes, a distance of 374 kilomètres, is $23\cdot05$ francs. If a kilomètre be equal to $1093\cdot\dot{3}$ yards, and the exchange be $25\cdot225$, compare this rate with the English parliamentary rate of 1*d*. per mile.

(766) *A* and *B* run a race, and *A* wins by 20 yards; *C* and *D* run over the same course, and *C* wins by 60 yards; *B* and *D* run over it, and *B* wins by 40 yards. If *A* and *C* run, which will win, and by how much, if the course is 1 mile in length?

(767) In what time will £732. 11*s.* 10*d.* amount to £1709. 7*s.* $7\frac{1}{3}$*d.* at $5\frac{1}{3}$ per cent. simple interest?

(768) What factor is wanted to make 1350 a perfect square?

(769) *A* has 5 houses let for £50 each; *B* has 6 houses let for £40 each; *C* has 7 houses let for £35 each. Divide a tax of £5. 2*s.* 1*d.* between them.

(770) By selling tea at 5*s.* 4*d.* a pound, a grocer clears $\frac{1}{8}$ of his outlay. He then raises th price to 6*s.* What does he clear per cent. by the latter price?

(771) Prove that $\frac{5+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$

(772) If 5 men can do as much as 8 boys in a day, how many days will it take 32 boys to finish a piece of work of which 15 men did a quarter in 16 days?

(773) A banker borrows money at $3\frac{1}{2}$ per cent., and pays the interest at the end of the year; he lends it out at 5 per cent., but receives the interest half-yearly, and by this means gains £200 a year: how much does he borrow?

(774) Saltpetre and sulphur are mixed together in a mass of 80 lb., so that for every 7 parts of saltpetre there are 3 parts of sulphur. How much saltpetre must be added to the mass, so that for every 11 parts of saltpetre there may be 4 parts of sulphur?

(775) A dealer sends out 1 cwt. of tea at 4*s.* per lb., and allows $2\frac{1}{2}$ per cent. on the price for the expense of carriage. Supposing the whole amount of carriage to amount to 16 shillings, how much will the customer have to pay?

(776) A bankrupt owes £515. 12*s.* 6*d.* to *A*, £407 to *B*, and £293. 6*s.* 8*d.* to *C*; his estate is worth £911. 19*s.* $4\frac{1}{2}$*d.* Find how much the bankrupt can pay in the pound.

(777) If a landlord received £864. 10*s.* as the net rental of an estate, after his agent had allowed out-goings at 1*s.* $0\frac{1}{2}$*d.* per pound, and charged 5 per cent. commission on the remainder, what was his gross income?

(778) If $5\frac{1}{2}$ per cent. is gained by selling at £2. 17*s*. 9*d*. per cwt., what will be the gain per cent. by selling at $7\frac{1}{2}$*d*. per lb.?

(779) A train, having to perform a journey of 250 miles, is obliged after 103 miles to reduce its speed by one-fifth. The result is that the train arrives at its destination 1 hr. 10 min. behind time. What is its ordinary rate?

(780) Find the square root of ·000904210312608I.

(781) Find the difference between the interest and discount on £1092. 17*s*. 6*d*. for $3\frac{2}{3}$ years, at $4\frac{1}{2}$ per cent. simple interest.

(782) What is the price of land per acre, when a piece 87 ft. $1\frac{1}{3}$ in. long and 47 ft. $4\frac{8}{13}$ in. broad costs £234. 10*s*.?

(783) A man, having bought a lot of goods for £150, sells $\frac{1}{3}$ at a loss of 4 per cent.; by what increase per cent. must he raise that selling price, in order that, by selling the rest at the increased rate, he may gain 4 per cent. on the whole transaction?

(784) In a cricket-match the scores in each successive innings are a quarter less than in the preceding innings, and the side which had the first innings wins by 50. What are the scores in each innings?

(785) A grocer sells one kind of tea at 2*s*. 6*d*. a lb., and loses 5 per cent., and another kind at 3*s*. 4*d*. a lb., and gains 14 per cent. He mixes the two together in equal proportions, and sells the mixture for 3*s*. a lb. What is now his gain per cent.?

(786) *A* and *B* start at the same time from X to Y and from Y to X respectively, each walking at the rate of $3\frac{1}{2}$ miles an hour. After meeting *B*, *A* increases his rate to $3\frac{3}{4}$ miles an hour, and arrives at Y in 2 hours from this time. After meeting *A*, *B* reduces his rate to 3 miles an hour. In what time will he reach X?

(787) What is the price of wheat when an additional profit of $3\frac{3}{4}$ per cent. would raise the price 1*s*. 9*d*. per quarter?

(788) When the income-tax was $7d.$ in the pound, a person had to pay £63 less than when it was $14d.$ in the pound, although his income had diminished in the interim by £225. What was his income at first?

(789) A man having a flock of sheep sold 8 per cent. of them to *A*, 90 to *B*, $3\frac{1}{2}$ per cent. of the remainder to *C*, and 29 to *D*. He then had 550 left. How many had he at first?

(790) Find the cube root of 128558238823.

(791) Find by Practice the value of 37 yd. 2 ft. 7 in. of silk at $5s.\ 3\frac{1}{2}d.$ a yard.

(792) A man buys 130 yards of a certain material, he sells half of it at a gain of 20 per cent., and the remainder at a gain of 15 per cent., and realised £95. $9s.\ 4\frac{1}{2}d.$ What was the cost price per yard?

(793) In England gunpowder is made of 75 parts nitre, 10 sulphur, and 15 charcoal; in France of 77 parts nitre, 9 sulphur, and 14 charcoal: if half a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

(794) A landlord has an estate that brings him in £3000 a year, but this gross income is liable to deductions for rates and repairs to the extent of 12 per cent. He sells it at 26 years' purchase on the gross income, and invests the price in the 3 per cents. at $97\frac{1}{2}$. What difference is caused in his income?

(795) A ship 40 miles from the shore springs a leak, which admits $3\frac{3}{4}$ tons of water in 12 minutes. 60 tons would suffice to sink her; but the ship's pumps can throw out 12 tons of water in an hour. Find the average rate of sailing that she may reach the shore just as she begins to sink.

(796) A grocer buys $1\frac{3}{4}$ cwt. of tea at $5s.\ 4\frac{1}{2}d.$ per lb., and $2\frac{1}{4}$ cwt. of tea at $3s.\ 3\frac{1}{2}d.$ per lb., and mixes them; he sells $2\frac{1}{2}$ cwt. at $4s.\ 6d.$ per lb.: at what rate must he sell the remainder to gain 20 per cent. on his outlay?

(797) The receipts of a railway company are apportioned in the following manner: 48 per cent. for the working expenses, 10 per cent. on one-fifth of the capital an the remainder, £32000, for division among the holders of the rest of the stock, being a dividend at the rate of 4 per cent.; find the capital and the receipts.

(798) If the discount on a sum due at the end of $2\frac{1}{2}$ years be $\frac{80}{87}$ of the simple interest, at what rate is that calculated?

(799) The value of a certain length of a material, A, is $\frac{9}{11}$ of the value of $\frac{1}{5}$ as much again of another, B; and the weight of 17 yd. 2 ft. of A is $\frac{53}{51}$ of the weight of 15 yd. 1 ft. 9 in. of B. If the value of 3 cwt. 27 lb. of A be £35. 2*s.*, what is the value of 1 cwt. 2 qr. 12 lb. of B?

(800) A square foot of paper weighed 104·68 grains, and when 320 figures had been written upon it, it weighed 105·155 grains. If a strip of this paper, $5\frac{3}{4}$ inches wide, be taken to have written upon it the circulating period of $\frac{1}{100103}$, which consists of 100102 figures, in two lines at the rate of 5 figures to an inch, find the weight of the whole.

(801) Add to one million fifty-three thousand one hundred and one the quotient of 94073 divided by 623.

(802) Reduce $47\cdot6\dot{7}53\dot{1}$ to a vulgar fraction, and test your result.

(803) If by selling wine at 12*s.* per gallon I lose 25 per cent., at what must I sell it per gallon to gain 25 per cent.?

(804) What will be the price of wainscoting a room, 24 ft. 9 in. long, 19 ft. 6 in. wide, and 10 ft. high, at 13*s.* 4*d.* per square yard?

(805) If a crew, which can row from A to B in 60 minutes can row from B to A in 55 minutes, compare the rates of the stream and boat.

(806) *A* does $\frac{3}{5}$ of a piece of work in 12 days, and then gets *B* to help him. They work together for 2 days, when *B* leaves, and *A* finishes the work in 3 days more. Find how long *B* would have taken to do the whole.

(807) Find the present worth of £2047. 13*s*. due $4\frac{4}{23}$ months hence at $5\frac{3}{4}$ per cent.

In a decimal coinage of florins, cents, and mils, find in what time £1387. 4 fl. will amount to £1599. 6 fl. 7 c. $2\frac{1}{5}$ m. at $3\frac{3}{5}$ per cent. per annum.

(808) A boat propelled by 8 oars which take 30 strokes per minute travels at the rate of $9\frac{1}{7}$ miles per hour: find the rate of a boat propelled by 6 oars which take 28 strokes per minute, the work done by each oar during one stroke in the latter case being a quarter as much again as in the former case.

(809) How many bricks, each 12 in. long, 3 in. wide, and 3 in. thick, will be required to build a wall 16 ft. 4 in. long, 12 ft. 6 in. high, and 9 in. thick, leaving in it a doorway 6 ft. 3 in. high and 2 ft. 4 in. wide?

(810) If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12; find how long 13 men, 14 children, and 15 women working together will take to do it.

(811) Simplify

(*a*) $$\frac{2+\cfrac{1}{3-\cfrac{1}{5+\cfrac{1}{6}}}}{1\frac{5}{8} \div (1\frac{3}{4} \times 14\frac{1}{2})}$$

(*β*) $$\frac{6\frac{3}{4} + 5\frac{1}{2} \times 3\frac{1}{7} \div 7\frac{1}{4}}{3\frac{1}{5} + 2\frac{1}{2} - 4\frac{1}{10}}$$

(812) Find the present worth of £1942. 13*s*. 5*d*. due $5\frac{3}{4}$ years hence at $4\frac{1}{2}$ per cent.

(813) The area of a square flag-stone is 1 sq. yd. 2 ft. 97 in.; find the length of its side.

(814) A person left a sum of money to be divided equally amongst 43 poor people, subject to a tax of 9*d*. in the pound, which caused a deduction of £19. 10*s*.: what did each receive?

(815) A foot-pound is the *work done in lifting 1 lb. one* foot high; what work (in foot-pounds) is done by a boy weighing 8 stone, who jumps 50 times over a bar 3 ft. 9 in. high?

(816) Water expands when freezing, so that a cubic foot of water becomes 1·089 feet of ice: find how many cubic feet of water there are in an iceberg which is estimated to be 900 ft. long, 88 ft. broad, and 220 ft. high.

(817) A person possessing £10000, 3 per cent. consols, sells out when they are at $93\frac{5}{8}$, and invests the proceeds in 4 per cent. stock at $101\frac{7}{8}$: find the change in his income, allowing $\frac{1}{8}$ per cent. commission on each transaction.

(818) Five men do $\cdot 600\dot{6}$ of a piece of work in 2·12 hours: how long will 6 boys take to finish it, it being known that 3 men and 7 boys have done a similar piece of work in 3 hours?

(819) A cistern without a top is 27 ft. long, 22 ft. wide, and 6 ft. 6 in. deep: what will it cost to paint it inside and out, at $1\frac{1}{2}d.$ a square yard?

(820) A creditor receives on a debt of £296, a dividend of 12*s.* 4*d.* in the pound, and he receives a further dividend, on the deficiency, of 3*s.* 9*d.* in the pound: what does he receive in all?

(821) Divide 1815·02 by ·601; 18·1502 by 6·01; and ·00181502 by ·00601.

(822) A man invests £15040 in 4 per cent. stock at $62\frac{13}{24}$, brokerage $\frac{1}{8}$ per cent.; determine his income.

(823) Find the compound interest on £2073. 15*s.* for 3 years at 4 per cent.

(824) The length and breadth of a wooden box are 6 ft. and 4 ft. respectively: find its height, if the cost of painting the outside at $1\frac{1}{2}d.$ a square foot amounts to 12*s.* $10\frac{1}{2}d.$

(825) A watch set accurately at 12 o'clock indicates 10 minutes to 5 at 5 o'clock: what is the exact time when the watch indicates 5 o'clock?

(826) If a job can be finished in 45 days by 35 men, and if the men drop off by 7 at a time at the end of every 15 days, how long will it be before the job is finished?

(827) The flooring of a room, 14 ft. 3 in. long by 13 ft. 4 in. broad, is composed of $\frac{1}{2}$ in. planks, each 8 in. wide and 10 ft. long. How many will be required; and what will be the weight of the whole, if 1 cubic inch of wood weighs half an ounce?

(828) A does $\frac{2}{5}$ of a piece of work in 20 days, and then gets B to help him. They work together for 2 days, when B leaves and A finishes the work in half a day more. How long would B have taken to do the whole?

(829) In rifle-shooting a bull's-eye counts 4, a centre 3, and an outer 2. Eleven boys fire five shots each at a target, and score 113; 8 misses are made and 3 bull's-eyes; find the number of centres and outers.

(830) The wages of 5 men, 3 women, and 1 child amount to £4. 10*s.*, a man receiving twice as much as a woman, and a woman three times as much as a child. What will be the wages of 6 men, 2 women, and 5 children?

(831) Find, by Practice, the value of 27 ac. 3 ro. $26\frac{2}{3}$ po., at £107. 10*s.* 6*d.* per acre.

(832) In what time will £1245 amount to £1618. 10*s.*, if in 3 years £124 amounts to £138·88?

(833) If 5 men with 7 boys can earn £7. 13*s.* in six days, and 2 men with 3 boys can earn £2. 2*s.* in four days, in what time will 6 men with 12 boys earn £60?

(834) A fishmonger pays £3 per bushel for oysters, and sells them at 1*s.* 4*d.* a dozen: find his gain per cent., assuming that a bushel contains 1150 oysters.

(835) In a mile race A wins, B being 11 yards behind, and C 53 yards behind B: by how much would B beat C in a three-mile race in which A does not run?

(836) A person receives as a legacy $\frac{3}{11}$ of an estate, of which he transfers $\frac{1}{3}$ to his son. The son pays £5. 10*s.* for income-tax (the tax being 6*d.* in the pound). What was the value of the estate?

(837) Which is the better investment, the $5\frac{1}{2}$ per cents. at 105, or the $4\frac{1}{4}$ per cents. at $81\frac{1}{8}$? Explain the meaning of the above expressions.

(838) The rates at which A and B walk are in the proportion of 9 to 10, and B can walk 6 miles an hour: how many seconds' start must B give A, in order that he may just beat him in a twelve-mile race?

(839) If 6 per cent. be gained by selling a horse for £132. 10*s.*: how much per cent. is lost by selling him for £115?

(840) Three penny pieces, or 5 halfpenny pieces, weigh 1 oz. avoirdupois; find the number of pennies in 175 oz. troy. Also find the least number of halfpennies whose weight is an integral number of ounces troy.

(841) Subtract from three millions, one hundred and fifty thousand, five hundred and one, the product of 401 and 523.

(842) Find the square roots of $4957 \cdot 5681$ and $\frac{129 \cdot 4947}{60 \cdot 75}$

(843) At what rate will £157. 10*s.* amount to £189 in 5 years?

(844) A person invests £6825 in the 3 per cents. at 91; he sells out £5000 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85. How much does he gain or lose by the transaction?

If he invest the produce in $4\frac{1}{2}$ per cent. stock at par, what is the difference in his income?

(845) Two bills for £273. 15*s.* and £456. 17*s.* 6*d.* are due on the 2nd and 22nd July respectively. What is their value on the 12th July, interest being reckoned at the rate of 5 per cent. per annum?

(846) A speculator sells at a profit of 75 per cent.; but his purchaser fails, and only pays 5*s.* in the £. How much per cent. does the speculator gain or lose by his venture?

(847) *A* and *B* run a race. *A* starts at the rate of 400 yards a minute, and in every successive minute diminishes his pace by a yard a minute. *B* increases his pace by the same, and overtakes *A* in 4 minutes. What was *B*'s pace at starting?

(848) If a cask contain 3 parts wine and 1 part water, how much of the mixture must be drawn off and water substituted for the mixture in the cask to become half and half?

(849) Reduce £420. 6*s.* $1\frac{1}{2}$*d.* to dollars and cents, a dollar being worth 4*s.* 2*d.* and a cent one hundredth part of a dollar.

(850) If a kilomètre be taken to be $\frac{5}{8}$ of a mile, reduce 17 mi. 6 fur. 82 yd. $1\frac{1}{2}$ ft. to kilomètres.

(851) Find, by Practice, the value of 103 tons 3 qr. 16 lb. at £57. 6*s.* 3*d.* per ton.

(852) In what time will £1075. 10*s.* amount to £1559. 9*s.* 6*d.*, if in 4 years £176. 5*s.* amounts to £197. 8*s.*?

(853) A vessel containing 21·84375 gallons of water is emptied by a pitcher which contains when full ·078125 gallon. How many times can the pitcher be filled entirely, and what fraction of a pint will it contain when the last quantity of water is poured into it?

(854) Three tramps meet together for a meal: the first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 8 halfpence; how ought they to divide the money?

(855) If the discount on a bill due 8 months hence at $2\frac{1}{2}$ per cent. per annum be £44. 3*s.* 8*d.*, what is the amount of the bill?

(856) A person invests £5012 in the 3 per cent. consols when they are at $89\frac{3}{8}$, the broker's commission being $\frac{1}{8}$ per cent.: find his income, and the diminution of it arising from the income-tax being increased from 4*d.* to 5*d.* in the pound.

(857) Estimate the cost of 9 quarts of strawberries at 4*s.* 8*d.* a gallon, a pint and a half of cream at 20*d.* a quart, 4 lb. of sugar at $6\frac{1}{2}$*d.* a lb., with sixpenny-worth of ice: if 5 boys had to pay for this, and 2 of them paid twice as much as each of the other 3, what change out of a half-sovereign ought the 2 together to receive?

(858) A certain quantity of bran will last 3 rabbits or 5 guinea pigs for a month: how long would it last 9 rabbits and 5 guinea pigs?

(859) A man sells two horses for £100 each, and by so doing gains 25 per cent. on one horse and loses 25 per cent. on the other. What did the horses cost him? Does he gain or lose on the whole?

(860) The difference between the interest and the discount on a certain sum of money for 6 months, at 4 per cent., is £2: what is that sum?

(861) Multiply $59 \cdot 2\dot{1}\dot{2}$ by $4 \cdot 6\dot{5}\dot{1}$.

(862) *A*, *B*, *C*, and *D* enter into trade; *A* started business with £500: after two months *B* and *C* joined him, *B* contri-

buting £700, *C* £650 : after two months more *D* joined with a capital of £800. At the end of the year they had gained £932. 8*s.* : what was the share of each ?

(863) A person has £500 to invest. He lends £250 to a company, which pays him 5 per cent. per annum for his money ; and he buys with the remainder Cape of Good Hope 6 per cent. stock at 105. What income does he receive from each source ? In 5 years he is paid off at *par*; find which will have been the better investment.

(864) The railway fare in France is 6 centimes a kilomètre. If 8 kilomètres be equal to 5 miles, and 2520 centimes be equal to £1, compare this rate with the English parliamentary rate of 1*d.* per mile.

(865) The train which leaves London at 3.10 P.M. arrives at Winchester at 5 o'clock ; and the train which leaves Winchester at 3.30 P.M. arrives in London at 5.42 : when do they pass each other ?

(866) Find the discount on £2034. 5*s.* due $6\frac{6}{17}$ months hence at $5\frac{2}{3}$ per cent. ?

(867) In a decimal coinage of florins, cents, and mils, find at what rate per cent. £1387. 4 fl. will amount in three years to £1574. 6 fl. 9 c. 9 m.

(868) A boat propelled by eight oars which take 27 strokes per minute travels at the rate of $9\frac{3}{8}$ miles per hour : find the rate of a boat propelled by 6 oars, which take 34 strokes per minute, the work done by each oar during one stroke in the former case being a quarter as much again as in the latter.

(869) How many bricks, each 12 in. long, 4 in. wide, and 3 in. thick, will be required to build a wall 18 ft. 8 in. long, 12 ft. 6 in. high, and 9 in. thick, leaving in it a doorway 6 ft. 3 in. high, and 2 ft. 8 in. wide ?

(870) If 3 men and 5 women do a piece of work in 8 days, which 2 men and 6 children, or 5 women and 3 children, can *do in 12 days, find the relative strength of men, women, and children.*

(871) Find, by Practice, the tax on £2005. 6*s.* 8*d.*, at 3*s.* $10\frac{1}{2}$*d.* in the £.

(872) A man sells out £15240 of 5 per cent. stock at 62[illegible], brokerage $\frac{1}{8}$ per cent. ; what money does he receive ?

(873) The debts of a company are seven millions and a half sterling, and its assets are four millions six hundred and eighty-seven thousand five hundred pounds. How much can the company pay in the pound, and what will be the debt on which they can pay £1466. 12*s.* $9\frac{3}{4}$*d.*?

(874) Find the value of 27 yd. 2 ft. 9 in. of one material, if 17 yd. 1 ft. 11 in. of another worth $\frac{1}{7}$ as much again cost £5. 5*s.* 10*d.*

(875) A purse and its contents are worth £3. 6*s.* $5\frac{1}{2}$*d.*, and the value of the contents is to that of the purse as 9 to 2. Find the value of the contents of the purse.

(876) The price of standard silver is 5*s.* 2*d.* per oz., the duty on manufactured silver is 1*s.* 6*d.* per oz., and the cost of workmanship is 1*s.* per oz. If a dozen silver forks are sold for £14. 4*s.* $3\frac{1}{2}$*d.*, and the silversmith makes $8\frac{1}{3}$ per cent. on the transaction, how many ounces of silver are there in one of the forks?

(877) A sum of money amounts in 10 years at $3\frac{1}{2}$ per cent. simple interest to £506. 15*s.* $1\frac{1}{2}$*d.* In how many years will it amount to £703. 16*s.* $6\frac{3}{4}$*d.*?

(878) If I owe £403. 17*s.* 8*d.*, to be paid in 4 months' time, and I pay £150. 10*s.* now, what extension of time ought to be allowed me for the payment of the remainder, reckoning money to be worth 5 per cent. per annum simple interest?

(879) A grocer has 225 lb. of tea, of which he sells 45 lb. at 4*s.* 6*d.* per lb., and only gains $7\frac{1}{2}$ per cent. at this price. He now raises the price so as to gain 10 per cent. on the whole outlay. What is the price when raised?

(880) *A* and *B* barter: *A* has 27 tons of coals worth £1. 2*s.* 6*d.* a ton, but insists on having £1. 5*s.* a ton: *B* has hops worth £2. 14*s.* a pocket, which he raises in price in proportion to *A*'s demand. *A* receives 6 pockets of hops: what cash does he get besides?

(881) Simplify

(*a*) $3 - \cfrac{1}{2 - \cfrac{1}{6 + \frac{10}{5}}}$ divided by $1 + \cfrac{1}{4 + \cfrac{1}{3 - \cfrac{1}{3 + \frac{1}{2}}}}$

(β) $\dfrac{5\frac{1}{3} - 3\frac{1}{2} \div 1\frac{8}{13} + 2\frac{1}{2}}{4\frac{2}{7} - 3\frac{1}{2} + 2\frac{3}{14}}$

(882) Find the square roots of 1706·5161 and $\frac{86 \cdot 9211}{54 \cdot 75}$

(883) Find the Compound Interest on £3245. 6*s*. for 3 years at 5 per cent.

(884) A man buys 2100 fives balls at the rate of 7 for one shilling, and sells them at the rate of 12 for half-a-crown; what is his whole gain and also his gain per cent.?

(885) *A* and *B* run a mile race: at first *A* runs 11 yards to *B*'s 10; but after *A* has run half a mile he tires and runs 9 yards in the time in which he at first ran 11, *B* running at his original rate. Which wins, and by how much?

(886) Which investment gives the better income, Indian 10 per cents. at 215, or Colonial 6 per cents. at 105? Find the income produced by investing £4515, half in each of these securities.

(887) An ordinary train on the Eastern Counties Railway is 1 hour and 57 minutes in travelling between Wymondham and Ely, and the express trains take 54 minutes less. If an express leaves Cambridge at 9 A.M., and arrives in London just as an ordinary train is leaving, which arrives in Cambridge at 2 P.M., find how long the express is in going to London.

(888) A woman buys a certain number of eggs at 21 a shilling, and the same number at 19 a shilling: she mixes them together and sells them at 20 a shilling: how much does she gain or lose per cent. by the transaction?

(889) A man has a certain number of apples: he sells half the number and one more to one person, half the remainder and one more to a second person, half the remainder and one more to a third person, and half the remainder and one more to a fourth person, by which time he has disposed of all that he had. How many had he?

(890) A room is 20 feet 10 inches long, and 16 feet broad: what length of carpet 2 feet wide will it require, so as to leave a margin of 1 foot in width uncarpeted all round the room?

(891) Find the discount on £1344. 8*s*. due 5 years hence at 4 per cent.

(892) Reduce 72·35621 to a vulgar fraction, and test your result.

(893) A square box whose depth is 10 inches is in cubic content 3 cub. ft. 106 in.; find the length of its side.

(894) A stream, which flows at a uniform rate of $1 \cdot 109$ miles an hour, is 20 yards wide, the average depth at a certain ferry from bank to bank being 6 feet: how many gallons pass the ferry in a minute? N.B. Each gallon contains approximately $277\frac{1}{4}$ cubic inches.

(895) A person has £1170 stock in a 4 per cent. stock; he sells out at 90, and buys 6 per cent. stock at $108\frac{1}{3}$: find the change in his income.

(896) A bag contains a certain number of sovereigns, three times as many shillings, and four times as many pence; and the whole sum in the bag is £280: find how many sovereigns, shillings, and pennies it contains?

(897) The road between two towns A and B, distant 15 miles, goes over a hill whose summit is 3 miles from A. Two pedestrians set out at the same time from A and B, the former going 4 miles an hour uphill, and $5\frac{1}{4}$ down, the latter $3\frac{1}{2}$ uphill, and $4\frac{1}{2}$ down: how far will the slower one have to walk when the first arrives at his journey's end?

(898) If 8 men and 5 boys can reap 29 acres in 3 days, and 6 men and 7 boys can reap 50 acres in 6 days, how long will it take 3 men and 6 boys to reap 15 acres?

(899) A grocer buys some tea at 4*s.* in the pound and some at 5*s.* 6*d.*: in what proportion must he mix them, that when he sells the tea at 6*s.* per pound he may be making a profit of 20 per cent.?

(900) Which is the better investment, the New Zealand 5 per cents. at 98, or the Victoria Government 6 per cents. at $113\frac{1}{2}$?

(901) Simplify

$$\frac{4\frac{5}{6} + 1\frac{11}{45} - 5\frac{11}{18}}{6\frac{1}{7} \times 3\frac{1}{2} - \frac{2\frac{1}{3}}{\frac{2}{7}} \times 1\frac{2}{7} + 1\frac{3}{5}}, \quad \frac{5}{6\frac{7}{8}} \times \left(1\frac{4}{7} \times 5\frac{1}{4}\right) + \frac{1}{3} + \frac{1}{1\frac{10}{17}}$$

and find their sum.

(902) A poor rate of 1*s.* and an improvement rate of 1*s.* 9*d.* in the pound produce £81. 19*s.*: what is the property assessed at?

(903) Which gives the best return, $3\frac{1}{2}$ per cent. stock at $96\frac{3}{4}$, or 3 per cent. stock at $93\frac{1}{8}$? and what will be the difference on an investment of £5000?

(904) Two boys, *A* and *B*, come into school punctually by their own watches, which are quite right at 9 o'clock on Monday morning. *A*'s watch gains two minutes, and *B*'s watch loses a minute and a half every day: find how much later *B* will be than *A* at Friday afternoon school, 2 P.M.

(905) Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 acres respectively. The first gang works 7 hours in the day, and the latter 8 hours. If the first gang complete their work in 12 days, in how many days will the second gang complete theirs?

(906) When wheat is 15*s.* per bushel, 8 men can be fed for 12 days at a certain cost. For how many days can 6 men be fed for the same cost, when wheat is 12*s.* per bushel?

(907) A four-wheeled carriage travels round on a circular railway. The circumferences of the two wheels of the carriage, and of the two circles of rails, are proportional to 6, 7, 7000, 7014. Find the number of revolutions made by each of the four wheels in a complete circuit.

(908) If either 5 oxen or 7 horses will eat up the grass of a field in 87 days, in what time will 2 oxen and 3 horses eat up the same?

(909) A tree grows each year one inch less than it did the previous year, and it grew a yard during its first year; the value of the tree at any time is equal to the number of pence in the cube of the number of yards in its height: find what the tree is worth when it has done growing.

(910) Find the alteration in income occasioned by shifting £3200 stock from the 3 per cents. at $86\frac{3}{8}$ to a 4 per cent. stock at $114\frac{7}{8}$; the brokerage being $\frac{1}{8}$ per cent. in each case.

(911) Simplify $(\cdot 006$ of £2. 1*s.* 8*d.* $+ 3\cdot 4\dot{5}\dot{4}$ of £3. 6*s.*$) \times 5\frac{5}{11}$

(912) If *A* can do as much work in 5 hours as *B* can do in 6 hours, or as *C* can do in 9 hours, how long will it take *C* to complete a piece of work, one-half of which has been done by *A* working 12 hours and *B* working 24 hours?

(913) A traveller meets two Arabs in the desert, one of whom has eight loaves and the other five, and the loaves are shared equally by the three. In what proportion should the Arabs be paid?

(914) A squad of 11 boys fired 10 shots each at a target, and scored 286; 20 bull's-eyes were made and 11 misses; how many centres and outers were there? (N.B. A bull's-eye scores 4, a centre 3, and an outer 2.)

(915) The length, breadth and height of a wooden box are 4 ft., $2\frac{1}{2}$ ft., 3 ft. respectively. Find the cost of painting the outside at 1*s.* 3*d.* a square yard.

(916) Find the decimal of a shilling which differs from a penny by less than the millionth part of a shilling.

(917) Alfred owed Robert two-thirds of the amount that Robert owed Charles; and to settle matters, Robert gave 10*d.* to Alfred, who then paid Charles; what did Robert owe Charles?

(918) On July 26th, the sun rises at 4 hr. 15 min. A.M., and sets at 7 hr. 54 min. P.M.; what o'clock is it when the sun is due south?

(919) A person invests in the 3 per cents. so as to obtain $3\frac{1}{4}$ per cent. clear on his investment when there is an income-tax of 6*d.* in the pound, the brokerage being $\frac{1}{8}$ per cent. At what price must he buy? What percentage clear will he obtain if the tax is afterwards raised to 1*s.* 3*d.*?

(920) A train 88 yards long overtook a person walking along the line at the rate of 4 miles an hour and passed him completely in 10 seconds; it afterwards overtook another person and passed him in 9 seconds. At what rate per hour was this second person walking?

(921) Divide 21·3962 by ·527; 213·962 by 52·7; and 213962 by ·000527.

(922) *Multiply* $6\cdot9\dot{5}\dot{4}$ *by* $5\cdot3\dot{0}\dot{3}$.

(923) Find the discount on £3073. 19*s.* 2*d.* due 4 years 10 months hence at $4\frac{3}{4}$ per cent.

(924) A bankrupt can pay 6*s.* 8*d.* in the £: if his assets were £500 more he could pay 7*s.* 4*d.* Find his debts and his assets.

(925) Wishing to pay 18 Kreuzers, I give 1 Thaler, and receive in change 22 Kreuzers, 10 Silver-Groschen, and half a Gulden. I know that 1 Thaler is 30 Silver-Groschen, and 1 Gulden is 60 Kreuzers. Find for me how many Gulden are worth 4 Thalers.

(926) A man walks a certain distance, and rides back in 3 hours 45 min.: he could ride both ways in $2\frac{1}{2}$ hours. How long would it take him to walk both ways?

(927) In a constituency, in which each elector may vote for two candidates, half of the constituency vote for *A*, but divide their votes among *B*, *C*, *D*, *E*, in the proportions of 4, 3, 2, 1; of the remainder, half vote for *B*, and divide their votes among *C*, *D*, *E*, in the proportions of 3, 1, 1; two-thirds of the remainder vote for *D* and *E*, and 540 do not vote at all; find the order on the poll, and the whole number of electors.

(928) An English mile is ·2136 of a German mile. What time will a man, who walks 4 English miles an hour, take to walk a German mile?

(929) A person bought a French watch, bearing a duty of 25 per cent.; and sold it at a loss of 5 per cent.; had he sold it for £3 more, he would have cleared 1 per cent. on his bargain. What had the French maker for the watch?

(930) I have to be at a certain place in a certain time, and I find that, if I walk at the rate of 4 miles per hour, I shall be five minutes too late, if at the rate of 5 miles per hour, I shall be ten minutes too soon. How far have I to go?

(931) On Monday, March 3, 1862, a man commenced to subscribe for a daily penny paper; what had he spent by Saturday, Nov. 16, 1872?

(932) *A*, *B*, *C*, and *D* enter into partnership: *A* and *B* contribute £1390, *B* and *C* £1590, *C* and *D* £1810, *A* and *D* £1610, *A* and *C* £1500: they gain £1152: what is the share of each?

(933) A person sells out £1150 stock (4 per cents.) at 92, and buys $3\frac{1}{2}$ per cents.: he gains £10 per annum by the change: find the price he paid for the latter stock.

(934) On a stream, B is intermediate to and equidistant from A and C; a boat can go from A to B and back in 5 hr. 15 min., and from A to C in 7 hr. How long would it take to go from C to A?

(935) I have a certain sum of money, wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 a penny I shall spend 5*d.* too much, if at the rate of 50 a penny 10*d.* too little. How much have I to spend?

(936) A ditch is being dug at the rate of 81 ft. per day by 54 men: after 13 days' work 8 of them are replaced by boys, and the work goes on for 11 days more, at the end of which the whole length dug is 1889 feet. How much work per day do the boys do?

(937) Find the expense of building a wall 108 yd. long, 3 ft. 8 in. high, 14 in. thick, at £14. 13*s.* 4*d.* per rod; a rod of brickwork consisting of $272\frac{1}{4}$ superficial feet, the work being 14 inches thick.

(938) *A* pays £9. 3*s.* 4*d.* more rates than *B*, their incomes being equal: living in different towns they are rated at 2*s.* and 1*s.* 4*d.* in the pound respectively: what is their income?

(939) A ditch is dug, which in half of its length has an average depth of 4 ft. 6 in., in the remaining half 5 ft. 3 in.: find the cost of excavation, the ditch being 270 yd. long, and 6 ft. 3 in. broad, at 1*s.* 3*d.* per cubic yard.

(940) In running a 3 mile race on a course $\frac{1}{3}$ of a mile round, *A* overlaps *B* at the middle of the 7th round. By what distance will *A* win at the same rate of running?

(941) Divide 225·3213 by ·0267, 2253·213 by ·000267, and 22·53213 by 2·67.

(942) Multiply $65\cdot4\dot{2}\dot{9}$ by $\cdot02\dot{7}$.

(943) Simplify $\cdot073\dot{2}$ of 8 ac. 0 ro. 7 po. + ·012625 ac. — ·02 po.

(944) The sum of the ages of *A* and *B* is now 80 years, and their ages 10 years ago were as 7 to 5. Find their present ages.

(945) If 201 men and 335 boys do in 210 days the same amount of work that 70 men and 96 boys do in 665 days: compare the average daily work done by each man with that done by each boy.

(946) *A* gives away in charity $\frac{1}{8}$ of his income, and pays $\frac{1}{10}$ of it in rates and taxes; with these deductions he has £473. 13*s.* 1*d.* left. What is his gross income?

(947) The incomes of *A* and *B* are in the proportion of 4 and 5; *B* pays £9. 1*s.* 3*d.* in income-tax at 5*d.* in the pound. What is *A*'s income?

(948) Two boats row a race over a straight course 1 mile 995 yards long, their rates of speed being 12 miles and $11\frac{23}{29}$ miles an hour respectively. Assuming that sound travels at the rate of 1140 feet in a second, find how much the faster boat will be ahead of the other when the sound of the gun fired at starting is heard at the winning-post.

(949) At what o'clock will a train, which leaves London for Rugby at 2.45 P.M., and goes at the rate of 41 miles an hour, meet a train which leaves Rugby for London at 1.45 P.M., and goes at the rate of 25 miles an hour, the distance between London and Rugby being 80 miles?

(950) A clock, which was 12 minutes fast at a quarter to 11 P.M. on Nov. 28, was exactly right at 11.30 P.M. on the following day. How many minutes was it slow at a quarter to 2 P.M. on Dec. 7?

(951) Find, by Practice, the dividend on £1726. 4*s.*, at 4*s.* $5\frac{3}{4}$*d.* in the £.

(952) Simplify

$$\frac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}, \qquad \frac{3\frac{2}{3}}{4\frac{1}{7}} \times \left(3\frac{5}{8} \times 5\frac{4}{7}\right) - 17\frac{3}{4},$$

and find their sum.

(953) A pond, with vertical sides, whose area is 1 ac. 3 ro. 22 po. 14 yd. $8\frac{1}{2}$ ft., has a square tower in the middle whose sides are 18 feet and 16 feet; how many gallons must be pumped out to reduce the depth 18 inches? (N.B. A gallon measures $277\frac{1}{4}$ cubic inches nearly.)

(954) After paying income-tax at the rate of 4*d.* in the pound, a man has £476. 18*s.* 4*d.* remaining. What sum did his income-tax amount to?

(955) The wages of *A* and *B* together for $22\frac{1}{2}$ days amount to the same sum as the wages of *A* alone for $38\frac{4}{7}$ days. For how many days will this sum pay the wages of *B* alone?

(956) *A*, *B*, and *C* run a 300 yards race, and *A* beats *B* by 20 yards, and *C* by 34 yards. By how many yards can *B* beat *C* in a 100 yards race?

(957) Two boats start to row in a race at 3 o'clock. The winning boat comes in at $6\frac{3}{4}$ minutes past 3, 40 yards ahead of the other. At 4 minutes past 3 the losing boat was 1140 yards from the winning-post. Find the length of the course, and the speed of the winning boat in miles per hour.

(958) A clock, which was 1·4 minutes fast at a quarter to 11 P.M. on Dec. 2, was 8 minutes slow at 9 A.M. on Dec. 7. When was it exactly right?

(959) Two pipes, *A* and *B*, would fill a cistern in 25 minutes and 30 minutes respectively. Both pipes being opened, find when the first pipe must be turned off, that the cistern may be just filled in 15 minutes.

(960) A man, having lived at the rate of £500 a year for 6 years, finds himself in debt, and reduces his expenditure to £450. He is out of debt in 4 years. What is his income?

(961) Multiply $43{\cdot}13\dot{8}$ by $2{\cdot}3\dot{4}\dot{8}$ to three places of decimals.

(962) Divide 257·153 by ·00151, 2·57153 by ·0151, and ·0257153 by ·0000151.

(963) In a certain sea-side borough the following rates are levied: Improvement Rate 2*s.* 8*d.* in the pound, Lighting Rate 1*s.*, Water Rate 3*s.* 4*d.*, General Sewers Rate 5*d.*, Special Sewers Rate 3*d.*, Poor's Rate 1*s.* 6*d.*, General Purposes Rate 4*s.*, various other rates amounting to 2*s.* $10\frac{1}{2}$*d.* in the pound; the total paid on a house is £32. 1*s.* 8*d.*; determine the rateable value of the house.

(964) A rectangular cistern, whose length is equal to its breadth, is $5\frac{5}{7}$ feet deep, and contains 5 tons of water. If a cubic foot of water weighs 1000 oz., find the dimensions of the cistern.

(965) A fortified town contains provisions enough to support the population and garrison for 90 days. An army $2\frac{1}{3}$ times more numerous than the occupants of the town is intercepted and compelled to take refuge within its walls. How long will the store of provisions last?

(966) A person invests £2700 in $3\frac{1}{2}$ per cent. stock, so as to receive an income of 100 guineas: what is the price of the stock?

(967) The sum, which will pay *A*'s wages for $61\frac{1}{4}$ days, will pay *B*'s wages for $81\frac{2}{3}$ days. For how many days will it pay the wages of *A* and *B* together?

(968) *A* can beat *B* by 5 yards in a 100 yards race, and *B* can beat *C* by 10 yards in a 200 yards race. By how much can *A* beat *C* in a 400 yards race?

(969) Two pipes, A and B, would fill a cistern in 25 minutes and 30 minutes respectively. Both are opened together, but at the end of $8\frac{2}{5}$ minutes the second is turned off. In how many minutes will the cistern be filled?

(970) A man for 5 years spends £40 a year more than his income. If he, at the end of that time, reduce his expenditure 10 per cent., in 4 years he will have paid off his debts and saved £120. Find his income.

(971) Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4} + \left(5\frac{1}{2} + \dfrac{\cdot 24 + \cdot 5\dot{3}}{2\cdot 2 - \cdot 6\dot{4}}\right)$

(972) A bag contains sixpences, shillings, and half-crowns; the three sums of money expressed by the different coins are the same; if there are 102 coins in the bag, find the number of sixpences, shillings, and half-crowns.

(973) Assuming that the circumference of a circle is to its diameter as 22 to 7, and that the circumference of the earth is to its diameter as 160 mètres to 167 feet, determine to five places of decimals the ratio of a mètre to a foot.

(974) At the siege of Sebastopol it was found that a certain length of trench could be dug by the soldiers and navvies in 4 days, but that when only half the navvies were present, it required 7 days to dig the same length of trench. Show that the navvies did six times as much work as the soldiers.

(975) The interest on a certain sum of money for two years is £71. 16*s*. $7\frac{1}{2}$*d*., and the discount on the same sum for the same time is £63. 17*s*., simple interest being reckoned in both cases. Find the rate per cent. per annum, and the sum.

(976) If 100 lb. of tea be bought at 4*s*. 4*d*. and sold at 5*s*. per lb., and 100 lb. of sugar bought at 6*d*. and sold at 7*d*. per lb. what profit per cent. will be realised on the whole outlay?

(977) *A* barters some sugar with *B* for flour, which is worth 2*s.* 3*d.* per stone, but uses a false stone-weight of $13\frac{1}{2}$ lb.; what value should *B* set upon his flour, that the exchange may be fair?

(978) If 5 men and 7 boys can reap a field of corn of 125 acres in 15 days, in how many days will 10 men and 3 boys reap a field of corn of 75 acres, each boy's work being half that of a man?

(979) If 8000 mètres be equal to 5 miles, and if a cubic fathom of water weigh six tons, and a cubic mètre of water 1000 kilogrammes, find the ratio of a kilogramme to a pound avoirdupois.

(980) Find to three decimal places the cube root of 267.

(981) Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{[illegible]} \text{ of } \frac{5}{[illegible]} \div 10\frac{1}{[illegible]}} \times \frac{2}{5}$ of $\dfrac{1\frac{1}{2} [illegible]}{12\frac{7}{[illegible]} [illegible]}$

(982) A person has £10666. 13*s.* 4*d.* invested at 5 per cent.; he saves each year one-quarter of his income, which he invests at the same rate; what will his income be after 3 years?

(983) There is three times as much sea as land on the earth's surface, and three-fourths of the whole land is in the northern hemisphere; how much of the northern hemisphere is land, and how much water? Also how much of the southern hemisphere is land, and how much water?

(984) Ash saplings after five years' growth are worth 1*s.* 3*d.*, and increase in value 1*s.* 3*d.* each year afterwards. Each is allowed 40 sq. yd. of ground, and they are cut after 20 years' growth; what will then be the value of an acre?

(985) Four apples are worth as much as five plums, three pears as much as seven apples, eight apricots as much as fifteen pears, and five apples sell for twopence: I wish to buy an equal number of each of the four fruits, and to spend an exact number of pence; find the least sum I can spend.

(986) A boy of 18 can run 100 yd. in $10\frac{1}{2}$ sec., and a boy of 13 in 13 sec. How much start in distance must the elder give the younger in order that they may run a dead heat?

(987) A usurer borrows £1000 at $4\frac{1}{2}$ per cent., and lends half of it at 10 per cent., half at 15 per cent. What does he gain in $2\frac{1}{2}$ years?

(988) A room is 21 ft. long, 15 ft. 6 in. wide, 10 ft. high; it contains 3 windows, the recesses of which reach to the ceiling, and are 4 ft. 6 in. wide; there are in it 4 doors, each 6 ft. 6 in. high and 3 ft. 3 in. wide; the fire-place is 6 ft. wide and 4 ft. high; a skirting 1 ft. 8 in. deep runs round the walls; find the expense of papering the room at 9*d.* a square yard.

(989) If the discount on £678. 8*s.*, which is due at the end of a year and a half, be £38. 8*s.*, what is the rate per cent. at simple interest?

(990) *A* and *B* barter: *A* has 59 sheep worth £2. 5*s.* each, but insists on £2. 12*s.* 6*d.* each: *B* has beer worth 1*s.* 6*d.* a gallon: how much must he raise his price so that *A* gets £98. 3*s.* 6*d.* and 18 barrels of beer?

(991) Find, by Practice, the cost of 5 tons 12 cwt. 2 qr. 14 lb. at £1. 6*s.* 8*d.* per ton.

(992) Find the number of miles in the radius of the earth, having given that it is the least number which is divisible by 2, 3, 4, 5, 6, 8, 9, 10, 11, and 12.

(993) The imperial gallon contains 277·27 cubic inches, and a cubic foot of water at its maximum density weighs 62·42 lb.; find the weight of a pint of water correctly to two places of decimals.

(994) The sum of £177 is to be divided among 15 men, 20 women, and 30 children, in such a manner that a man and a child may receive together as much as two women, and all the women may together receive £60; what will they each respectively receive?

(995) A wine merchant buys 12 doz. of port at 84*s.* a dozen, and 60 doz. more at 48*s.* a dozen; he mixes them, and sells the mixture at 72*s.* per dozen. What profit per cent. does he make?

(996) An Australian squatter bought 160 ewes: if one-half have one lamb, three-eighths two lambs, and the rest none on a yearly average, and of these three-fifths are males, how many sheep will he have at the end of the third lambing-season, if the lambs are allowed to breed when two years old?

(997) A farmer rents a piece of land for £120 a year. He lays out £625 in buying 50 bullocks. At the end of the year he sells them, having expended £12. 10*s.* in labour. How much per head must he gain by them, in order to realise his rent and expenses, and 10 per cent. upon his original outlay?

(998) A merchant in London owes another in Petersburg a debt of 460 rubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 23 francs for £1, and between Paris and Petersburg 2 francs for 1 ruble. The remittance is delayed until the rates of exchange are 24 francs for £1, and 3 francs for 2 rubles. What does the broker gain or lose by the transaction?

(999) A mixture of soda and potash, dissolved in 2540 grains of water, took up 980 grains of aqueous sulphuric acid, and the weight of the compound solution was 4285 grains. Find how much potash and how much soda the mixture contained, assuming that aqueous sulphuric acid unites with soda in the proportion of 49 grains to 32, and with potash in the proportion of 49 to 48.

(1000) Two houses are building: the first takes half as long again as the second, twice as many men are employed, the wages are one-third per hour higher, and they work six full days a week, and ten hours a day; the others work only eight hours a day, and at the rate of four full days and two half-days in a week: the cost of the workmen's wages in the second is £500, and this is three-eighths of the entire expense. What will be the entire cost of the first house?

TABLES OF MEASURES.

Measures of Money.

4 farthings make 1 penny.
12 pence make 1 shilling
20 shillings make 1 pound.

Measures of Time.

60 seconds make 1 minute.
60 minutes make 1 hour.
24 hours make 1 day.
7 days make 1 week.
For months and years, see page 130.

Measures of Length.

12 inches make 1 foot.
3 feet make 1 yard.
5½ yards make 1 pole, rod, or perch.
40 poles make 1 furlong.
8 furlongs make 1 mile.
3 miles make 1 league.

Note.—6 feet make 1 fathom.

Cloth Measure.

2¼ inches make 1 nail.
4 nails make 1 quarter.
4 quarters make 1 yard.
5 quarters make 1 ell.

MEASURES OF SURFACE.

144	square inches	make	1 square foot.
9	square feet	make	1 square yard.
30¼	square yards	make	1 square pole, rod, or perch.
40	square poles	make	1 rood.
4	roods	make	1 acre.

NOTE.— 1 acre = 4840 square yards.
640 acres = 1 square mile.

CHAIN MEASURE.

22 yards	make	1 chain.
100 links	make	1 chain.
10 square chains	make	1 acre.

MEASURES OF SOLIDITY.

1728 cubic inches	make	1 cubic foot.
27 cubic feet	make	1 cubic yard.

MEASURES OF CAPACITY.

2 pints	make	1 quart.
4 quarts	make	1 gallon.
2 gallons	make	1 peck.
4 pecks	make	1 bushel.
8 bushels	make	1 quarter.

NOTE.—1 barrel of beer contains 36 gallons.

TROY WEIGHT.

24 grains	make	1 pennyweight.
20 pennyweights	make	1 ounce.
12 ounces	make	1 pound.

AVOIRDUPOIS WEIGHT.

16 drachms	make	1 ounce.
16 ounces	make	1 pound.
14 pounds	*make*	*1 stone.*
28 pounds	*make*	*1 quarter.*
4 quarters	make	1 hundredweight.
20 hundredweight	make	1 ton.

The pound Avoirdupois contains 7000 grains Troy.

Apothecaries' Weight.

1. *Measures of Weight.*

437½ grains	make	1 ounce.
16 ounces	make	1 pound

The grain is the same as the grain Troy.
The ounce is the same as the ounce Avoirdupois.

20 grains	make	1 scruple.
60 grains	make	1 drachm.

2. *Measures of Capacity.*

60 minims	make	1 fluid drachm.
8 fluid drachms	make	1 fluid ounce.
20 fluid ounces	make	1 pint.
8 pints	make	1 gallon.

ANSWERS.

Ex. (i), p. 5.

(1) Seven; thirteen; forty-five; fifty-nine; three hundred and twenty-six; four thousand, five hundred and seventy-eight.

(2) Ninety; one hundred and ten; two hundred and seven; four thousand, three hundred; four thousand and thirty-six; four thousand, three hundred and six.

(3) Seven hundred and eighty; six hundred and nine; five thousand, three hundred and sixty; two thousand and twenty; one thousand, one hundred and one.

(4) Thirty-six thousand, four hundred and ninety-seven; forty-nine thousand, five hundred and thirty-two; six hundred and fifty-four thousand, three hundred and twenty-one; seven hundred and forty-three thousand, two hundred and sixty-nine.

(5) Forty-five thousand; thirty-two thousand, six hundred; seventy-five thousand, two hundred and thirty; five hundred thousand.

(6) Eight millions, five hundred and seventy-two thousand, nine hundred and fourteen; three millions, four hundred and sixty-nine thousand, two hundred and eighteen; four millions, six hundred and twenty-nine thousand, eight hundred and seventeen.

(7) Nine millions; twenty-nine millions; seven hundred and fifteen millions.

(8) Nine hundred and ten millions, three hundred and seven thousand, two hundred and forty; three hundred and seven millions, four thousand two hundred and five; three hundred and eighty millions, five hundred and three thousand and forty.

(9) Two hundred and forty-three thousand seven hundred and fifty-nine millions, two hundred and sixty-eight thousand three hundred and forty-two; three hundred and seven thousand four hundred and five millions, six thousand, two hundred and seventy.

(10) Four hundred and seventeen billions, two hundred and thirty-five thousand six hundred and eighty-two millions, seven hundred and nineteen thousand, four hundred and thirty-five; two hundred and three billions, fifty-six thousand three hundred millions, seventy-two thousand and ten.

Ex. (ii), p. 6.

(1) 9; 12; 17; 19; 13; 16; 11.

(2) 23; 27; 35; 38; 44; 40; 26; 34.

(3) 67; 75; 62; 83; 74; 92; 68; 95.

(4) 76; 22; 50; 15; 28; 61; 49; 18; 90; 73.

(5) 107; 130; 246; 372; 608; 740; 990.

(6) 836; 747; 410; 913; 750; 384.

(7) 818; 808; 206; 430; 512; 787.

(8) 7845; 9637; 12000; 8400; 6003; 85040.

(9) 5470; 3650; 8780; 1247; 4808.

(10) 6004; 7022; 3500; 9047; 2017; 19402.

(11) 70007; 60060; 14014; 70017; 12303; 16005.

(12) 356728; 640842; 900000; 800040.

(13) 7000000; 4576865; 75806940.

(14) 315000000; 5040000; 8000700; 18000020; 700000002

(15) 315674018003; 35600000520.

(16) 7000000000000; 5800000600047; 8000000043007.

(17) 305005004006003; 53000053053053.

(18) 9000000000009; 9000000000900; 19000000019000; 1000001001101.

Ex. (iii), p. 9.

1.

(1) Twenty-seven. (2) Forty-nine. (3) Sixty-eight.

(4) Seventy-three. (5) Ninety-two.

(6) One hundred and forty-four.

(7) One hundred and sixty-three.

(8) One hundred and ninety-nine.

(9) Six hundred and sixty-four.

(10) One thousand eight hundred and seventy-two.

2.

(1) XXXVII. (2) LIX. (3) LXII.

(4) LXXXVII. (5) XCV. (6) CXXXIX.

(7) CXLV. (8) CLXXIX. (9) DCCCXLVI.

(10) MDCCLXIII.

Ex. (iv), p. 10.

(1)	11; 16; 20; 36.	(2)	98.	(3)	67.
(4)	60.	(5)	1409.	(6)	949.
(7)	738.	(8)	4971.	(9)	23406.
(10)	74338.	(11)	2008.	(12)	3310.
(13)	1671.	(14)	880.	(15)	28493.
(16)	33633.	(17)	28206.	(18)	18526.
(19)	208.	(20)	1163.	(21)	9289.
(22)	12932.	(23)	106384.	(24)	59313.
(25)	284271.	(26)	1450741.	(27)	680077891.
(28)	1843088.	(29)	1979628.	(30)	1184946.
(31)	3782272.	(32)	3476908.	(33)	799819.
(34)	50333150.	(35)	20826857.	(36)	14621293.
(37)	112251.	(38)	764368.	(39)	1825947.
(40)	227656697.				

Ex. (v), p. 14.

(1)	7.	(2)	8.	(3)	19.
(4)	29.	(5)	34.	(6)	54.
(7)	35.	(8)	29.	(9)	45.
(10)	66.	(11)	509.	(12)	82.
(13)	3808.	(14)	2112.	(15)	4228.
(16)	6222.	(17)	61471.	(18)	108.
(19)	2779.	(20)	28828.	(21)	2761.
(22)	381.	(23)	46.	(24)	32.
(25)	2042.	(26)	6457.	(27)	5780.
(28)	51195.	(29)	10999.	(30)	1096.
(31)	18467.	(32)	60023.	(33)	775772.
(34)	1.	(35)	999000.	(36)	99900000.
(37)	9999998999.	(38)	8.	(39)	26.
(40)	610.	(41)	593.	(42)	159.
(43)	619.				

Ex. (vi), p. 19.

(1) 45. (2) 304. (3) 490.
(4) 870. (5) 684. (6) 861.
(7) 9273. (8) 11364. (9) 50; 75; 175; 225.
(10) 635; 1016; 1270; 1397; 2540; 8890.
(11) 9868; 14802; 27137; 29604; 1233500; 17269000.
(12) 336861; 411719; 449148; 1871450000; 2994320000000.

Ex. (vii), p. 18.

(1)	345.	(2)	1073.	(3)	1620.
(4)	1820.	(5)	3000.	(6)	13734.
(7)	8815.	(8)	30086.	(9)	93940.
(10)	1546992.	(11)	7417784.	(12)	579826952.

Ex. (viii), p. 20.

(1)	173432.	(2)	123904.	(3)	409354.
(4)	372302.	(5)	2274048.	(6)	2667640.
(7)	39342154.	(8)	51212122.	(9)	319766614.
(10)	152847420.	(11)	58376823669.		
(12)	348087421500.	(13)	38871923744.		
(14)	3340400440.	(15)	2959990965442.		
(16)	609435012763918.	(17)	13426705851000.		
(18)	703004503.	(19)	3590386740.		
(20)	3454309838.	(21)	49300038124.		
(22)	611103346167.	(23)	14070096216.		
(24)	24259354428.	(25)	248155914760.		
(26)	135755555747.	(27)	249493596792.		

Ex. (ix), p. 21.

(1) 6840. (2) 1909680. (3) 1121111043844000.

Ex. (x), p. 22.

(1)	225.	(2)	576.	(3)	1600.
(4)	3249.	(5)	4761.	(6)	5184.
(7)	7569.	(8)	10000.	(9)	12996.
(10)	56169.	(11)	390625.	(12)	804609.
(13)	622521.	(14)	1331.	(15)	2197.
(16)	15625.	(17)	103823.	(18)	314432.
(19)	804357.	(20)	1000000.	(21)	16974593.
(22)	45118016.	(23)	156590819.	(24)	348913664.
(25)	961504803.				

Ex. (xi), p. 25.

(1) 3. (2) 3. (3) 12. (4) 11.
(5) 14. (6) 14. (7) 24. (8) 103.
(9) 108. (10) 13. (11) 528. (12) 1032.
(13) 56285. (14) 241248. (15) 458097. (16) 7589523.
(17) 2104. (18) 17553. (19) 24000729. (20) 2019.
(21) 56169. (22) 5678094. (23) 4348432. (24) 5072.
(25) 317649. (26) 391525. (27) 39876548. (28) 30207.
(29) 3469805. (30) 68274625. (31) 472304974.
(32) 5642300741. (33) 8462974231. (34) 90807.
(35) 300071. (36) 29970.

Ex. (xii), p. 25.

(1) 3426. (2) 6487. (3) 64008924.

Ex. (xiii), p. 27.

(1) 3826. (2) 241987. (3) 2162558.
(4) 1749864. (5) 1243904. (6) 500603.
(7) 79267440. (8) 396547. (9) 659372.
(10) 444513674545. (11) 694754611. (12) 3007490200467.
(13) 2131962, 1421308, 1065981.
(14) 310218774, 206812516, 155109387.
(15) 13770459132, 9180306088, 6885229566
(16) 9035784, 5816115, 5169880.
(17) 196350840, 122719275, 109083800.
(18) 46913400, 29320875, 26063000.
(19) 1138764, 724668, 664279.
(20) 4224924, 2688588, 2464539.
(21) 962341116, 612398892, 561365651.

Ex. (xiv), p. 28.

(1)	13688871.	(2)	139456322.	(3)	1555094.
(4)	61556570.	(5)	10855676.	(6)	15839856.

Ex. (xv), p. 30.

(1)	3, 9.	(2)	2, 3, 4, 8, 9.	(3)	3, 5, 9, 11.
(4)	2, 3, 5, 10.	(5)	2, 3, 4, 8, 9.	(6)	3, 5, 9.
(7)	2, 4, 8.	(8)	5.	(9)	2, 3, 4.
(10)	2, 3, 4, 5, 8, 9.	(11)	2, 11.		

Ex. (xvi), p. 30.

(1)	2, 3, 3.	(2)	2, 2, 2, 3.	(3)	3, 3, 3.
(4)	2, 2, 2, 2, 2.	(5)	2, 2, 3, 3.	(6)	3, 13.
(7)	2, 3, 7.	(8)	3, 17.	(9)	2, 3, 3, 3.
(10)	3, 19.	(11)	2, 2, 2, 3, 3.	(12)	5, 17.
(13)	7, 13.	(14)	3, 3, 11.	(15)	2, 2, 5, 5.
(16)	3, 5, 7.	(17)	2, 2, 3, 3, 3.	(18)	2, 2, 2, 2, 7.
(19)	2, 2, 3, 11.	(20)	2, 2, 2, 2, 11.		
(21)	2, 2, 2, 2, 2, 3, 3.	(22)	2, 2, 2, 2, 3, 3, 3.		
(23)	3, 5, 5, 7.	(24)	5, 5, 5, 5.		
(25)	3, 3, 3, 3, 3, 3.	(26)	3, 3, 3, 37.		
(27)	2, 2, 2, 2, 3, 3, 3, 3.	(28)	2, 2, 2, 2, 2, 5, 11.		
(29)	2, 2, 2, 2, 2, 2, 2, 3, 3, 5.				

Ex. (xvii), p. 31.

(1)	4858.	(2)	9306.	(3)	147474.
(4)	305892.	(5)	420077.	(6)	1594432.
(7)	3104199.	(8)	11976096.	(9)	32661000.
(10)	4342356.	(11)	48482280.	(12)	10138680000.

Ex. (xviii), p. 32.

(1) 2472.	(2) 452736.	(3) 41798032.
(4) 42370218.	(5) 8642934.	(6) 42765328.
(7) 74232657.	(8) 14287262.	(9) 204074.
(10) 436876.	(11) 3781076.	(12) 784978.
(13) 37296.		

Ex. (xix), p. 33.

(1) 94, rem. 14.	(2) 11860, rem. 36.
(3) 18573, rem. 37.	(4) 878, rem. 22.
(5) 105531, rem. 35.	(6) 844380, rem. 85.
(7) 849, rem. 20.	(8) 2392, rem. 134.
(9) 11447, rem. 72.	(10) 965316, rem. 718.
(11) 10005, rem. 3549.	(12) 10002276, rem. 4507.

Ex. (xx), p. 33.

(1) 276, rem. 13.	(2) 36724, rem. 11.
(3) 2378, rem. 9.	(4) 20174, rem. 18.
(5) 28998, rem. 22.	(6) 21074998, rem. 25.
(7) 85629, rem. 23.	(8) 246925, rem. 21.
(9) 7429, rem. 7.	(10) 129295, rem. 33.
(11) 2987635, rem. 19.	(12) 4236, rem. 57.
(13) 423, rem. 72.	(14) 504, rem. 123.
(15) 5687, rem. 207.	

Ex. (xxi), p. 35.

(1) 1.	(2) 472369.	(3) 624.	(4) 3012.

Examination Papers. (Page 35.)

(A)

(1) Four millions, two hundred and thirty-seven thousand, four hundred and ninety-six; 653812.

(2) 196181. (3) 7829.

(4) 4253111; 15362894. (5) 935977; 7429.

(B)

(1) 25257630; four hundred and two millions, fifty thousand four hundred and seven.

(2) 16992009. (3) 26438313; 99914800.

(4) 338091, rem. 53. (5) 1175427; 130603.

(C)

(1) Ten thousand and ten millions, two hundred and one thousand, four hundred and one; 1023001; 1001122440?; 2046002.

(2) 1546478344; 15779188816. (3) 2237069, rem. 11.

(4) 31405999. (5) 5226, rem. 33.

(D)

(1) 1888. (2) 7482229, rem. 93.

(3) 2, 2, 2, 7; 2, 3, 13; 2, 3, 19. (4) 12000590.

(5) 999899.

(E)

(1) 65299476, rem. 5346. (2) 88652792964.

(3) 2, 2, 2, 5; 2, 3, 3, 5; 2, 3, 3, 7.

(4) xxiv; xlvii; clxxviii. (5) 12000.

Ex. (xxii), p. 37.

(1) 2. (2) 6. (3) 20. (4) 18. (5) 48.
(6) 7. (7) 3. (8) 16. (9) 16. (10) 3.

Ex. (xxiii), p. 38.

(1) 48. (2) 32. (3) 3. (4) 3. (5) 3453.
(6) 36. (7) 936. (8) 355. (9) 237. (10) 2345.

Ex. (xxiv), p. 39.

(1) 4. (2) 2. (3) 73. (4) 29. (5) 41 (6) 37

Ex. (xxv), p. 40.

(1) 54. (2) 2376. (3) 2532. (4) 9555.
(5) 17000. (6) 85800. (7) 23400. (8) 16128.
(9) 31759.

Ex. (xxvi), p. 41.

(1) 360. (2) 1320. (3) 288. (4) 5040.
(5) 36036. (6) 27324. (7) 3570. (8) 2340.
(9) 27720. (10) 228150.

Examination Papers. (Page 41.)

I.

(1) 9242002. (2) 1548913132560. (3) 143.
(4) 11088. (5) 67219841.

II.

(1) 590895546. (2) 50951, rem. 31. (3) 25.
(4) 198. (5) 83772.

III.

(1) 1182. (2) 1267, rem. 54.

(3) Ten millions, twenty thousand and thirty; ten billions, twenty thousand and thirty millions, forty thousand and fifty.

(4) 96. (5) 672.

IV.

(1) 1457. (2) 1834. (3) 19.

(4) $13 \times 17 \times 19 \times 21$. (5) 1302.

V.

(1) 13. (2) 595259. (3) 2003. (4) 936.

(5) 270728547466; two hundred and seventy thousand seven hundred and twenty-eight millions, five hundred and forty-seven thousand, four hundred and sixty-six.

VI.

(1) 25031007; one hundred and seventy-six millions, five hundred thousand three hundred and one.

(2) 470046066583; four hundred and seventy thousand and forty-six millions, sixty-six thousand five hundred and eighty-three.

(3) 56725. (4) 40320. (5) 1035.

VII.

(1) Three groups of ten units and four single units.

(2) 201096342; 3702. (3) 5844, rem. 114. (4) 26196.

(5) 2, 2, 7; 3, 3, 5; 3, 3, 3, 5; 3, 3, 3, 3, 3.

VIII.

(1) 33363727089; 1011022033; one thousand and eleven millions, twenty-two thousand and thirty-three.

(2) 3596, rem. 52. (3) 1312500000.

(4) 3, 29; 2, 2, 2, 2, 11; 2, 2, 2, 2, 2, 2, 3, 3. (5) 4.

IX.

(1) 6464064064; 101001001; one hundred and one millions, one thousand and one.

(2) 7, rem. 155. (3) 203. (4) 3 and 6.

(5) 13800.

Ex. (xxvii), p. 47.

(1) $\frac{3}{10}$ (2) $\frac{9}{35}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$ (5) $\frac{3}{4}$

(6) $\frac{7}{8}$ (7) $\frac{13}{15}$ (8) $\frac{29}{517}$ (9) $\frac{211}{347}$ (10) $\frac{14}{23}$

Ex. (xxviii), p. 48.

(1) $\frac{21}{28}, \frac{20}{28}$ (2) $\frac{24}{54}, \frac{15}{54}, \frac{14}{54}$ (3) $\frac{231}{385}, \frac{220}{385}, \frac{210}{385}$

(4) $\frac{100}{240}, \frac{156}{240}, \frac{51}{240}, \frac{38}{240}$ (5) $\frac{408}{714}, \frac{630}{714}, \frac{364}{714}, \frac{455}{714}$

(6) $\frac{30}{90}, \frac{54}{90}, \frac{15}{90}, \frac{25}{90}$ (7) $\frac{324}{1080}, \frac{200}{1080}, \frac{84}{1080}, \frac{33}{1080}$

Ex. (xxix), p. 49.

The fractions are arranged in *descending* order.

(1) $\frac{4}{5}, \frac{9}{13}, \frac{3}{7}$ (2) $\frac{5}{6}, \frac{7}{9}, \frac{12}{17}$ (3) $\frac{13}{15}, \frac{9}{11}, \frac{17}{21}$

(4) $\frac{7}{40}, \frac{11}{70}, \frac{3}{20}$ (5) $\frac{7}{33}, \frac{9}{43}, \frac{11}{53}$ (6) $\frac{5}{34}, \frac{7}{51}, \frac{2}{17}$

Ex. (xxx), p. 50.

(1) $\frac{19}{35}$ (2) $\frac{37}{42}$ (3) $\frac{13}{21}$ (4) $\frac{58}{63}$ (5) $\frac{125}{156}$

(6) $\frac{31}{32}$ (7) $\frac{1018}{1485}$ (8) $\frac{3043}{3465}$ (9) $\frac{2231}{2880}$

Ex. (xxxi), p. 51.

(1) $\frac{1}{30}$ (2) $\frac{48}{133}$ (3) $\frac{1}{156}$ (4) $\frac{48}{119}$ (5) $\frac{1}{756}$

(6) $\frac{13}{418}$ (7) $\frac{218}{1911}$ (8) $\frac{1}{450}$ (9) $\frac{1061}{7059}$

Ex. (xxxii), p. 53.

(1) $\frac{5}{21}$ (2) $\frac{135}{308}$ (3) $\frac{2}{5}$ (4) $\frac{98}{165}$ (5) $\frac{9}{76}$

(6) $\frac{275}{1164}$ (7) $\frac{1287}{2320}$ (8) $\frac{32}{405}$ (9) $\frac{50}{243}$

Ex. (xxxiii), p. 54.

(1) $\frac{4}{7}$ (2) $\frac{5}{6}$ (3) $\frac{5}{12}$ (4) $\frac{12}{29}$ (5)

(6) $\frac{49}{54}$ (7) $\frac{29}{77}$ (8) $\frac{5}{6}$ (9) $\frac{20}{27}$

Ex. (xxxiv), p. 56.

(1) $\frac{67}{9}$	(2) $\frac{1096}{47}$	(3) $\frac{6277}{29}$	(4) $\frac{173019}{1000}$
(5) $42\frac{7}{16}$	(6) $3\frac{477}{1000}$	(7) $31\frac{46}{137}$	(8) $928\frac{55}{71}$

Ex. (xxxv), p. 58.

(1) $1\frac{16}{119}$	(2) $1\frac{2}{5}$	(3) $1\frac{2965}{3132}$	(4) $65\frac{5}{9}$
(5) $43\frac{1}{3}$	(6) $178\frac{5}{7}$	(7) $5\frac{7}{12}$	(8) $12\frac{953}{1155}$
(9) $38\frac{2}{15}$	(10) $2\frac{8}{35}$	(11) $8\frac{41}{72}$	(12) $\frac{163}{182}$

Ex. (xxxvi), p. 59.

(1) $27\frac{3}{5}$	(2) 744.	(3) $718\frac{3}{4}$	(4) $\frac{16}{35}$
(5) $1\frac{1}{3}$	(6) 4.	(7) $2\frac{3}{4}$	(8) 26.

Ex. (xxxvii), p. 62.

(1) $1\frac{1}{23}$	(2) $\frac{77}{206}$	(3) $\frac{12}{95}$	(4) $\frac{165}{254}$
(5) $1\frac{7}{19}$	(6) $\frac{23}{64}$	(7) $\frac{26}{73}$	(8) $1\frac{7}{23}$
(9) $2\frac{28}{31}$	(10) $\frac{12}{19}$		

Ex. (xxxviii), p. 63.

(1) $\frac{21}{25}$	(2) $\frac{2}{11}$	(3) $1\frac{4}{19}$	(4) $\frac{3}{5}$
(5) $5\frac{37}{41}$	(6) $\frac{1}{2}$	(7) $\frac{685}{756}$	(8) $20\frac{103}{120}$
(9) $1\frac{11}{108}$	(10) $1\frac{11}{17}$	(11) $7\frac{7}{20}$	(12) $\frac{3}{4}$

Ex. (xxxix), p. 63.

(1) $1\frac{43}{88}$	(2) $1\frac{6}{35}$, $\frac{20}{63}$	(3) $\frac{1}{9}$, $6\frac{2}{3}$
(4) $\frac{13}{37}$, $\frac{111}{13}$, 3.	(5) $4\frac{1}{5}$	(6) $11\frac{11}{12}$, 20.
(7) $\frac{15}{284}$, $\frac{1}{4}$	(8) 3.	(9) $7\frac{1}{9}$
(10) 3.	(11) $10\frac{146}{171}$	(12) $1\frac{1}{2}$, $1\frac{1}{4}$
(13) $\frac{3}{4}$, $\frac{7}{51}$	(14) $\frac{304}{357}$, $1\frac{2}{7}$	(15) 1, $\frac{3}{4}$
(16) 5, 18.	(17) 66.	(18) $14\frac{82}{193}$
(19) $\frac{75}{77}$	(20) 1.	(21) $\frac{1}{2}$
(22) $\frac{226}{299}$	(23) 2.	

Ex. (xl), p. 68.

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) $\frac{3}{8}$
(5) $\frac{243}{100000}$ (6) $\frac{29}{4000000}$ (7) $\frac{74}{5}$ (8) $\frac{20847}{200}$
(9) $\frac{125001}{2500}$ (10) $\frac{100001}{1000}$ (11) ·9. (12) ·37.
(13) ·4579. (14) ·003. (15) 172·95. (16) ·0000059.
(17) ·025679. (18) 3·25793. (19) ·0019.

Ex. (xli), p. 71.

(1) ·7. (2) ·2464. (3) ·0012.
(4) 758·279832. (5) 385·260863. (6) 8741·2062.
(7) 6964·72672. (8) 970·17047.

Ex. (xlii), p. 72.

(1) 51·211. (2) 1·543. (3) 48·2293. (4) ·001.
(5) ·0607. (6) 579·1274. (7) ·0000014.
(8) ·004385. (9) 9·9998. (10) ·00101.

Ex. (xliii), p. 74.

(1) 35·25. (2) 18·9326. (3) ·100345.
(4) ·00041588. (5) 12·08980432. (6) ·9.
(7) 14977·92625425. (8) ·0000465131.
(9) ·0577468988228045. (10) 203·175662750726562.
(11) ·00984126. (12) 1·01. (13) ·00031304.
(14) ·15205806. (15) ·1009981674. (16) 20·570824.
(17) 150·0625.

Ex. (xliv), p. 78.

(1) 12. (2) 14400. (3) ·0013.
(4) 12700. (5) 43·078. (6) 10000.
(7) 430. (8) 147. (9) ·0000002004.
(10) 98·476. (11) ·0065839. (12) 876540000.
(13) ·0000771039. (14) 299846000. (15) ·20162.
(16) 2469300000. (17) 3596. (18) ·00000029.
(19) 1290. (20) 3·59. (21) 457·61.
(22) 76·371. (23) 905741000.

Ex. (xlv), p. 80.

(1) $23\cdot 28125$. (2) $1\cdot 119296875$. (3) $3\cdot 4608$.
(4) $33035\cdot 44$. (5) $\cdot 00192$. (6) $\cdot 0001736$.

Ex. (xlvi), p. 80.

(1) $26\cdot 654875$. (2) $\cdot 0010902475$. (3) $14498\cdot 8$.
(4) $\cdot 000016I4$. (5) $175\cdot 0309875$. (6) $\cdot 0000926$.
(7) $154468\cdot 75$. (8) 25000000. (9) $\cdot 00001$.
(10) $\cdot 0000005005005$.

Ex. (xlvii), p. 82.

(1) $18478\cdot 260$. (2) $\cdot 249$. (3) $\cdot 092$.
(4) $8658146\cdot 964$. (5) $\cdot 095$. (6) $32714\cdot 285$.

Ex. (xlviii), p. 85.

(1) $\cdot 35$. (2) $\cdot 44$. (3) $\cdot\dot{8}5714\dot{2}$. (4) $\cdot\dot{0}\dot{1}$.
(5) $\cdot\dot{0}0\dot{1}$. (6) $\cdot\dot{0}243\dot{9}$. (7) $\cdot\dot{5}2380\dot{9}$. (8) $\cdot 21\dot{6}$.
(9) $\cdot 012\dot{3}\dot{6}$. (10) $2\cdot 3\dot{4}\dot{5}$.

Ex. (xlix), p. 87.

(1) $\frac{2}{3}$ (2) $\frac{3}{11}$ (3) $\frac{5}{111}$ (4) $\frac{347}{1111}$
(5) $\frac{8}{1111}$ (6) $\frac{447}{1111}$ (7) $\frac{6}{11111}$ (8) $\frac{1}{11111}$

Ex. (l), p. 88.

(1) $\frac{421}{990}$ (2) $\frac{1178}{2475}$ (3) $4\frac{19}{75}$ (4) $\frac{211}{49500}$
(5) $53\frac{9}{3700}$ (6) $7\frac{2009}{9990}$ (7) $2\frac{1751}{3300}$

Ex. (li), p. 89.

(1) $15\cdot 8\dot{4}3\dot{0}$. (2) $20\cdot 51\dot{6}6202\dot{5}$. (3) $1\cdot 7\dot{7}8005\dot{2}$.
(4) $\cdot 02\dot{0}6724\dot{9}$. (5) $20\frac{676}{891}$ (6) $\frac{13}{3960}$
(7) $\frac{310}{369}$ (8) $\frac{37}{297}$

Ex. (lii), p. 90.

(1) $\cdot 82008$; $2701\cdot 8$. (2) $49\cdot 070701$; 2401800.

(3) $\cdot 08\dot{6}$; $\cdot 0\dot{3}\dot{4}$ $1\cdot\dot{8}\dot{4}$. (4) $\cdot 01\dot{5}$; $\cdot 1\dot{8}\dot{7}$; $\cdot 8\dot{0}\dot{3}$.

(5) 2040000 ; 36500. (6) 60320000 ; 20500.

(7) $982\cdot 99883$; $270\cdot 3\dot{3}0063396\dot{7}$.

(8) $6\cdot 01$; 60100 ; 601000. (9) $230\cdot 625593$; 501.

(10) $121\cdot 1145$. (11) $14\cdot\dot{8}5714\dot{2}$.

(12) $883\cdot 7304$. (13) $652\cdot 20834$; $\cdot 4$.

(14) $\cdot 83$; 83 ; 830000. (15) $241\cdot 16047$; $22\cdot 3\dot{6}$.

(16) $6\cdot 91$; 69100 ; 691. (17) $\cdot 0359$; 4000000.

(18) $5\cdot 1296$; $3\cdot\dot{6}05\dot{1}$; $\frac{7}{64}$; $\frac{31}{54}$

(19) $\cdot 0000029$; 80000000.

(20) $4\cdot 05536$; $5\cdot 07\dot{9}6\dot{2}$; $\frac{15}{16}$; $\frac{133}{270}$

(21) $8\cdot 424$; $\cdot 0000129$. (22) $1\cdot 965$; 800.

(23) $\cdot 000\dot{6}$. (24) $\cdot\dot{5}$. (25) $\cdot\dot{0}4761\dot{9}$.

(26) $1\cdot 4\dot{4}\dot{0}$. (27) $\cdot 6848$. (28) $2\cdot 9\dot{9}\dot{1}$.

(29) $1\cdot 37070$. (30) $1\cdot 88872$. (31) $73\cdot 52\dot{4}9422\dot{1}$.

(32) $73\cdot 52\dot{4}8639\dot{5}$.

(33) $4\cdot 21\dot{2}9\dot{5}$. (34) $15\cdot\dot{3}2098765\dot{4}$. (35) $10\cdot 08\dot{3}$.

Ex. (liii), p. 95.

(1) 14. (2) 23. (3) 32. (4) 75.

(5) 297. (6) 345. (7) 327. (8) 867.

(9) 440. (10) 835. (11) 6031. (12) 4698.

(13) 23456. (14) 72500. (15) 2031. (16) 739000.

(17) 5678. (18) 437962.

Ex. (liv), p. 96.

(1) $4\cdot 1$. (2) $16\cdot 79$. (3) $\cdot 95$. (4) $\cdot 51$.

(5) $\cdot 25$. (6) $\cdot 027$. (7) $131\cdot 31$. (8) $1\cdot 001$.

(9) $210\cdot 75$. (10) $137\cdot 65$.

Ex. (lv), p. 98.

(1) 4·4721 (2) 5·4772. (3) ·9486. (4) ·3478.
(5) ·4110. (6) ·1264. (7) ·0252. (8) ·0347.
(9) 4·0305. (10) ·9999. (11) ·5025. (12) 6·4833.

Ex. (lvi), p. 99.

(1) $\frac{6}{7}$ (2) $\frac{8}{11}$ (3) $\frac{17}{25}$ (4) $\frac{37}{87}$
(5) $\frac{123}{427}$ (6) $2\frac{1}{4}$ (7) $2\frac{2}{5}$ (8) $1\frac{10}{13}$
(9) $8\frac{1}{9}$ (10) $6\frac{1}{5}$ (11) $4\frac{1}{5}$ (12) $3\frac{3}{7}$
(13) ·7905. (14) ·6454. (15) 2·5298. (16) 3·0822.
(17) 8·7649.

Ex. (lvii), p. 101.

(1) 16. (2) 32. (3) 42. (4) 79.
(5) 85. (6) 64. (7) 34. (8) 73.
(9) 58. (10) 99. (11) 39. (12) 63.

Ex. (lviii), p. 103.

(1) 245. (2) 531. (3) 307. (4) 670.
(5) 128. (6) 179. (7) 463. (8) 103.
(9) 256. (10) 579. (11) 438. (12) 507.
(13) 686. (14) 708. (15) 888. (16) 512.
(17) 4968. (18) 8765.

Ex. (lix), p. 104.

(1) ·73. (2) ·364. (3) 30·02. (4) $\frac{11}{12}$
(5) $\frac{5}{7}$ (6) $1\frac{5}{7}$ (7) $7\frac{2}{5}$ (8) 1·709.
(9) 8·320. (10) ·495. (11) 2·516. (12) ·822.
(13) ·908. (14) ·693. (15) 1·966. (16) 1·473.

Ex. (lx), p. 105.

(1) 27. (2) 45. (3) 6·3. (4) 13.
(5) 54. (6) 8·1.

Ex. (lxi), p. 105.

(1) 32.	(2) 90.	(3) 68 ; 58.	(4) 792.
(5) 37.	(6) 23.	(7) 31 ; 17.	(8) 275 ; 103.
(9) 31.	(10) 1517.	(11) 31.	(12) 49.
(13) 60.	(14) 96.	(15) 240.	(16) 280.
(17) 700.	(18) 120.	(19) 60.	(20) 960.
(21) 1897.	(22) 3456 ; 2304.		(23) 1400 ; 400.
(24) 18.	(25) 42 ; 18.		(26) 80 ; 49.
(27) ·53.	(28) 165·135.		(29) 288.
(30) 600.	(31) $\frac{1}{40}$	(32) 784.	(33) 14.
(34) 3375.	(35) 39.	(36) $13\frac{1}{2}$	(37) 67.
(38) $3\frac{22}{49}$	(39) 23.	(41) 72.	(42) 224.

Ex. (lxii), p. 110.

(1) 13 ; 30 ; 36 ; 47.	(2) 108 ; 270 ; 615 ; 845.
(3) 3456 ; 4800 ; 2762 ; 16535.	(4) 72 ; 58 ; 94 ; 105 ; 163.
(5) 960 ; 1228 ; 4253 ; 14087.	(6) 41880 ; 103870 ; 305973.

Ex. (lxiii), p. 111.

(1) $14\frac{1}{4}$*d.*	(2) $43\frac{1}{4}$*d.*	(3) $49\frac{1}{4}$*d.*
(4) 7*s.* $5\frac{1}{4}$*d.*	(5) 9*s.* $11\frac{3}{4}$*d.*	(6) 15*s.* $6\frac{3}{4}$*d.*
(7) £4. 8*s.* $3\frac{1}{2}$*d.*	(8) £391. 19*s.* $4\frac{1}{4}$*d.*	(9) £564. 19*s.* 7*d*

Ex. (lxiv), p. 112.

(1) 221.	(2) 2240.	(3) 4578.	(4) 3625.
(5) 3762.	(6) 2583.	(7) 177975.	

Ex. (lxv), p. 113.

(1) $1\frac{1}{4}$*d.*	(2) $1\frac{3}{4}$*d.*	(3) $2\frac{3}{4}$*d.*	(4) $3\frac{3}{4}$*d.*
(5) $4\frac{1}{4}$*d.*	(6) $4\frac{3}{4}$*d.*	(7) $5\frac{1}{4}$*d.*	(8) $6\frac{3}{4}$*d.*
(9) $7\frac{1}{2}$*d.*	(10) $8\frac{3}{4}$*d.*	(11) 9*d.*	(12) $9\frac{3}{4}$*d.*
(13) $10\frac{1}{2}$*d.*	(14) $11\frac{3}{4}$*d.*	(15) $14\frac{3}{4}$*d.*	(16) $15\frac{3}{4}$*d.*

(17) $17\frac{1}{4}d.$	(18) $18\frac{3}{4}d.$	(19) $21\frac{3}{4}d.$	(20) $23\frac{1}{2}d.$
(21) 1*s*. 7*d*.	(22) 1*s*. 11*d*.	(23) 2*s*. 3*d*.	(24) 2*s*. 9*d*.
(25) 3*s*. 3*d*.	(26) 3*s*. 7*d*.	(27) 4*s*. 9*d*.	(28) 5*s*. 8*d*.
(29) 6*s*. 2*d*.	(30) 7*s*. 2*d*.	(31) 8*s*. 3*d*.	(32) 8*s*. 9*d*.
(33) 9*s*. 9*d*.	(34) 10*s*. 6*d*.	(35) 11*s*. 2*d*.	(36) 12*s*. 1*d*.
(37) 13*s*. 7*d*.	(38) 14*s*. 11*d*.	(39) 16*s*. 3*d*.	(40) 20*s*. 7*d*.
(41) £1. 7*s*.	(42) £1. 19*s*.	(43) £2. 17*s*.	(44) £3. 19*s*.
(45) £4. 13*s*.	(46) £5. 7*s*.	(47) £6. 9*s*.	(48) £7. 5*s*.
(49) £8. 16*s*.	(50) £9. 18*s*.	(51) £11. 15*s*.	(52) £12. 7*s*.
(53) £12. 18*s*.	(54) £13. 13*s*.	(55) £14. 17*s*.	(56) £17. 5*s*.
(57) £18. 13*s*.	(58) £20. 12*s*.	(59) £21. 17*s*.	(60) £22. 19*s*.

Ex. (lxvi), p. 115.

(1) 10*d*.	(2) $11\frac{1}{2}d.$	(3) $9\frac{3}{4}d.$	(4) 11*d*.
(5) 19*s*. 10*d*.	(6) 19*s*. 3*d*.	(7) 18*s*. 4*d*.	(8) 18*s*. 5*d*.

(9) 13*s*. $10\frac{3}{4}d.$	(10) 18*s*. $1\frac{1}{2}d.$	(11) 18*s*. $6\frac{3}{4}d.$
(12) 18*s*. $10\frac{1}{4}d.$	(13) £21. 15*s*. 1*d*.	(14) £31. 8*s*. 0*d*.
(15) £32. 15*s*. 2*d*.	(16) £31. 12*s*. 9*d*.	(17) £23. 13*s*. $8\frac{1}{4}d.$
(18) £33. 18*s*. $4\frac{3}{4}d.$	(19) £32. 9*s*. $3\frac{3}{4}d.$	(20) £32. 6*s*. 5*d*.
(21) £169. 5*s*. 1*d*.	(22) £181. 18*s*. 6*d*.	(23) £240. 19*s*. 7*d*.
(24) £168. 11*s*.	(25) £200. 17*s*. $11\frac{1}{4}d.$	(26) £220. 6*s*. $9\frac{3}{4}d.$

(27) £3602. 17*s*. 6*d*.	(28) £14801. 0*s*. $2\frac{3}{4}d.$
(29) £21906. 1*s*. 1*d*.	(30) £10841. 11*s*. $7\frac{1}{2}d.$
(31) £34. 17*s*. $7\frac{3}{4}d.$	(32) £341. 2*s*. $3\frac{1}{4}d.$
(33) £403. 15*s*. $9\frac{3}{4}d.$	(34) £3298. 11*s*. $6\frac{1}{4}d.$
(35) £4393. 2*s*. $9\frac{1}{2}d.$	(36) £5218. 9*s*. $3\frac{3}{4}d.$
(37) £5042. 17*s*. 4*d*.	(38) £14203. 13*s*. 2*d*.
(39) £15170. 7*s*. 10*d*.	(40) £46110. 1*s*. 2*d*.
(41) £52597. 13*s*. 11*d*.	(42) £29275. 3*s*. 10*d*.
(43) £33739. 19*s*. 10*d*.	(44) £31713. 12*s*. 10*d*.
(45) £28702. 7*s*. 8*d*.	(46) £49097. 6*s*. 10*d*.
(47) £26061. 13*s*. 2*d*.	(48) £37974. 8*s*. 6*d*.
(49) £36009. 19*s*. 8*d*.	(50) £27924. 2*s*. 6*d*.
(51) £62725. 11*s*. 9*d*.	

Ex. (lxvii), p. 119.

(1) £36. 3s. 5d.	(2) £28. 0s. 10d.
(3) £8. 18s. 11d.	(4) £238. 17s. $10\frac{1}{4}$d.
(5) 14s. $1\frac{3}{4}$d.	(6) £1. 16s. $7\frac{3}{4}$d.
(7) $1\frac{3}{4}$d.	(8) £1519. 12s. $9\frac{1}{2}$d.
(9) £36108. 17s. $6\frac{3}{4}$d.	(10) £1219. 19s. $10\frac{1}{2}$d.

Ex. (lxviii), p. 121.

(1) £1. 9s.	(2) 5s. 10d.	(3) 3s. 9d.
(4) £3. 6s. 6d.	(5) 18s. 8d.	(6) £1. 2s. 1d.
(7) £22. 14s. 8d.	(8) £14. 11s.	(9) £12. 5s.
(10) £5. 18s. $1\frac{1}{2}$d.	(11) £21. 12s.	(12) £15. 15s.
(13) £111. 6s. 8d.	(14) £5. 18s. $1\frac{1}{2}$d.	(15) £122. 9s. 4d.
(16) £104. 12s.	(17) £5. 12s. 6d.	(18) £8. 0s. $10\frac{1}{2}$d.
(19) £3. 14s. 8d.	(20) £48.	(21) £36. 6s.
(22) £39. 7s. 6d.	(23) £11. 3s. 6d.	(24) £68. 5s.
(25) £43. 12s. 8d.	(26) £43. 10s.	(27) £8. 12s.
(28) £6. 2s. 11d.	(29) £12. 5s. $5\frac{1}{4}$d.	(30) £3. 5s. 4d.
(31) £7. 14s.	(32) £96.	(33) £37. 6s. 8d.

Ex. (lxix), p. 123.

(1) £6. 10s. 6d.	(2) £24. 9s. $1\frac{1}{2}$d.
(3) £3. 12s. $5\frac{1}{2}$d.	(4) £5. 18s. 4d.
(5) £29. 13s. 4d.	(6) £34. 1s. $7\frac{1}{2}$d.
(7) £167. 19s. 2d.	(8) £15212. 12s. 6d.
(9) £6189. 5s. $7\frac{1}{2}$d.	(10) £6022. 0s. $7\frac{1}{2}$d.
(11) £8615. 3s. 9d.	

Ex. (lxx), p. 125.

(1) $6\frac{3}{4}$d.	(2) 1s. $5\frac{1}{2}$d.	(3) 2s. $9\frac{1}{4}$d.
(4) 5s.	(5) 4s. $10\frac{1}{2}$d.	(6) 8s. $7\frac{1}{2}$d.
(7) 3s. 7d.	(8) 2s. $11\frac{3}{4}$d.	(9) 3s. 9d.

(10) 6s. $3\frac{1}{4}$d.	(11) 5s. $5\frac{1}{4}$d.	(12) 10s. $6\frac{1}{2}$d.
(13) 4s. 3d.	(14) 7s. 6d.	(15) 9s. 9d.
(16) 11s. 3d.	(17) 14s. 6d.	(18) 16s. 9d.
(19) 17s. 3d.	(20) 19s. 9d.	(21) 4s. $10\frac{1}{2}$d.
(22) 6s. $1\frac{1}{2}$d.	(23) 9s. $8\frac{1}{4}$d.	(24) 11s. 4d.
(25) 7s. $9\frac{1}{2}$d.	(26) 14s. $7\frac{1}{2}$d.	(27) 13s. $0\frac{3}{4}$d.
(28) 9s. 7d.	(29) 5s. 5d.	(30) 9s. 11d.
(31) 15s. $1\frac{1}{2}$d.	(32) £1. 10s. $0\frac{3}{4}$d.	(33) £1. 6s. $5\frac{3}{4}$d.
(34) £1. 10s. $5\frac{1}{2}$d.	(35) 10s. $8\frac{1}{4}$d.	(36) £1. 1s. $3\frac{1}{2}$d.
(37) £1. 10s. $9\frac{3}{4}$d.	(38) £2. 1s. 3d.	(39) £3. 14s. $9\frac{1}{4}$d.

Ex. (lxxi), p. 126.

I.	(1) 7s. $10\frac{1}{2}$d.	(2) £5. 12s. 6d.	(3) 18s. $7\frac{1}{2}$d.
	(4) £3. 19s. 4d.	(5) 12s. $3\frac{1}{2}$d.	(6) £1. 17s. $7\frac{1}{2}$d.
II.	(1) £1. 16s. $6\frac{1}{6}$d.	(2) 4s. 3d.	(3) 5s. 6d.
	(4) 2s. 4d.	(5) 1s. $6\frac{1}{2}$d.	(6) 19s. 10d.
III.	(1) £1. 3s. 2d.	(2) 3s. $4\frac{1}{4}$d.	(3) £1. 4s. $10\frac{1}{4}$d.
	(4) £4. 4s. $3\frac{1}{4}$d.	(5) 6s. $4\frac{3}{4}$d.	(6) £1. 3s. $9\frac{12}{39}$d.
IV.	(1) £1. 10s. $9\frac{1}{2}$d.	(2) £7. 17s. $10\frac{1}{4}$d.	(3) 13s. $8\frac{1}{4}$d.
	(4) $7\frac{1}{4}$d.	(5) 9s. $7\frac{1}{4}$d.	(6) 1s. 2d.
	(7) 2s. $4\frac{1}{4}$d.	(8) £2. 2s. 4d.	(9) 14s. $9\frac{1}{4}$d.
	(10) £1. 8s. 7d.	(11) 11s. $5\frac{41}{125}$d.	(12) 11s. $1\frac{122}{216}$d.
	(13) £4. 6s. $8\frac{53}{126}$d.		

Ex. (lxxii), p. 128.

(1) 100.	(2) 22.	(3) 42.	(4) 79.
(5) 231.	(6) 10.		

Ex. (lxxiii), p. 129.

(1) 3s. $6\frac{3}{4}$d.	(2) 4s. $5\frac{3}{4}$d.	(3) 6s. $6\frac{3}{4}$d.
(4) 1s.	(5) 1s. $9\frac{3}{4}$d.	(6) £24. 16s. 8d.
(7) 10s. 6d.	(8) 14s. 8d.	(9) £13. 6s. 6d.
(10) £48. 1s. $4\frac{5}{7}$d.	(11) £73. 17s. $10\frac{14}{45}$d.	

(12) £157. 10s. 8d. (13) £107. 2s. $7\frac{1}{4}d$. (14) £198. 8s. 5d.
(15) £310. 0s. 10d. (16) £71. 7s. $4\frac{3}{4}d$. (17) £77. 5s.
(18) £1. 15s. $0\frac{3}{4}d$. (19) £8. 3s. $3\frac{1}{4}d$. (20) £8. 12s. 1d.

Ex. (lxxiv), p. 130.

(1) £1412. 11s. 8d. (2) £3226. 0s. 6d.
(3) £28299. 1s. 10d. (4) £31282. 8s. 5d.
(5) £18873. 1s. 6d. (6) £27877. 13s. 3d.

Ex. (lxxv), p. 131.

(1) 22645 sec. ; 61243 sec.
(2) 107020800 sec. ; 544324 min.
(3) 33 da. 17 hr. 27 min. ; 6 hr. 32 min. 56 sec.
(4) 8 da. 14 hr. 13 min. 12 sec. ;
2 da. 0 hr. 24 min. 56 sec.
(5) 118 ; 151 ; 286 ; 120 ; 151.
(6) 76 hr. 34 min. 36 sec. (7) 136 da. 1 hr. 42 min.
(8) 26 wk. 2 da. 2 hr. (9) 22 yr. 293 da. 1 hr.
(10) 77 hr. 3 min. 41 sec.
(11) 250 da. 23 hr. 1 min. 13 sec.
(12) 2 hr. 54 min. 48 sec. (13) 83 da. 17 hr. 47 min.
(14) 6 da. 22 hr. (15) 298 da. 21 hr.
(16) 1 yr. 331 da. 21 hr.
(17) 5 da. 9 hr. 36 min. 46 sec.
(18) 463 hr. 35 min. 5 sec. ; 740 hr. 46 min. 57 sec.
(19) 2 da. 6 hr. 14 min. ; 12 min. 17 sec.

Ex. (lxxvi), p. 133.

(1) 132 in. ; 23166 ft. (2) 446418 in. ; 5499 in.
(3) 13513 po. $3\frac{1}{2}$ yd. ; 306 fur. 0 po. 4 yd. 2 ft. 6 in.
(4) 137 mi. 36 po. 3 yd. 1 ft. ;
1309 mi. 4 fur. 32 po. 4 yd. 2 ft. 8 in.
(5) 107 yd. 1 ft. 8 in. (6) 154 mi. 2 fur. 20 po.
(7) 23 fur. 21 po. $4\frac{1}{4}$ yd. (8) 75 yd. 8 in.
(9) 50 mi. 2 fur. 35 po. (10) 35 po. 3 yd.

(11) 87 yd. 3 in.; 932 mi. 1 fur. 32 po.
(12) 1858 po. 3 yd.; 1783 mi. 3 fur. 5 po. 1 yd.
(13) 6 yd. 1 ft. 2 in.; 5 fur. $6\frac{5}{27}$ po.
(14) 2 yd. 1 ft. $5\frac{1}{2}$ in.; 1 fur. $29\frac{12}{55}$ po.

Ex. (lxxvii), p. 136.

(1) 36751875 sq. in. (2) 44425044 sq. in.
(3) 1210000 sq. yd.; 94608 sq. in.
(4) 4 sq. yd. 55 sq. in.; 3 ac. 28 po. 9 sq. yd.
(5) 1148 po. 2 sq. yd.; 14 po. 10 sq. yd. 7 sq. ft. 110 sq. in.
(6) 284 ac. 2 ro. 25 po.
(7) 163 sq. yd. 7 sq. ft. 91 sq. in.
(8) 112 ac. 3 ro. 33 po. $15\frac{1}{4}$ sq. yd.
(9) 1158 ac. 3 ro. 27 po. (10) 1686 ac. 1 ro. 30 po.
(11) 4760 ac. 0 ro. 28 po. (12) 27 ac. 2 ro. 36 po.
(13) 5 sq. yd. 8 sq. ft. 129 sq. in.
(14) 1 ac. 2 ro. 16 po. (15) 3 ac. 1 ro. 30 po.
(16) 6 sq. yd. 7 sq. ft. 22 sq. in.
(17) 66 ac. 3 ro. 36 po.
(18) 88 ac. 2 ro.; 931 ac. 3 ro. 9 po.
(19) 1 ro. 18 po.; 1 ro. 27 po.

Ex. (lxxviii), p. 137.

(1) 202 cub. ft.; 1175183 cub. in.; 654558 cub. in.
(2) 43 cub. ft. 21 cub. in.; 9 cub. yd. 11 cub. ft. 372 cub. in.
(3) 244944 cub. in.; 149904 cub. in.
(4) 270 cub. yd. 26 cub. ft. 1143 cub. in.
(5) 195 cub. yd. 3 cub. ft. 298 cub. in.
(6) 3558 cub. yd. 10 cub. ft. 284 cub. in.
(7) 8 cub. yd. 20 cub. ft. 1545 cub. in.
(8) 8 cub. yd. 1634 cub. in.
(9) 27 cub. yd. 7 cub. ft. 1472 cub. in.
(10) 707 cub. yd. 1323 cub. in.;
25049 cub. yd. 17 cub. ft. 518 cub. in.
(11) 6 cub. yd. 14 cub. ft. 1029 cub. in.; 8 cub. yd. 24 cub.

Ex. (lxxix), p. 138.

(1) 59 pts.; 109792 pts.

(2) 8 qr. 2 bus. 1 gall. 2 pt.; 47 qr. 4 bus. 3 pk. 1 gall.

(3) 41 gall. 1 pt. (4) 20 bus. 1 pk. 1 gall.

(5) 197 qr. 3 bus. (6) 2 qt. 1 pt.

(7) 3 pk. 1 gall. (8) 6 qr. 7 bus. 3 pk.

(9) 342 qr. 4 bus. 2 pk.; 1115 qr. 4 bus. 1 pk.

(10) 3 qt. 1 pt.; 8 qr. 3 pk.

Ex. (lxxx), p. 139.

(1) 12960 gr. (2) 1680 dwt.; 3420 dwt.; 6185 dwt

(3) 22253 gr.; 42663 gr.

(4) 6 oz. 11 dwt. 1 gr.; 7 lb. 4 oz. 18 dwt.

(5) 12 lb. 6 oz. 19 dwt. 13 gr.; 13 lb. 6 oz. 6 dwt.

(6) 74 lb. 7 oz. (7) 30 oz. 4 dwt. 9 gr.

(8) 87 lb. 7 oz. 12 dwt. 18 gr. (9) 3 oz. 4 dwt. 21 gr.

(10) 7 lb. 9 oz. 13 dwt. (11) 9 oz. 12 dwt. 23 gr.

(12) 89 lb. 5 oz. 8 dwt.; 141 lb. 7 oz. 19 dwt.

(13) 401 oz. 7 dwt. 11 gr.; 148 lb. 9 oz. 5 dwt. 21 gr.

(14) 2 lb. 12 dwt.; 6 oz. 6 dwt. $11\frac{2}{3}$ gr.

(15) 5 dwt. 8 gr.; 2 oz. 19 dwt. 20 gr.

Ex. (lxxxi), p. 140.

(1) 19712 oz.; 4352 dr.; 11200 lb.

(2) 227584 oz.; 33656 lb. (3) 87632 dr.; 8407 lb.

(4) 2 cwt. 2 qr. 17 lb. 11 oz.; 1 ton 13 cwt. 1 qr. 25 lb

(5) 4 cwt. 16 lb. 8 oz.; 8 cwt. 2 qr. 13 lb. 15 oz. 14 dr

(6) 53 lb. 12 oz. 1 dr. (7) 45 qr. 13 lb. 15 oz.

(8) 88 cwt. 2 qr. 8 lb. (9) 150 tons 10 cwt. 1 qr.

(10) 186 cwt. 1 qr. 26 lb.

(11) 1559 tons 9 cwt. 2 qr. 15 lb.

(12) 2 lb. 1 oz. 9 dr. (13) 2 qr. 25 lb. 8 oz.

(14) 1 cwt. 1 qr. 14 lb. (15) 7 tons 19 cwt. 3 qr.

(16) 6 lb. (17) 34 tons 18 cwt. 1 qr. 16 lb.
(18) 120 cwt. 55 lb. 2 oz.; 186 cwt. 93 lb.
(19) 156 cwt. 1 qr. 3 lb.; 390 oz. 13 dr.
(20) 1 cwt. 24 lb.; 16 cwt. 1 qr. $14\frac{7}{10}$ lb.
(21) 1 qr. 1 lb.; 2 tons 3 cwt. 3 qr. 7 lb.

Ex. (lxxxii), p. 142.

(1) $5\frac{1}{4}d.$ (2) $4d.$ (3) $9\frac{3}{4}d.$ (4) $5\frac{3}{4}d.$
(5) $3\frac{1}{4}d.$ (6) $4s.$ (7) $7s.$ (8) $12s.$
(9) $14s.$ (10) $9s.$

Ex. (lxxxiii), p. 143.

(1) 13 cwt. $25\frac{1}{3}$ lb. (2) 13 lb. 14 oz. 12 dr.
(3) 80 mi. 1 fur. 22 po. (4) 679 yd. 1 ft. 6 in.
(5) 166 ac. 3 ro. 32 po. (6) 757 ac. 2 ro. 12 po.
(7) 78 sq. yd. 7 sq. ft. 6 sq. in.

Ex. (lxxxiv), p. 143.

(1) 2 cwt. 4 lb. (2) 10 oz. 5 dr.
(3) 1 mi. 5 fur. 8 po. (4) 3 yd. 6 in.
(5) 5 ac. 3 ro. 4 po. (6) 1 ac. 3 ro. 8 po.
(7) 5 sq. yd. 7 sq. ft. 87 sq. in.

Ex. (lxxxv), p. 144.

(1) 13s. 4d.; £1. 11s. 3d.; £2. 10s. 9d.
(2) 6 fur. 16 po.; 30 po.; 2 qr. 14 lb.
(3) £152. 11s. $0\frac{4}{5}d.$; £1. 13s. 9d.; 2 mi. 2 fur.
(4) £514. 16s.; 15s. 9d. (5) £1. 2s. $10\frac{1}{2}d.$
(6) 13s. 6d. (7) 9 ac. 2 ro. $13\frac{1}{3}$ po.
(8) 16 da. 3 hr. 35 min. (9) 2 fur. 37 yd. $1\frac{1}{2}$ in.
(10) 4 cwt. 2 qr. 12 lb.

Ex. (lxxxvi), p. 146.

(1)	$\frac{1}{46}$	(2)	$\frac{224}{247}$	(3)	$\frac{11}{42}$	(4)	$\frac{158}{211}$
(5)	$\frac{613}{2016}$	(6)	$\frac{5}{8}$	(7)	$\frac{3}{5}$	(8)	$\frac{3}{7}$
(9)	$\frac{3}{11}$	(10)	$\frac{1}{3}$	(11)	$\frac{2880}{17}$	(12)	$\frac{11}{7}$
(13)	$\frac{1}{56}$	(14)	$\frac{112}{9}$	(15)	$\frac{3}{2}$	(16)	$\frac{40}{63}$
(17)	$\frac{1}{20}$	(18)	$\frac{59}{8640}$	(19)	$\frac{30}{67}$	(20)	$\frac{2523}{2200}$
(21)	$\frac{35}{48}$						

Ex. (lxxxvii), p. 148.

(1)	12*s.* 6*d.*	(2)	£15. 5*s.* 6*d.*
(3)	2·3436*d.*	(4)	3 qr. 21 lb.
(5)	12 dr.	(6)	£16. 0*s.* 6*d.*
(7)	1*s.* 5*d.*	(8)	£2. 16*s.* 9·375*d.*
(9)	4*s.* 2*d.*	(10)	£2. 10*s.* 7·6*d.*
(11)	24 lb. 8 oz.	(12)	£7. 16*s.* $2\frac{1}{4}$*d.*
(13)	4 tons 16 cwt. 19·488 lb.		
(14)	£26. 17*s.* $10\frac{23}{30}$*d.*	(15)	£2. 5*s.* 9·8*d.*
(16)	16*s.* 7*d.*	(17)	£1. 14*s.* 8*d.*

Ex. (lxxxviii), p. 150.

(1)	·815625.	(2)	$\cdot\dot{8}4126\dot{9}$.	(3)	$\cdot 133680\dot{5}$.
(4)	·0625.	(5)	·2953125.	(6)	$1\cdot 15\dot{6}$.
(7)	$\cdot 0013\dot{8}$.	(8)	$\cdot 14\dot{6}\dot{9}$.	(9)	$\cdot\dot{7}$.
(10)	$\cdot 3173958\dot{3}$.	(11)	·328125.	(12)	$\cdot 00208\dot{3}$.
(13)	·1875.	(14)	·43.	(15)	14·49.
(16)	·234375.	(17)	2·64.	(18)	1·37744140625.
(19)	$\cdot 002\dot{7}$.	(20)	$1\cdot 43\dot{1}\dot{8}$.	(21)	·3.
(22)	$\cdot 0009187\dot{5}$.	(23)	$2\cdot 44591\dot{6}$.	(24)	$1\cdot 791\dot{6}$.
(25)	·1406....				

Ex. (lxxxix), p. 151.

(1) ·64125. (2) 1*s.* 6*d.*

(3) ·25 ; 59·52. (4) £9. 2*s.* 1*d.*

(5) £21. 10*s.* 10½*d.* (6) $\frac{279}{10}$

(7) ·32265625. (8) £20. 6*s.* 3*d.*

(9) $\frac{139}{36960}$ (10) ·11125.

(11) ·3125. (12) $\cdot172569\dot{4}$.

(13) $\cdot20\dot{4}$. (14) $\cdot23708\dot{3}$.

(15) $\cdot218\dot{3}$. (16) £34. 6*s.* $4\frac{4}{5}$*d.*

(17) £18. 19*s.* 11*d.* (18) 1*s.* $8\frac{1}{5}$*d.*

(19) 250 lb. (20) $\frac{2}{3}$ (21) $29\frac{1}{5}$

(22) 13*s.* 10½*d.* (23) 9*s.* 9¼*d.* ; $\cdot48854\dot{1}\dot{6}$.

(24) 1*s.* 3¾*d.* ; ·065625. (25) ·09765625.

(26) $\cdot632523\dot{1}4\dot{8}$. (27) $\frac{9}{1000}$ (28) $\frac{1}{24}$

Ex. (xc), p. 154.

(1) 72600 m. (2) 6456 m. (3) 45325 m.

(4) 7030 m. (5) 18004 m. (6) 19035 m.

(7) £126. 7 c. 4 m. (8) £823. 2 f. 2 c. 2 m.

(9) £1. 7 f. 4 c. 7 m. (10) 5 m.

(11) £719. 7 f. 3 c. 4 m. (12) £10441. 4 f. 8 c. 2 m.

(13) £11. 6 f. 9 c. 5 m. (14) £8. 8 f. 6 c. 2 m.

Ex. (xci), p. 156.

(1) 5 f. (2) 25 c. (3) 125 m. (4) 5 c.

(5) 25 m. (6) $4\frac{1}{6}$ m. (7) 34 f. (8) 465 c.

(9) £7. 7 f. 2 c. 5 m. (10) £13. 7 f. 6 c. $4\cdot58\dot{3}$ m.

(11) £42. $2\cdot08\dot{3}$ c. (12) £54. 1 f. 5 c. 6·25 m.

(13) $2\frac{2}{5}$*d.* (14) $\frac{24}{25}$ q. (15) £14. 10*s.*

(16) £3. 9*s.* (17) £7. 17*s.* 3·36*d.*

(18) £423. 8*s.* (19) £437. 5*s.* (20) 9*s.* 4·158*d.*

(21) £5. 8*s.* 7 8*d.* (22) £7. 8*s.* 5·622*d.*

Ex. (xcii), p. 159.

(1)	£243. 15*s.* $6\frac{1}{4}$*d.*	(2)	£142. 7*s.* 3*d.*
(3)	£340. 8*s.* $4\frac{1}{2}$*d.*	(4)	£95. 9*s.* $1\frac{1}{4}$*d.*
(5)	£606. 16*s.* $10\frac{1}{4}$*d.*	(6)	£395. 16*s.* 8*d.*
(7)	£1832. 16*s.* $5\frac{1}{2}$*d.*	(8)	£5324. 19*s.* $0\frac{3}{4}$*d.*
(9)	£15214. 7*s.* $9\frac{3}{4}$*d.*	(10)	£8615. 3*s.* 9*d.*
(11)	£3329. 5*s.* 9*d.*	(12)	£4375. 17*s.* $9\frac{1}{2}$*d.*
(13)	£6189. 5*s.* $7\frac{1}{2}$*d.*	(14)	£8061. 7*s.* $3\frac{3}{4}$*d.*
(15)	£6022. 0*s.* $7\frac{1}{2}$*d.*	(16)	£5158. 2*s.* $8\frac{1}{4}$*d.*
(17)	£53761. 15*s.* 10*d.*	(18)	£83720. 19*s.* $5\frac{1}{2}$*d.*
(19)	£61386. 16*s.* 7*d.*	(20)	£169567. 17*s.* $11\frac{1}{2}$*d.*
(21)	£1164. 9*s.* 9*d.*	(22)	£839. 10*s.* $7\frac{1}{2}$*d.*
(23)	£1457. 0*s.* $0\frac{15}{32}$*d.*		

Ex. (xciii), p. 161.

(1)	£99. 5*s.* $11\frac{1}{4}$*d.*	(2)	£310. 3*s.* $2\frac{3}{14}$*d*
(3)	£227. 14*s.*	(4)	£146. 7*s.* $9\frac{23}{56}$*d.*
(5)	£190. 0*s.* $4\frac{13}{56}$*d.*	(6)	£76. 1*s.* $5\frac{1}{7}$*d.*
(7)	£440. 1*s.* $3\frac{13}{14}$*d.*	(8)	£36. 3*s.* $1\frac{31}{56}$*d.*
(9)	£43. 10*s.* $7\frac{3}{7}$*d.*	(10)	£13. 17*s.* $4\frac{7}{8}$*d.*
(11)	£231. 0*s.* $8\frac{112}{121}$*d.*	(12)	£50. 12*s.* $6\frac{1}{32}$*d.*
(13)	£32. 2*s.* $7\frac{1}{2}$*d.*	(14)	£496. 1*s.* $9\frac{27}{128}$*d.*
(15)	£805. 4*s.* $4\frac{1}{2}$*d.*	(16)	£3127. 0*s.* $7\frac{1}{2}$*d.*
(17)	£63. 6*s.* $5\frac{3}{4}$*d.*	(18)	£1066. 1*s.* $6\frac{3}{4}$*d.*
(19)	£311. 8*s.* 0*d.*	(20)	£839. 14*s.* $4\frac{1}{2}$*d.*

Ex. (xciv), p. 162.

(1)	£1. 6*s.* 0*d.*	(2)	£2. 5*s.* $3\frac{1}{4}$*d.*
(3)	£8. 19*s.* $11\frac{3}{4}$*d.*	(4)	£13. 17*s.* $4\frac{3}{4}$*d.*
(5)	£19. 2*s.* $1\frac{3}{4}$*d.*	(6)	£29. 6*s.* $6\frac{1}{2}$*d.*

Ex. (xcv), p. 164.

(1)	6*d.*	(2)	$15\frac{1}{2}$ miles.	(3)	24 days
(4)	$1\frac{1}{2}$*d.*	(5)	306 days.	(6)	6*d.*

Ex. (xcvi), p. 167.

(1)	6.	(2)	$1\frac{1}{2}$	(3)	£6. 19*s.* 9*d.*
(4)	£9555.	(5)	11*s.* 8*d.*	(6)	7*s.* 6*d.*
(7)	£2. 2*s.*	(8)	25.	(9)	£301.
(10)	203 miles.	(11)	288 gr.	(12)	£55.
(13)	£648.	(14)	£1. 8*s.*	(15)	£277. 9*s.* 3*d.*
(16)	£5. 9*s.* 3*d.*	(17)	£31. 10*s.*	(18)	54 days.
(19)	£16. 16*s.*	(20)	4*s.*	(21)	£102. 1*s.* 8*d.*
(22)	£2. 5*s.* 9*d.*	(23)	£17.	(24)	£3. 12*s.*
(25)	£18. 16*s.* 8*d.*	(26)	£11. 2*s.* $4\frac{3}{4}$*d.*	(27)	11*s.* 3*d.*
(28)	6 cwt. 3 lb.	(29)	£1. 19*s.* 3*d.*		

Ex. (xcvii), p. 169.

(1)	£7833. 6*s.* 8*d.*	(2)	£5040.
(3)	$18\frac{7}{10}$	(4)	5*s.* $9\frac{3}{10}$*d.*
(5)	$\frac{14}{165}$	(6)	27 min.
(7)	£236. 8*s.* 6*d.*	(8)	$286\frac{2}{13}$ miles.
(9)	£100.	(10)	£1. 17*s.* 4·128*d.*
(11)	The first.	(12)	$22\frac{1}{2}$ cwt.

Ex. (xcviii), p. 172.

(1)	$3\frac{3}{5}$ hr.	(2)	$13\frac{37}{191}$ days.	(3)	$2\frac{22}{73}$ days.
(4)	$2\frac{2}{3}$ min.	(5)	10 days.	(6)	4 hr.
(7)	18 days.	(8)	$\frac{29}{36}$		

Ex. (xcix), p. 174.

(1) $21\frac{9}{11}$ min. past 4. (2) $32\frac{8}{11}$ min. past 6.

(3) $49\frac{1}{11}$ min. past 9.

(4) $5\frac{5}{11}$ min. and $38\frac{2}{11}$ min. past 4.

(5) $21\frac{9}{11}$ min. and $54\frac{6}{11}$ min. past 7.

(6) $10\frac{10}{11}$ min. and $43\frac{7}{11}$ min. past 11.

(7) $38\frac{2}{11}$ min. past 1. (8) $54\frac{6}{11}$ min. past 4.

(9) $10\frac{10}{11}$ min. past 8.

Ex. (c), p. 175.

(1)	1800 lb.	(2)	5*s*. 6*d*.	(3)	£1. 2*s*. 6*d*.
(4)	£4590.	(5)	£6. 12*s*.	(6)	£260. 12*s*.
(7)	£14. 1*s*. 3*d*.	(8)	£615.	(9)	£250. 10*s*. 6*d*.
(10)	£740.	(11)	7*d*.	(12)	90.
(13)	69.	(14)	17*s*. 6*d*.	(15)	£46. 13*s*. 4*d*.
(16)	$39\frac{19}{459}$ cub. ft.			(17)	£67. 6*s*. 8*d*.
(18)	£196. 8*s*. $1\frac{1}{2}$*d*.			(19)	£602. 2*s*.
(20)	$2\frac{28}{43}$ lb.	(21)	1*s*. $3\frac{8}{723}$*d*.	(22)	£77. 1*s*. $4\frac{4}{5}$*d*.
(23)	£11. 3*s*. 10*d*.	(24)	300.	(25)	£22. 10*s*.
(26)	7 months.	(27)	324.	(28)	£450.
(29)	6 cwt. 1 qr. 14 lb.			(30)	112.
(31)	£611. 3*s*. 4*d*.			(32)	£8. 1*s*. 8*d*.

Ex. (ci), p. 177.

(1)	480.	(2)	£230. 8*s*.	(3)	11268.
(4)	4000.	(5)	£1. 16*s*. 9*d*.	(6)	£18. 4*s*. 1*d*.
(7)	15 cwt.	(8)	£8.	(9)	16.
(10)	20.	(11)	£16. 7*s*. 6*d*.	(12)	27.
(13)	75.	(14)	6 weeks.	(15)	£88. 4*s*. $1\frac{1}{2}$*d*.
(16)	7 weeks.	(17)	4.	(18)	$155\frac{3}{4}$
(19)	10.	(20)	84.	(21)	10 days.
(22)	2 days.	(23)	660.	(24)	£120.

Ex. (cii), p. 181.

(1) £825. (2) £1160. (3) £364. 10*s*.
(4) £2294. 10*s*. (5) £651. 7*s*. (6) £718. 8*s*. 7·68*d*.
(7) £93. (8) £87. 4*s*. 7·9675*d*.

Ex. (ciii), p. 183.

(1) £4. 13*s*. 7·2*d*. (2) £5. 14*s*. 4*d*.
(3) £46. 8*s*. 9$\frac{15}{73}$*d*. (4) £56. 7*s*. 6$\frac{6}{73}$*d*.
(5) 5*s*. 4$\frac{88}{9125}$*d*. (6) £3. 7*s*. 3$\frac{33}{365}$*d*.
(7) £5. 12*s*. 6*d*. (8) £5. 10*s*. 10$\frac{4}{5}$*d*
(9) £86. 2*s*. 5$\frac{55}{73}$*d*. (10) £4. 9*s*. 2$\frac{7189}{7300}$*d*.

Ex. (civ), p. 184.

(1) £1364. 9*s*. (2) £132. 5*s*. 6*d*.
(3) £1728. 9*s*. 9*d*. (4) £233. 12*s*. 2$\frac{1}{2}$*d*.
(5) £43. 3*s*. 7$\frac{1}{2}$*d*. (6) £229. 7*s*. 1$\frac{1}{2}$*d*.

Ex. (cv), p. 185.

(1) £53. 16*s*. 2*d*. (2) £112. 16*s*. 11*d*.
(3) £129. 12*s*. 10*d*. (4) £98. 3*s*. 1*d*.
(5) £564. 13*s*. (6) £1094. 16*s*. 3*d*.

Ex. (cvi), p. 186.

(1) 4$\frac{1}{2}$ 2) 4$\frac{1}{2}$ (3) 4$\frac{1}{3}$
(4) 16 months. (5) 3$\frac{1}{3}$ yr. (6) 3$\frac{3}{4}$ yr.
(7) £375. (8) 3$\frac{3}{4}$ yr. (9) £750. 2*s*. 6*d*.

Ex. (cvii), p. 189.

(1) £102. (2) £152. 5*s*.
(3) £630. 10*s*. (4) £705. 16*s*. 7·35*d*.
(5) £1087. 3*s*. 6·9*d*. (6) £670. 1*s*. 10·084608*d*.
(7) £342. 17*s*. 6·37056*d*. (8) £589. 19*s*. 7·776*d*.
(9) £269. 2*s*. 0·375*d*. (10) £175. 9*s*. 6·90816*d*.
(11) £409. 10*s*. 2$\frac{1}{4}$*d*. (12) £218. 10*s*. 10·896*d*.
(13) £631. 0*s*. 11·68896*d*. (14) £5191. 0*s*. 7·5297*d*.

Ex. (cviii), p. 192.

(1) £4335·305625. (2) £7968·990625. (3) £39021·53216
(4) £504·723175. (5) £4796·8137984.

Ex. (cix), p. 193.

(1) £2706·0804. (2) £6958·16050927734375.
(3) £2913·442946465625.

Ex. (cx), p. 195.

(1) £4600. (2) £2758. (3) £725.
(4) £346. 15*s*. (5) £7850. (6) £1084. 10*s*.
(7) £88. 9*s*. 3*d*. (8) £114. 6*s*. $7\frac{31}{92}$*d*. (9) £756.
(10) £80. 12*s*. 6*d*.

Ex. (cxi), p. 196.

(1) £22. 0*s*. 3*d*. (2) £5600. (3) £10400.
(4) $4\frac{1}{5}$ per cent. (5) £7. 10*s*. $11\frac{1}{4}$*d*. (6) £3. 0*s*. $4\frac{1}{2}$*d*.
(7) £245. (8) £25. 6*s*. 3*d*. (9) £640.
(10) $\dot{4}$ per cent. (11) £502. 13*s*. 4*d*. (12) £32. 10*s*.
(13) £702. 13*s*. 4*d*. (14) £2. 11*s*. (15) 5 per cent.
(16) 9*s*. $7\frac{23}{25}$*d*.

Ex. (cxii), p. 197.

(1) £6000. (2) £274. 1*s*. (3) £7. 10*s*. $10\frac{110}{169}$*d*.
(4) £1050. (5) £19. 1*s*. 4*d*. (6) £17. 1*s*. 11*d*.
(7) £2. 6*s*. (8) £5. 7*s*. 4*d*. (9) £1. 1*s*. 1*d*.
(10) £4. 8*s*. 9*d*. (11) £18. 9*s*. 10*d*.

Ex. (cxiii), p. 199.

(1) 21·25. (2) $738\cdot\dot{5}7142\dot{8}$. (3) $56087\cdot\dot{6}$.
(4) 26·9625. (5) 10·154875.

Ex. (cxiv), p. 200.

(1) £3. 6s. 9d. (2) 15s. $7\frac{1}{2}d$. (3) £350. 14s.
(4) £3. 3s. $11\frac{1}{4}d$. (5) £149. 9s. (6) £22. 12s. 8·01d.
(7) £120. (8) £173. 15s. (9) £211. 14s. 6d.
(10) £163. 15s. 9d. (11) £107. 10s. (12) £86. 19s. 6d.
(13) £130. 18s. 3d. (14) £106. 19s. $4\frac{1}{2}d$. (15) £24. 10s.
(16) £827. 12s. (17) £32. 9s. (18) £72. 17s. 6d.
(19) £5. (20) £9. 10. (21) £11. 11s. 3d.
(22) £12. 19s. $4\frac{1}{2}d$.

Ex. (cxv), p. 202.

(1) 25. (2) £500. (3) £1. 1s. $9\frac{1}{3}d$.
(4) £1. 1s. 1d. per gall. (5) $3\frac{13}{17}$ gain. (6) $33\frac{1}{3}$
(7) 8 per cent. (8) 10 per cent. (9) 5.
(10) 23. (11) he gains £13. 18s. $7\frac{3}{4}d$.
(12) 10. (13) £75 (14) 3s. $7\frac{1}{4}d$. per lb. (15) 4 lb.
(16) 17s. 3d. per gall. (17) 18s. (18) $33\frac{1}{3}$

Ex. (cxvi), p. 206.

(1) £3542. (2) £441. 15s. (3) £523. 16s. 9d.
(4) £2400. (5) £6000. (6) £776. 13s. 4d.
(7) £55. 17s. 6d. (8) £5. 12s. 6d. (9) £21. 1s. $10\frac{1}{2}d$.

Ex. (cxvii), p. 209.

(1) Nothing. (2) £5. 12s. 6d. (3) £104. (4) 90.
(5) £9. (6) $91\frac{1}{4}$ (7) £6. 5s.
(8) £3678. 6s. (9) £196. 13s. 4d. (10) £24960.
(11) £26. 13s. 4d. (12) £4725. (13) $1\frac{4}{5}$

Ex. (cxviii), p. 213.

(1) $\frac{4}{9}$ is the greater. (2) $\frac{10}{41}$ is the greater.
(3) $\frac{4}{7}$ is the greatest, and $\frac{8}{15}$ the least.
(4) 45 : 364. (5) 21
(6) $\frac{10}{21}$ (7) ·048. (8) 28.
(9) $\frac{5}{27}$ (10) ·171 (11) $A : C = 25 : 39$.
(12) A gets £552; B £460; C £345; D £230.

Ex. (cxix), p. 215.

(1) £33 and £27. (2) £250; £375; £875; £1000.
(3) £3300; £2200; £1650; £1320.
(4) 9 cwt. of saltpetre; 1 cwt. $22\frac{2}{5}$ lb. of sulphur; 1 cwt. 3 qr. $5\frac{3}{5}$ lb. of charcoal.
(5) 120 yd.; 160 yd.; 200 yd.
(6) £240 to *A*; £80 to *B*; £320 to *C*.
(7) First 28; second 32; third 40.
(8) *A* £102. 3*s*. 9*d*.; *B* £132. 16*s*. $10\frac{1}{2}$*d*.; *C* £183. 18*s*. 9*d*.
(9) 113; 339; 678; 791. (10) 30.
(11) $57\frac{41}{87}$, $40\frac{20}{87}$, $91\frac{83}{87}$, $10\frac{30}{87}$ (12) *A* 11*s*.; *B* 14*s*. 8*d*.; *C* 29*s*. 4*d*. (13) Men £1. 5*s*.; women 15*s*.; boys 12*s*.
(14) Men £45. 13*s*. 6*d*.; women £45. 13*s*. 6*d*.; children £38. 1*s*. 3*d*.

Ex. (cxx), p. 216.

(1) 1 : 2. (2) 8 : 7. (3) 3 : 4. (4) 24 : 12 : 15.

Ex. (cxxi), p. 217.

(1) *A* £656; *B* £287. (2) *D* £20; *E* £50.
(3) *A* £87. 10*s*.; *B* £120; *C* £202. 10*s*.
(4) £7. 10*s*.; £12; £7. (5) £3. 16*s*. 6*d*.; £3. 11*s*. 3*d*.
(6) *A* £245; *B* £225.

Ex. (cxxii), p. 220.

(1) $7\frac{1}{2}$ mo. (2) $4\frac{7}{12}$ mo. (3) $4\frac{3}{5}$ mo. nearly.
(4) $4\frac{1}{5}$ mo. (5) 28th May.

Ex. (cxxiii), p. 222.

(1) 2 fr. 13 cent. (2) 1760 copeks. (3) 9 fl. 20 kr.
(4) £4 17*s*. 6*d*. (5) 3 fr. 84 cent. (6) I lose 10 per cent
(7) 55. (8) 960. (9) $9\frac{3}{8}$*d*. on £1.
(10) 520·425.

Ex. (cxxiv), p. 227.

(1) 35 sq. ft. (2) 135 sq. ft. (3) $300\frac{3}{8}$ sq. ft.
(4) 12 sq. ft. (5) $452\frac{5}{8}$ sq. ft. (6) 224 sq. ft.
(7) 608 sq. ft. (8) $150\frac{3}{16}$ sq. ft. (9) $402\frac{1}{2}$ sq. ft.
(10) $30\frac{1}{4}$ sq. yd (11) $1387\frac{9}{16}$ sq. yd. (12) $315\frac{1}{16}$ sq. ft.
(13) $870\frac{1}{4}$ sq. ft. (14) 91 sq. ft. 121 sq. in.
(15) $11\frac{1}{9}$ sq. ft. (16) 502 sq. ft. 73 sq. in.
(17) 2232 sq. ft. 81 sq. in. (18) 16 ft.
(19) 7 ft. 5 in. (20) 8 ft. 9 in. (21) 11 yd.
(22) 88 yd. (23) 99 yd. (24) 9 ft.
(25) 16 ft. (26) 103 ft. (27) 405 yd.
(28) 255 yd. (29) 360·5 yd. (30) 163·25 yd. nearly
(31) $5\sqrt{2}$ in. (32) $625\sqrt{2}$ ft.

Ex. (cxxv), p. 229.

(1) $28\frac{8}{9}$ (2) $46\frac{8}{27}$ (3) 67.
(4) 58. (5) $142\frac{52}{81}$ (6) £8. 8*s.*
(7) £11. 7*s.* 4*d.* (8) £22. 6*s.* $7\frac{23}{24}$*d.* (9) £11. 9*s.* 8*d.*

Ex. (cxxvi), p. 231.

(1) 630. (2) 855. (3) $875\frac{7}{8}$
(4) 798. (5) £6. 8*s.* (6) £2. 16*s.* 9*d.*
(7) £6. 13*s.* $2\frac{1}{3}$*d.* (8) £6. 6*s.* $9\frac{9}{10}$*d.*

Ex. (cxxvii), p. 231.

(1) 12*s.* $4\frac{1}{9}$*d.* (2) £67. 16*s.* $8\frac{5}{64}$*d.* (3) 210 ft.
(4) 135 ft. (5) £13. 10*s.* (6) £1. 4*s.* $9\frac{1}{2}$*d.*
(7) 5*s.* 6*d.* (8) 12 ft. (9) 62 yd. 1 ft.
(10) $22\frac{1}{2}$ (11) $17\frac{1}{2}$ ft. (12) 1 ft. $9\frac{1}{3}$ in.
(13) £1. 5*s.* (14) 5952. (15) £2. 5*s.*
(16) £3. 7*s.* 6*d.* (17) 429 yd.; 715 yd.
(18) $10511\frac{1}{9}$ sq. yd.; $2955\frac{5}{9}$ sq. yd.
(19) 26. (20) £6. 16*s.* $1\frac{8}{9}$*d.* (21) £7.
(22) £23. 9*s.* 4*d.* (23) 2 ft. (24) 300.

Ex. (cxxviii), p. 235.

(1) 336 cub. ft. (2) $548\frac{5}{8}$ cub. ft. (3) $83\frac{73}{96}$ cub. ft.
(4) $850\frac{185}{216}$ cub. ft. (5) $1058\frac{17}{432}$ cub. ft. (6) 9600.
(7) $12579\frac{183}{448}$ tons. (8) 500 men. (9) 1 ft. 7 in.
(10) $2031\frac{1}{4}$ lb. (11) 14 lb. (12) $5\frac{1}{2}$ ft.
(13) 160. (14) $3\frac{3}{4}$ ft. (15) £38. 19*s*. 2*d*.

ANSWERS TO EXAMINATION PAPERS,
p. 236.

(1) Twenty millions one hundred and three thousand and seven; 19001006.
(2) 220015. (3) 4845. (4) 6472352; 18518250.
(5) 326; 7846.
(6) 225003010; nine hundred and ninety-nine millions nine hundred and ninety thousand and ninety.
(7) 7929931. (8) 36645813; 2975850.
(9) 83983, rem. 288; 970052. (10) 35.
(11) 549079. (12) 8086, rem. 50.
(13) 19. (14) 68673151750. (15) 93007, rem. 193.
(16) 3463733. (17) 106. (18) 27; 4.
(19) £50627. 16*s*. 10*d*. (20) £43. 8*s*. $6\frac{1}{4}$*d*.
(21) 1174575. (22) 118941 farthings. (23) 69.
(24) 5544. (25) 82; 66. (26) $\frac{5}{9}$; $\frac{11}{32}$
(27) 1992648. (28) £2135. 6*s*. $1\frac{1}{2}$*d*. (29) 720.
(30) £3750000. (31) 71810685. (32) 1505.
(33) 23. (34) 2520. (35) 14198399.
(36) $\frac{3044}{4953}$; $\frac{1}{20}$
(37) 20718414756800; 262258, rem. 32756800.
(38) £40. (39) 17*s*. 8*d*. (40) 12476, rem. 34.
(41) £12. 18*s*. $6\frac{1}{2}$*d*. (42) $2\frac{1}{8}$ (43) 3260 farthings.
(44) £15. 6*s*. $4\frac{3}{4}$*d*. (45) 23477724. (46) 113.
(47) $\frac{7}{9}$ (48) $16\frac{13}{70}$ (49) £33642. 16*s*. $9\frac{1}{4}$*d*.; £27. 9*s*. $3\frac{1}{4}$*d*.
(50) 245 tons 8 cwt. 12 lb. 14 oz. (51) 362880; 3456.

(52) £12. 10*s*. (53) 31; $\frac{11}{13}$; $\frac{24}{29}$; $\frac{7}{9}$
(54) 111·4806; 292·6. (55) $6\frac{1}{2}$*d*.
(56) 14272 oz.; 217 tons 15 cwt. 2 qr. 27 lb. 3 oz.
(57) £11298. 13*s*. $1\frac{1}{2}$*d*. (58) 24 times.
(59) $10\frac{37}{40}$ (60) ·021.
(61) 107020060; twenty-one millions six hundred and thirty-seven thousand and thirty-seven.
(62) £374. 10*s*. $11\frac{3}{4}$*d*. (63) 9·67995; ·0735.
(64) £10. 15*s*. 3*d*. (65) 30 inches.
(66) £100. 2*s*. 6*d*.; £1. 2*s*. 6*d*.
(67) 1 cwt. 1 qr. 1 lb. 1 oz.
(68) 6 of each. (69) 2. (70) 6000; ·0006.
(71) 22400000 lb.; 4 tons 9 cwt. 1 qr. 4 lb.
(72) 619 ac. 2 ro. 6 po.
(73) £78155. 3*s*. $2\frac{1}{2}$*d*.; £39077. 11*s*. $7\frac{1}{4}$*d*. (74) £4.
(75) $\frac{25}{144}$ (76) 37 ac. 2 ro. 31 po. $2\frac{3}{4}$ yd. 3 ft. 30 in.
(77) ·02; 2000; ·000002; 2000·020002; $\frac{1000010001}{500000}$.
(78) £552. 1*s*. 3*d*. (79) 7899 mi. 1 fur. 25 po. 3 ft. 6 in.
(80) 15 cwt. 1 qr. $3\frac{7}{72}$ lb. (81) 38; 2736.
(82) 320009009; ninety-nine-thousand-and-two *hundred-millionths*.
(83) 45 miles. (84) 20; 2000; $\cdot 000\dot{2}$.
(85) $1\frac{37}{200}$; $\frac{13}{200}$; $\frac{7}{20}$ (86) 153. (87) $\cdot 4\dot{3}\dot{2}$.
(88) $21\frac{42}{61}$ times. (89) £125. 5*s*.
(90) 1081; $999\frac{5}{12}$ (91) 40300; 14·79816.
(92) $\frac{3}{400}$; $\frac{3}{22}$; $\frac{1}{111}$ (93) £48. 8*s*.
(94) 9405 steps. (95) 512896 dr.
(96) £4. 3*s*. 9*d*.; £3. 2*s*. $9\frac{3}{4}$*d*. (97) $\frac{55}{2898}$; ·0189.
(98) 3365·67; 3290. (99) £78. 16*s*. (100) 14
(101) 2064260 dr. (102) £1129.
(103) 128; 2304. (104) 12345. (105) £3. 5*s*.
(106) ·04976767; 180·712. (107) £233. 7*s*. 2*d*.
(108) 17695260 in.; $\frac{331}{352}$ (109) $7702\frac{1}{5}$ in.
(110) 168·1. (111) 162 dollars.
(112) £2. 14*s*. 5*d*.; 11*d*. (113) $\frac{16}{21}$; $1\frac{29}{56}$ (114) $\frac{1}{2}$

(115) 23046. (116) £291. 17*s*. $8\frac{19}{28}$*d*. (117) 5 yd.

(118) ·54 ; ·000054. (119) £3089. 15*s*. $8\frac{1}{8}$*d*.

(120) 19712014 ; nineteen millions seven hundred and twelve thousand and fourteen.

(121) 143360000000. (122) 140·9540 ; ·025 ; 25.

(123) 12 cwt. 2 qr. 14 lb. $10\frac{2}{3}$ oz. ; 1·71875.

(124) £25. 11*s*. $10\frac{1}{2}$*d*. (125) 86·42. (126) £94. 10*s*.

(127) 247131012 ; one billion two hundred and forty-seven thousand six hundred and eighty-one millions three hundred and fifty-four thousand two hundred and eleven.

(128) £52. 0*s*. $9\frac{21}{106}$*d*. (129) 1.

(130) ·02496 ; 476 ; £5. 12*s*. 4·05*d*. (131) £1. 2*s*. 9*d*.

(132) £1000. (133) 108. (134) $\frac{3}{4}$; ·75.

(135) 29400000. (136) 16*s*. $0\frac{1}{2}$*d*. (137) £37. 1*s*. $4\frac{1}{14}$*d*.

(138) £721. 4*s*. (139) ·65 ; 13*s*. (140) 1 ; $\frac{38}{49}$

(141) $\frac{9601}{400}$; $\frac{13}{16000}$; ·21 ; 2100. (142) 19*s*. $7\frac{1}{2}$*d*. ; $\cdot 87\dot{2}$

(143) £142. 12*s*. 6*d*. ; £42. 15*s*. 9*d*. ; £14. 5*s*. 3*d*.

(144) £2. 13*s*. $9\frac{23}{40}$*d*. (145) 12·96 ; $\frac{23}{49}$ (146) $\cdot 1\dot{6}$.

(147) $\frac{197}{1687}$; twice. (148) £49. 10*s*. ; £49. 10*s*.

(149) £54. 13*s*. 4*d*. (150) 5 h. 48 min.

(151) £1632 ; £81. 12*s*. (152) ·778125 ; $\cdot 30\dot{5}$.

(153) $\frac{115}{162}$ (154) 13*s*. $10\frac{1}{2}$*d*. ; $\frac{37}{84}$

(155) 12*s*. $3\frac{1}{2}$*d*. less. (156) 981·5988 fr.

(157) £4. 17*s*. 3·98625*d*. (158) £228. 5*s*. 8·4*d*.

(159) £5. 4*s*. $9\frac{3}{421}$*d*. (160) 6 ; 6.

(161) 82 ; 820 ; 1053. (162) 21 yd. 2 ft. $2\frac{5}{6}$ in.

(163) $\frac{22}{125}$; ·176 ; $\frac{1}{96}$ (164) 371 ; ·6324.

(165) As 1875 is to 1951. (166) £21. 5*s*. $8\frac{92}{101}$*d*.

(167) £21. 10*s*. (168) £3. 5*s*. $11\frac{1}{4}$*d*. ; £2. 3*s*. $11\frac{1}{2}$*d*.

(169) £3. (170) £3. 4*s*. 9*d*. ; $\frac{21}{47}$

(171) 11*s*. $4\frac{3}{4}$*d*. less. (172) 121·98.

(173) £723. 19*s*. 2*d*. (174) 7*s*. less. (175) 3 ft.

(176) $24\frac{6}{7}$; 16. (177) $\frac{1}{2}$ (178) $13\frac{179}{192}$ per cent. gain.

(179) £1. 6*s*. $8\frac{14}{17}$*d*. (180) 4 per cents. ; £12480.

(181) 1341 ; 1·581 .... (182) 55·3 ... ft. (183) $\cdot\dot{1}$.

(184) £$136\frac{58}{97}$; £$163\frac{89}{97}$; £$229\frac{47}{97}$ (185) $2\frac{11}{36}$

(186) $3\frac{1}{2}$ (187) £1. 9*s*. $1\frac{227}{1536}$*d*. (188) 52·425 sq. ft.

(189) $\frac{6667}{3630}$ (190) 10 h. $34\frac{7}{17}$ min.

(191) 0. (192) ·176875 ; $\frac{283}{1600}$ (193) £21. 7*s*.

(194) 218 mi. 4 fur. 24·8 po. ; £3. 12*s*. 1*d*. (195) $7\frac{9}{13}$ kts.

(196) 36. (197) $75\frac{1}{6}$ (198) 1. (199) £4200.

(200) 16*s*. 6*d*. (201) 8*s*. 2*d*. (202) $2691\frac{13}{17}$ yd.

(203) 270 ft. (204) ·0000289 ... (205) $\frac{829}{44800}$

(206) ·007257 ; 1·4514. (207) 256.

(208) £296. 15*s*. $11\frac{5}{8}$*d*. (209) £$53\frac{63}{509}$

(210) 200 ; 189 ; 101. (211) £6825.

(212) 25·2664 ; $\frac{31583}{1250}$

(213) (*a*) 20·1 ; 20100. (*β*) ·625. (*γ*) ·74375 ; ·0074375.

(214) 90 miles. (215) (*a*) 13*s*. 4*d*. (*β*) 4.

(216) £5. 15*s*. $0\frac{1}{4}$*d*. (217) £4732. 1*s*. 6*d*. ; £303. 11*s*. $5\frac{5}{7}$*d*.

(218) $\frac{2}{3}$; $\cdot\dot{6}$. (219) $\frac{7}{495}$; $\frac{1049}{9990}$; 13·569.

(220) £735 ; £88. 14*s*. 6*d*. (221) £2. 6*s*. $3\frac{5}{24}$*d*.

(222) £315. 10*s*. 8*d*. (223) £8282. 3*s*. $1\frac{1}{2}$*d*.

(224) £8. 12*s*. 9*d*. (225) 188 tons 13 cwt. 2 qr. 19 lb. 14 oz.

(226) £4567. 17*s*. $10\frac{1}{2}$*d*. (227) 1*s*. $3\frac{3}{4}$*d*.

(228) 130 yd. (229) 1. (230) ·000321 ...

(231) £955. 8*s*. (232) 36 days.

(233) 360138569 ; three hundred and sixty millions one hundred and thirty-eight thousand five hundred and sixty-nine.

(234) $\frac{144}{625}$ (235) 8*s*. 4*d*. ; $\cdot41\dot{6}$.

(236) ·045796 ; 16200 ; 11*s*. $10\frac{1}{5}$*d*. (237) £269. 19*s*. $8\frac{1}{4}$*d*.

(238) $2\frac{2}{35}$ hr. (239) £32. 10*s*. (240) £9. 7*s*. 6*d*.

(241) 7. (242) ·1225 ; ·009 ; 1 lb. 8·9 oz.

(243) £3733. 6*s*. 8*d*. (244) 4 ft. (245) 5*s*. 1*d*.

(246) £2. 3*s*. 9*d*. (247) £14. 0*s*. 6*d*.

(248) 26 sec. loss. (249) $\frac{2}{3}$ (250) $\frac{1}{4}$

(251) £1. 5*s*. (252) 7*s*. $10\frac{1}{2}$*d*. (253) 1·007 ; $13\frac{1}{4}$

(254) £1. 10*s*. (255) 1 day. (256) 3 hr.

(257) £103. 6*s.* 2*d.* (258) 33·6 in.
(259) £1163. 12*s.* $8\frac{8}{11}$*d.* (260) £567. 2*s.* $2\frac{1}{4}$*d.*
(261) ·02479538.... (262) $\frac{18}{419375}$ in. (263) $7\frac{7}{20}$
(264) $\frac{1}{1120}$ (265) ·41$\dot{6}$; 4·991 d.
(266) *A* takes £266. 13*s.* 4*d.* ; *B* £333. 6*s.* 8*d.* ; *C* £400.
(267) 50 ft. (268) 1 hr. (269) $16\frac{1}{2}$ ft.
(270) $2\frac{4}{5}$ (271) £494. 9*s.* (272) £5. (273) ·4$\dot{8}$.
(274) $3\frac{1}{2}$ (275) £1. 3*s.* $10\frac{1}{5}$*d.* more. (276) 8 hr.
(277) 11 sq. ft. (278) $1\frac{35}{76}$ (279) ·5375 ; $\frac{5}{12}$
(280) £8. 14*s.* $4\frac{1}{5}$*d.* (281) £17. 6*s.* 8*d.*
(282) $\frac{43}{90}$; $\frac{34213}{49950}$; 1 qr. $18\frac{302}{999}$ lb.
(283) tin $37\frac{1}{2}$ lb. ; copper $112\frac{1}{2}$ lb. (284) 95 yd.
(285) $1\frac{183}{670}$ (286) 9999 ; 1111000.
(287) £67. 0*s.* $7\frac{1}{2}$*d.* (288) 5·$\dot{3}$; $\frac{83}{144}$; 7·97$\dot{6}$*d.*
(289) £5. 8*s.* $3\frac{33}{112}$*d.* (290) £4. 1*s.* $6\frac{23}{27}$*d.* (291) 75.
(292) $\frac{28}{33}$ (293) ·0001. (294) $\frac{5}{12}$
(295) ·1875 ; 3*s.* $9\frac{27}{32}$*d.* (296) 340 cub. ft.
(297) £122. 9*s.* 3*d.* (298) 16 days.
(299) The eldest £9825 ; the second £4825 ; the third £3250.
(300) (1) 1. (2) 1. (301) 4*s.* 1*d.* ; $\frac{1079}{2240}$
(302) 5 ; 10000000 ; 11·1 (303) 3 cwt. 2 qr. ; 3·7996*d.*
(304) £1416. (305) (*a*) 200 sq. ft. (*β*) 96 cub. ft.
(306) $89\frac{81}{196}$; ·000365. (307) £13. 2*s.* 6*d.* ; £3. 15*s.*
(308) $2\frac{25}{29}$ (309) £1. 11*s.* $11\frac{49}{72}$*d.* (310) $107\frac{85}{193}$ days.
(311) $52\frac{43}{66}$ per cent. loss.
(312) £824. 6*s.* $5\frac{31}{37}$*d.* ; £329. 14*s.* $7\frac{5}{37}$*d.* ; £65. 18*s.* $11\frac{1}{37}$*d.*
(313) 12 hr. (314) 4 hr. 32 min.
(315) £1041. 13*s.* 4*d.* ; £41. 13*s.* 4*d.* (316) 4·225 lb. ; ·065.
(317) $\frac{6}{37}$ (318) £4. 9*s.* 7·2*d.*
(319) 22 lb. of nitre ; $4\frac{2}{3}$ lb. of charcoal ; $3\frac{1}{3}$ lb. of sulphur.
(320) £6998. 17*s.* $9\frac{3}{4}$*d.* (321) 5·9375*d.*
(322) 417 ; $11\frac{8}{13}$ (323) £23. 9*s.* 4*d.* (324) $3\frac{4}{15}$; $3\frac{19}{24}$
(325) $\frac{27}{80}$; $84\frac{1}{2}$*d.* (326) £2130 ; £58. 1*s.* 8*d.*
(328) £66. 13*s.* 4*d.* (329) The $4\frac{3}{4}$ per cents. ; £12480.

(330) 2312; 1·264...
(331) (*a*) £3545. 1*s*. 2*d*. (*β*) £450. 10*s*. $1\frac{1}{10}$*d*.
(332) (1) $\frac{2}{3}$ (2) 6·33403. (333) 45 ft.
(334) 2*s*. 6*d*.; 75 per cent. (335) 300·51; 2·74.
(336) 2250. (337) $38\frac{2}{11}$ min. past 7. (338) Bank Stock.
(339) 1300005001; five thousand and thirty millions ninety thousand five hundred and one.
(340) £233. 4*s*. 6*d*.; 6·775*d*. (341) £9000.
(342) 1·2535... lb. (343) £13. 18*s*. $5\frac{1}{4}$*d*.; £4. 5*s*. $1\frac{1}{4}$*d*.
(344) 8 days. (345) $93\frac{1}{3}$ days. (346) $3\frac{1}{2}$
(347) £37. 0*s*. 8*d*. (348) 16850890551050.
(349) 31116. (350) 1127; 60060.
(351) £1; $\cdot\dot{7}4\dot{0}$. (352) 529.
(353) $1\frac{3}{32}$ min. to 12. (354) 4 and 5 per cent.
(355) 4 per cent. (356) 1360.
(357) £141. 13*s*. 4*d*. (358) 1.
(359) £46. 4*s*. $10\frac{1}{2}$*d*. (360) 10·1 and 2·5 approximately.
(361) £201. 5*s*. (363) 900.
(364) £410.; £800. (365) £380. 2*s*.
(366) $9\frac{29}{30}$ (367) 42238274625; 1959 and rem. 39.
(368) £17516. 19*s*. $11\frac{1}{3}$*d*.; £26508. 5 f. 5 c. 2 m.
(369) 19*s*. $0\frac{25}{216}$*d*.
(370) *A* in $6\frac{22}{27}$ days; *B* in $9\frac{13}{19}$ days; *C* in $14\frac{2}{13}$ days.
(371) 90 miles. (372) 10*s*. $1\frac{241}{254}$*d*.
(373) She loses 4 per cent. (374) 583.
(375) 9374 tons 1 cwt. 2 qr. 17 lb.
(376) 265200. (377) £23398. 7*s*. 6*d*.
(378) 45 days from the commencement.
(379) 19*s*. $9\frac{63}{101}$*d*.; $6\frac{2}{33}$ (380) 1.
(381) £19. 2*s*. 6*d*. and £12. 15*s*. (382) £45.
(383) ·3937 in. (384) $37\cdot\dot{2}74883\dot{9}$ and ·0625.
(385) £135. 19*s*. 2*d*. (386) £214. (387) 1*s*. $1\frac{8}{9}$*d*.
(388) £4520. (389) 877.
(391) Greatest 35 : 49; least 17 : 24. (392) 6*s*. $2\frac{1}{10}$*d*.
(393) $3\cdot4\dot{2}$ and $342\cdot\dot{2}$. (394) $\cdot028\dot{3}$.

(395) £1016. 12*s*. $0\frac{48}{203}$*d*. (396) 14 minutes.

(397) 115 ft. 30 in. (398) 5 days.

(399) £1021. 0*s*. $6\frac{3}{50}$*d*. (400) 4 days.

(401) £61. 11*s*. $1\frac{1}{2}$*d*. (402) 547;6.

(403) 4*s*. 6*d*.; ·225. (404) 3·1404...

(405) 999·4... oz. (406) £491. 8*s*.

(407) On Tuesday evening, when one clock marks 9 h. 11 m. and the other 8 h. 54 m. 30 sec.

(408) One of the latter = two of the former.

(409) £10. 4*s*. (410) ·12. (411) 10*d*. $3\frac{8}{9}$ q.; 1*s*.

(412) ·000004; 250000; 8·25.

(413) $7\frac{1}{2}$; £50 less. (414) 65·108... yd.

(415) $\frac{1}{9}$ (416) 16. (417) 30 mi. and 25 mi. per hr.

(418) 140 yd.; £1. 6*s*. 3*d*. (419) £8250 each.

(420) 5427. (421) $12\frac{1}{8}$*d*. (422) $\cdot 00291\dot{6}$; 1*s*. 2*d*.

(423) £40. (424) *A* pays £10; *B*, £18; *C*, £15.

(425) 5*s*. 4*d*. (426) 30. (427) £$11111\frac{1}{9}$.

(428) 54 times.

(429) *B* walks a mile in $13\frac{3}{4}$ min.; he loses by $11\frac{1}{4}$ min., and by $\frac{45}{53}$ of a mile.

(430) $7\frac{1}{2}$ months. (431) 4497 times.

(432) £6. 15*s*.; £4. 10*s*.; £3. 7*s*. 6*d*.; £2. 5*s*.

(433) $\frac{70}{407}$ (434) ·076775, $\cdot 05829\dot{6}$, 135000, $\cdot 33\dot{8}5\dot{8}$.

(435) 4320 times; 45 miles. (436) 90; £465.

(437) 1200 gallons; 2 hours. (438) 90 quarters.

(439) $13\frac{11}{13}$ min. and $16\frac{4}{11}$ min. past 3.

(440) 16 tons 15 cwt. 3 qr. 0 lb. $2\frac{2}{3}$ oz. (441) 40.

(442) 4. (443) 38. (444) $4\frac{1}{21}$

(445) £12; £11. 19*s*. 11·712*d*.; $\frac{1}{10000}$

(446) £136. 9*s*. 2*d*.; $6\frac{1}{4}$ per cent. (447) 10.

(448) 46. (449) 2300·099. (450) $4\frac{1}{2}$

(451) 44 times. (452) 12.

(453) 1836·88305; ·385746. (454) $\cdot 7\dot{2}71825396\dot{}$; 10*s*.

(455) 84000. (456) £60000. (457) 4000 ft.

(458) 100 gallons. (459) 400 miles.

(460) The $3\frac{1}{4}$ per cents. (461) 420.
(462) £2. 15*s*. $10\frac{10}{21}$*d*. (463) 6·3 ; 2·50...
(464) $2762\frac{1}{2}$ cub. ft. (465) $87\frac{1}{2}$ (466) 3 times.
(467) £44. 13*s*. 3*d*. ; $\frac{1}{9}$ ft. (468) $3\frac{1}{2}$ days.
(469) £28. 11*s*. 6*d*. (470) 649. (471) $3\frac{3}{28}$
(472) ·02 ; 2000 ; ·000002 ; $\frac{1000010001}{500000}$ (473) £4. 6*s*. 8*d*.
(474) The Stock ; £25. 10*s*. ; £27. 13*s*.
(476) $6\frac{1}{4}$ per cent. (477) £39. 18*s*. ; £63. 12*s*.; £97. 10*s*.
(478) $28\frac{4}{7}$ days. (479) 19085159·25 tons.
(480) $111835\frac{3}{7}$ mètres. (481) 1*s*. 6*d*.
(483) 937 ; ·02268 of an inch. (484) 3 ft. $11\frac{1}{15}$ in.
(485) £259. 18*s*. $4\frac{100}{243}$*d*. (486) £11. 4*s*. 9*d*.
(487) $7\frac{1}{3}$ miles. (488) $8\frac{1}{3}$ min. (489) 113400.
(490) $10\frac{10}{11}$ m. and $43\frac{7}{11}$ m. past 5. (491) $938\frac{2}{3}$ mètres.
(492) $420\frac{60}{157}$ times. (493) £600. (494) $10\frac{82}{101}$*d*.
(495) $25\frac{9}{43}$ francs. (496) $16\frac{2}{3}$ miles. (497) £30000.
(499) $10\frac{10}{11}$ days.
(500) 24·9 francs = £1 ; 25·15 francs = £1.
(501) 2. (502) ·00000001 ; 10000 ; ·001.
(503) The former. (504) 45760.
(505) 9 of spirit to 31 of water.
(506) 11 days and 330 miles. (507) 10.
(508) The $3\frac{1}{4}$ per cents. ; £14550. 1*s*. (509) 7777 ; 729.
(510) 2·198 inches in a century. (511) 1.
(512) 2 and 4. (513) £12. 12*s*. $6\frac{1}{4}$*d*.
(514) 5 months hence. (515) 12*s*. 3*d*.
(516) $3721\frac{1}{35}$ lb. av. (517) 48 minutes. (518) 15 hr.
(519) 56 yd. (520) 10*s*. (521) 1.
(522) 103·67 ; 574. (523) 84 seconds.
(524) £$888\frac{8}{9}$; £14. 8*s*. $10\frac{2}{3}$*d*. increase.
(525) 1120 yd. ; $262\frac{50}{121}$ ac. (526) £286492500.
(527) 8 sovereigns, 12 half-crowns, 130 shillings.
(528) 6 hr. 30 min. P.M. (529) $4\frac{1}{2}$ tons.
(530) $\frac{1}{11}$ min. (531) 1540031966602440.

(532) 12*s.* 9*d.* (533) £32. 3 f. 5 c. 3·125 m.

(534) 316·2... yd. (535) $746\frac{2}{3}$ yd. (536) 6.

(537) £23. 3*s.* $7\frac{1}{8}$*d.* (538) $13\frac{29}{31}$ min.

(539) 1520 tons. (540) 1·0861578 nearly.

(541) 319 tons 1 qr. 12 lb. (542) £976. 13*s.*

(543) £1. 16*s.* (544) 2601. (545) £3200.

(546) £7320. (547) He loses 40 per cent.

(548) £4. 2*s.* $3\frac{1}{2}$*d.* (549) 15 masters and 345 boys.

(550) 1 o'clock. (551) ·07. (552) 3.

(553) $7908\frac{102}{121}$ miles. (554) $14 \cdot 58\dot{3}$*d.*

(556) £19. 16*s.* 8*d.* less. (557) $2\frac{1}{2}$

(558) *A* is to have 8*s.* 4*d.*; *B*, £1. 5*s.*; *C*, £2. 10*s.* *D*, £4. 3*s.* 4*d.*

(559) £47. 5*s.* (560) £12. 13*s.* 6*d.*; 7 yards.

(561) 505; 468750. (562) 10800.

(563) £5398. 0*s.* $1\frac{1}{2}$*d.* (564) £1. 14*s.* (565) £12. 10*s*

(566) 4 per cent. (567) 2*d.* (568) £2000.

(569) 6·144*d.* (570) £3750. (571) 75.

(572) ·00000503616. (573) £132. 1*s.* 8*d.* (574) £1. 5*s*

(575) Each child gets £1920. 12*s.*, and each brother £960. 6*s.*

(576) $117\frac{11}{17}$ ac. (577) $£254\frac{33}{103}$

(578) 33 lb. 7 oz. avoird. (579) $1\frac{9}{17}$ hr.

(580) 2 ft. 3 in. (581) $\frac{63}{100}$; $\frac{1}{256}$ (582) 27720.

(583) He gains $14\frac{6}{11}$ per cent. (584) 84.

(585) 499440. (586) $1\frac{1}{2}$*d.* (587) 2 : 7

(588) 50000000 quarters.

(589) 3 miles an hour; $1\frac{1}{2}$ miles an hour.

(590) £114. (591) 2. (592) ·1.

(593) $63\frac{5}{6}$ sq. ft.; £2. 2*s.* $6\frac{2}{3}$*d.* (594) 11 months.

(595) £2. 2*s.* $1\frac{5}{19}$*d.* (596) £187. 10*s.* (597) $4\frac{4}{5}$ days

(598) 20; 35; 17. (599) $37\frac{5}{3}$[illegible] gain per cent.

(600) 567. (601) ·00159; 730000; 16.

(602) 40707; 364. (603) 2176. (604) 6*s.* 8*d.*

(605) He gains £2. 0*s.* $10\frac{10}{11}$*d.* (606) 16.

(608) 1609·306 mètres. (609) £5. 17*s.*

(610) 792 yd. or 1572 yd. (612) £350. (613) 66 ft. 10 in.

(614) £417. 5*s.* 10*d.*; £2670. 13*s.* 4*d.*; £1001. 10*s.*

(615) £6400. (616) £9. 3*s.* 6*d.* (617) 1*s.* $0\frac{1}{2}$*d.*

(618) $2\frac{1}{2}$ and 5. (619) 3 pints. (620) 1·912.

(621) $\frac{7}{12}$; 910 gr. (622) £1. 16*s.* 6*d.* (623) 75.

(624) 4 per cent. (625) £2450.

(626) A half-quartern less. (627) 294000. (628) $22\frac{22}{49}$

(629) 56; 40; 24. (630) 474.

(631) £22668. 8*s.* 10*d.* (632) ·000038...

(633) £62. 6*s.* 8*d.* (634) £4. 17*s.* 6*d.*

(635) £475. (636) 6 dol. 16 skil. (637) 30 days.

(638) £1000. (639) 5*d.*

(640) 200 : 209; £109. 3*s.* 4*d.* (641) 2·175327.

(642) $77\frac{77}{87}$; $35353\frac{1}{23}$ (643) 90*s.* a quarter.

(644) The first is £10 more than the other.

(645) £3543. 15*s.* (646) 114.

(647) $7\frac{1}{5}$ hr.; 18 hr.; $5\frac{1}{7}$ hr. (648) £160. 9*s.* 4·553203125*d.*

(649) £2. 10*s.* (650) 124·001 and $\frac{8}{9}$ (651) £17. 1*s.* 3*d.*

(652) 3·6792; 168000. (653) 90. (654) 9000 men.

(655) 100 miles. (656) 30. (658) 10 hours.

(659) $8\frac{117}{736}$*s.* (660) 5 points.

(661) 180 and 300. (662) ·05.

(663) Length 32 ft.; breadth 16 ft.; height 8 ft.

(664) 6048. (665) £100. (666) 14*s.* 6*d.*; 15*s.* 6*d.*

(667) £376. (668) $\frac{1}{18}$ lb.

(669) A sheep, £2. 10*s.*; a pig, £1. 10*s.* (670) 48000.

(671) (a) £1. 17*s.* 6*d.*; (β) £2. 19*s.* 11*d.*

(672) £30. 14*s.* $11\frac{1}{8}$*d.* (673) *A* gets £6; *B*, £18; *C*, £45.

(674) $28\frac{41917}{89600}$ tons. (675) £6000. (676) 5*s.* $1\frac{1}{11}$*d.*

(677) 42000. (678) £26. 11*s.* $6\frac{3}{4}$*d.*

(680) £610260560. (681) 5*s.*

(682) 2, 2, 2, 2, 3, 3, 3, 5, 5, 7, 11; 118580.

(683) £589. 13*s.* 4*d.* (684) 33·75 dollars.

(685) £82. (686) $17\frac{1}{16}$ qr.

(687) Length, 27 ft.; breadth, 18 ft.; height, 12 ft.

(688) 1296. (689) £25. (690) 3*s*. 4*d*. and 5*s*. 4*d*.

(691) 715. (692) £56000; £48000; £42000.

(693) 160; 623. (694) 36 days. (695) $3\frac{9}{13}$ days.

(696) 82212. (697) $3\frac{2242}{9961}$ (698) $11\frac{1}{9}$ miles from P.

(699) Equal. (700) $101\frac{127}{448}$ tons. (701) 3*s*.

(702) £245. 16*s*. 8*d*. (703) $4\frac{1}{2}$ per cent.

(704) 2, 3, 5, 7; 2, 3, 7, 7; 2, 2, 2, 2, 3, 7; 2, 3, 6, 7, 14, 21, 42; 11760.

(705) £1. 11*s*. $4\frac{1}{5}$*d*. (706) $5\frac{1}{2}$ years.

(707) *A*, £20; *B*, £40. (708) 7 o'clock

(709) $4\frac{1}{3}$ days. (710) 6800 : 7221. (711) £87. 8*s*. $2\frac{133}{160}$*d*.

(712) $9\frac{60}{77}$ min. (713) The 3 per cents. (714) £725.

(715) 70. (716) £30. (717) 44 times.

(718) $4\frac{1}{2}$ per cent. (719) 15. (720) £50. 15*s*. less.

(721) 18*s*. $4\frac{1}{2}$*d*.; 1·75. (722) 24 days.

(723) £1180. (724) He increases it. (725) 2 boys.

(726) $1\frac{7}{11}$ miles per hour. (727) 15*s*. and 3*s*. 4*d*.

(728) £1176. (729) £110; 150 per cent.

(730) 3·0365. (731) 39989366616. (732) 9.

(733) £10. (734) $43\frac{1}{3}$ (735) £10. (736) $77\frac{1}{2}$

(737) £2376. 5*s*. (738) $301\frac{1}{3}$; $165\frac{65}{339}$ (739) *B*.

(740) 30 gall. (741) 6 ac. 3 ro. $17\frac{3}{5}$ po. (742) $\frac{3}{2}$

(743) 13*s*. $9\frac{2}{3}$*d*. (744) $2\frac{1}{2}$ hr.

(745) Loses 5 per cent. (746) $1\frac{1}{6}$ mile.

(747) £20. 6*s*. 3*d*. (748) £2577. 12*s*. 1*d*.

(749) £10. 13*s*. 4*d*.; £11. 13*s*. 4*d*. (750) 400 in.

(751) $7\frac{1}{2}$*d*. (752) $6\frac{3}{4}$ (753) £1. 1*s*. $9\frac{1}{3}$*d*.

(754) $3\frac{1}{3}$ yr. (755) £9150; £5820. (756) 1*s*. per lb.

(757) Loses £1. 4*s*. 3·84*d*.

(758) £553. 6*s*. 8*d*.; 5 per cent.

(759) £950; £27. 14*s*. 2*d*.

(760) Between ·0001 and ·0002.

(761) $\frac{11}{504}$; $\cdot 023\dot{8}2539\dot{6}$. (762) $\frac{1}{56448}$ of an inch.

(763) £3200; £4800; £6000; £7000.

(764) £280; 4 per cent. (765) 663840 : 703273.

(766) C; $\frac{1760}{3741}$ yd. (767) 25 yr. (768) 6.

(769) £1. 14*s*. $8\frac{2}{3}$*d*.; £1. 13*s*. 4*d*.; £1. 14*s*. $0\frac{1}{3}$*d*.

(770) $26\frac{9}{16}$ (772) 36. (773) £12800.

(774) 10 lb. (775) 4*s*. $9\frac{3}{5}$*d*. (776) 15*s*.

(777) £960. (778) $27\frac{20}{33}$ (779) $31\frac{1}{2}$ miles an hour.

(780) ·03007009. (781) £255. 4*s*. $8\frac{1}{4}$*d*.

(782) £$2474\frac{10273}{15168}$ (783) $12\frac{1}{2}$ per cent.

(784) 128; 96; 72; 54. (785) 8 per cent.

(786) $2\frac{1}{2}$ hr. (787) £2. 6*s*. 8*d*. (788) £1935.

(789) 750. (790) 5047.

(791) £9. 19*s*. $6\frac{103}{144}$*d*. (792) 12*s*. 6*d*.

(793) $15\frac{1}{5}$ cwt. of nitre; $1\frac{9}{10}$ cwt. of sulphur; $2\frac{9}{10}$ cwt. of charcoal.

(794) £240 less. (795) $4\frac{1}{2}$ miles an hour.

(796) 5*s*. $11\frac{3}{5}$*d*. per lb.

(797) Capital £1000000; receipts £100000. (798) $3\frac{1}{2}$.

(799) £16. 5*s*. (800) $41990\cdot52975347\dot{2}$ grains.

(801) 1053252. (803) £1. (804) £$65\frac{5}{9}$

(805) 1 : 23. (806) $13\frac{1}{3}$ days. (807) £2007. 10*s*.; $4\frac{1}{4}$ yr.

(808) 8 miles an hour. (809) 2275. (810) $1\frac{11}{13}$ days.

(811) (*a*) $36\frac{31}{39}$ (β) $10\frac{55}{112}$ (812) £1543. 6*s*. 8*d*.

(813) 41 inches. (814) £11. 12*s*. $9\frac{21}{43}$*d*. (815) 21000.

(816) 16000000. (817) £66. 13*s*. 4*d*. increase.

(818) $2\cdot\dot{8}52380\dot{9}$ hr. (819) £1. 14*s* $2\frac{1}{3}$*d*.

(820) £203. 16*s*. 2*d*.

(821) 3020; 3·02; ·302. (822) £960.

(823) £258. 18*s*. 8·8128*d*. (824) 2 ft. 9 in.

(825) 5 hr. $10\frac{10}{29}$ min. (826) 75 days.

(827) $28\frac{1}{2}$; $427\frac{1}{2}$ lb. (828) 32 days.

(829) 13 centres, 31 outers. (830) £5. 5*s*. 9*d*.

(831) £3001. 14*s*. $9\frac{1}{2}$*d*. (832) $7\frac{1}{2}$ yr. (833) $33\frac{1}{3}$ days.

(834) $112\frac{26}{27}$ (835) 160 yd. (836) £2420 per ann.

(837) The $4\frac{1}{4}$ per cents. (838) 799 sec.

(839) 8 per cent. (840) 576 ; 192. (841) 2940778.

(842) 70·41 ; 1·46. (843) 4 per cent.

(844) Loses £25 ; gains £81 income.

(845) £274. 2*s*. 6*d*.; £456. 5*s*. (846) Loses $56\frac{1}{4}$ per cent.

(847) 397 yd. per minute. (848) One-third.

(849) 2017 dol. 47 cents. (850) 28·475 kil.

(851) £5905. 14*s*. $11\frac{1}{16}$*d*. (852) 15 years.

(853) 279 ; $\frac{3}{8}$ (854) 7 and 1.

(855) £2695. 3*s*. 8*d*. (856) £168 ; 14*s*.

(857) 1*s*. $9\frac{1}{7}$*d*. (858) $\frac{1}{4}$ of a month.

(859) £80 and £133. 6*s*. 8*d*. ; he loses £13. 6*s*. 8*d*.

(860) £5100. (861) 275·41 . . .

(862) £216 ; £252 ; £234 ; £230. 8*s*.

(863) £12. 10*s*. ; £14. 5*s*. $8\frac{4}{7}$*d*. ; the former by £$2\frac{41}{42}$

(864) 32 : 35. (865) 4 hr. $19\frac{1}{11}$ min. (866) £59. 5*s*

(867) $4\frac{1}{2}$ (868) $7\frac{1}{12}$ miles per hr. (869) 1950.

(870) 15 : 9 : 5. (871) £388. 10*s*. 8*d*.

(872) £9550. 8*s*. (873) 12*s*. 6*d*. ; £2346. 12*s*. 6*d*.

(874) £7. 6*s*. $6\frac{3}{4}$*d*. (875) £2. 14*s*. $4\frac{1}{2}$*d*.

(876) $2\frac{2039}{2392}$ (877) 25 years. (878) $2\frac{5714}{15203}$ months.

(879) 4*s*. $7\frac{49}{86}$*d*. per lb. (880) £15. 15*s*.

(881) (*a*) $1\frac{2903}{2907}$ (*β*) $1\frac{8}{9}$ (882) 41·31 ; 1·26.

(883) £511·5404125 (884) £6. 17*s*. 6*d*. ; $45\frac{5}{6}$

(885) *B*, by 16 yd. (886) The 6 per cents. ; £234.

(887) 1 hr. 45 min. (888) $\frac{1}{4}$ (889) 30.

(890) $131\frac{5}{6}$ ft. (891) £224. 1*s*. 4*d*. (893) 23 inches.

(894) 218972·16 gall. (895) £11. 10*s*. $4\frac{4}{5}$*d*.

(896) 240 ; 720 ; 960. (897) $4\frac{3}{8}$ miles.

(898) 3 days. (899) 1 to 2.

(900) The 6 per cents. (901) $\frac{1}{27}$; $6\frac{26}{27}$; 7.

(902) £596. (903) The former ; £$19\frac{46403}{57663}$

(904) 14 min. $43\frac{3}{4}$ sec. (905) 9 days.

(907) $1166\frac{2}{3}$; 1169; 1000; 1002. (908) 105 days.

(909) £26. 7*s*. $7\frac{5}{8}$*d*. (910) Nothing. (911) £62. 5*s*.

(912) $57\frac{3}{5}$ hr. (913) 11 : 2. (914) 48 centres, 31 outers.

(915) 8*s*. $2\frac{1}{3}$*d*. (916) ·083333. (917) 2*s*. 6*d*.

(918) 4 min. 30 sec. P.M. (919) $89\frac{7}{8}$; $3\frac{1}{8}$

(920) 2 miles. (921) 40·6; 4·06; 406000000.

(922) 36·880165... (923) £573. 19*s*. 2*d*.

(924) £15000; £5000. (925) 7. (926) 5 hr.

(927) *A*, 3240; *B*, 2916; *D*, 2052; *C*, 1944; *E*, 1728; in all, 6480.

(928) 1 hr. $10\frac{20}{89}$ min. (929) £40.

(930) 5 miles. (931) £13. 19*s*. 6*d*.

(932) £234; £266. 8*s*.; £306; £345. 12*s*. (933) $66\frac{1}{8}$

(934) $3\frac{1}{2}$ hr. (935) 5*s*. 10*d*. (936) 7 ft.

(937) £64. (938) £275. (939) £57. 2*s*. $6\frac{15}{16}$*d*.

(940) $\frac{6}{13}$ of a mile. (941) 8439; 8439000; 8·439.

(942) 1·817... (943) 2 ro. 16·25 po.

(944) 45 and 35. (945) 3 : 2. (946) £611. 3*s*. 4*d*.

(947) £348. (948) $2\frac{1}{5}$ ft. (949) 3 hr. 35 min. P.M.

(950) 1 h. $28\frac{4}{11}$ min. (951) £386. 11*s*. $11\frac{1}{4}$*d*.

(952) $\frac{7}{8}$; $\frac{1}{8}$; 1. (953) 767232 gall. (954) £8. 1*s*. 8*d*.

(955) 54. (956) 5. (957) 2740 yd.; $13\frac{83}{99}$

(958) at 3 P.M. on Dec. 3. (959) in $12\frac{1}{2}$ min.

(960) £480. (961) 101·310...

(962) 170300; 170·3; 1703. (963) £40.

(964) $5\frac{3}{5}$ ft. in length and breadth. (965) 27 days.

(966) 90. (967) 35. (968) 39 yd.

(969) 18 min. (970) £1160. (971) $12\frac{122}{175}$

(972) 60, 30, 12. (973) 3·28035.

(975) $6\frac{1}{4}$; £574. 13*s*. (976) $15\frac{15}{29}$

(977) 2*s*. 4*d*. per stone. (978) $6\frac{15}{23}$ days.

(979) 27951 : 12500. (980) 6·439.

(981) $4\frac{23}{64}$ (982) £553. 11*s*. $8\frac{1}{4}$*d*.

(983) $\frac{3}{8}$ of northern hemisphere is land; $\frac{1}{8}$ of southern hemisphere is land.

(984) £121. (985) £4. 5*s*. 1*d*. (986) $19\frac{3}{13}$ yd.

(987) £200. (988) £1. 14*s*. 11*d*. (989) 4.

(990) 3*d*. a gallon. (991) £7. 10*s*. 2*d*. (992) 3960.

(993) 1·25. (994) £4. 4*s*.; £3; £1. 16*s*. (995) $33\frac{1}{3}$

(996) 860. (997) £3. 18*s*. (998) Gains £11. 5*s*.

(999) 375 grains of potash; 390 grains of soda.

(1000) £8000.

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